Dependability Analysis

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Main topics of the course

- Overview (1)
  - V&V techniques, Critical systems
- Static techniques (2)
  - Verifying specifications
  - Verifying source code
- Dynamic techniques: Testing (7)
  - Developer testing, Test design techniques
  - Testing process and levels, Test generation, Automation
- **System-level verification (3)**
  - Verifying architecture, **Dependability analysis**
  - Runtime verification

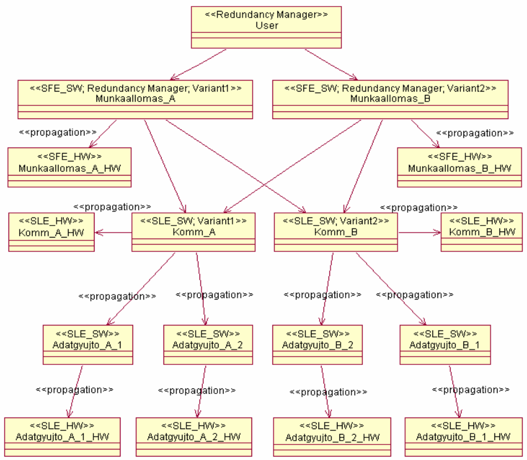
Table of Contents

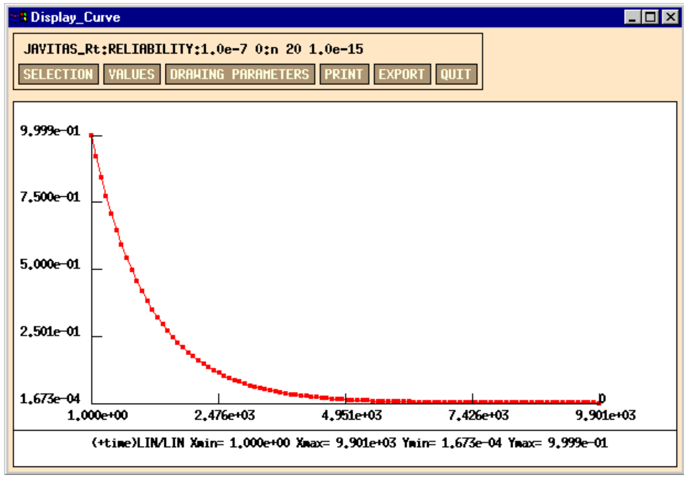
- Attributes of dependability
  - Reliability, availability
  - Safety, integrity, maintainability
- Combinatorial models for dependability analysis
  - Reliability block diagrams
- Stochastic models for dependability analysis
  - Markov models (CTMC)
  - Stochastic Petri-nets (SPN, GSPN)

Learning outcomes

- Explain the attributes of dependability and the objectives of dependability analysis (K2)
- Perform dependability analysis with reliability block diagrams (K3)
- Perform dependability analysis of simple redundancy structures with Markov chains (K3)
- Identify how stochastic Petri nets can be used for dependability analysis (K1)

Attributes of dependability





Characterizing the system services

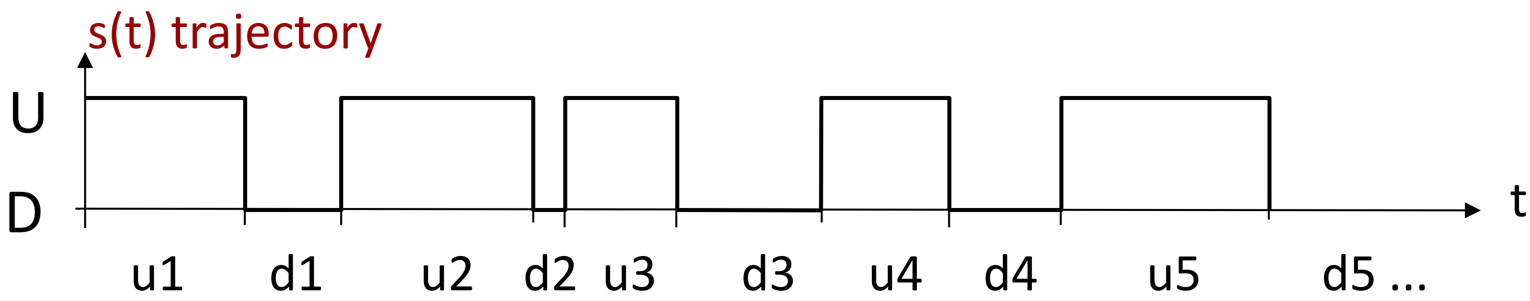
- Typical extra-functional characteristics
  - Reliability, availability, integrity, ...
  - Depend on the faults occurring during the use of the services
- Composite characteristic: Dependability
  - Definition: Ability to provide service in which reliance can justifiably be placed
    - Justifiably: based on analysis, evaluation, measurements
    - Reliance: the service satisfies the needs
- Role of dependability
  - Service Level Agreements (IT service providers)
  - Tolerable Hazard Rate (safety-critical systems)

Attributes of dependability

Attribute	Definition
Availability	Probability of correct service (considering repairs and maintenance) "Availability of the web service shall be 95%"
Reliability	Probability of continuous correct service (until the first failure) "After departure the flight control system shall function correctly for 12 hours"
Safety	Freedom from unacceptable risk of harm
Integrity	Avoidance of erroneous changes or alterations
Maintainability	Possibility of repairs and improvements

Dependability metrics: Mean values

- Basis: Partitioning the states of the system
  - Correct (U, up) and incorrect (D, down) state partitions



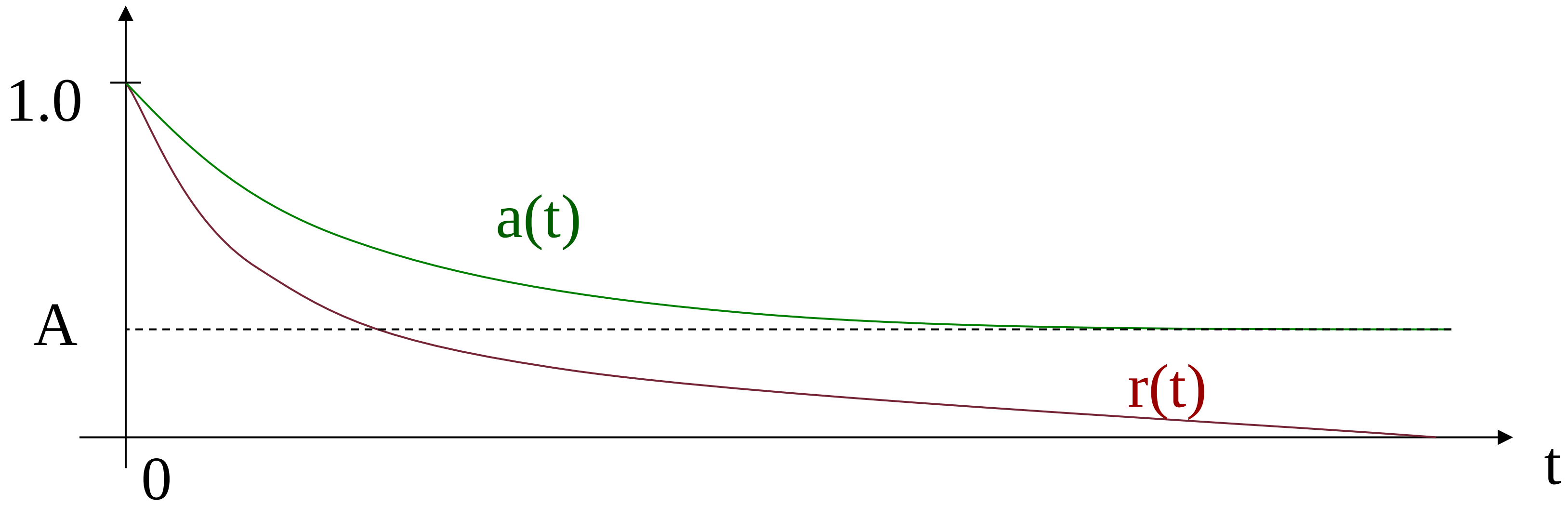
- Mean values:
  - Mean Time to First Failure: MTFF = E{u1}
  - Mean Up Time: MUT = MTTF = E{ui}
    (Mean Time To Failure)
  - Mean Down Time: MDT = MTTR = E{di}
    (Mean Time To Repair)
  - Mean Time Between Failures: MTBF = MUT + MDT

Dependability metrics: Probability functions

- Availability: $a(t) = P\{s(t) \in U\}$
- Asymptotic availability: $A = \lim_{t \to \infty} a(t)$

$$A = \frac{MTTF}{MTTF + MTTR}$$

- Reliability: $r(t) = P\{s(t') \in U, \forall t' < t\}$



Availability related requirements

Availability	Failure period per year
99%	~ 3,5 days
99,9%	~ 9 hours
99,99% („4 nines”)	~ 1 hour
99,999% („5 nines”)	~ 5 minutes
99,9999% („6 nines”)	~ 32 sec
99,99999%	~ 3 sec

Availability of a system built up from components, where
the availability of single a component is 95%,
all components are needed to perform the system function:

- system built from 2 components: 90%
- system built from 5 components : 77%
- system built from 10 components : 60%

Attributes of components

- Fault rate: $\lambda(t)$

  The probability that the component will fail in the interval Δt at time point t given that it has been correct until t is given by $\lambda(t)\Delta t$:

$$\lambda(t)\Delta t = P\left\{ s(t+\Delta t) \in D \mid s(t) \in U \right\} \text{ while } \Delta t \to 0$$

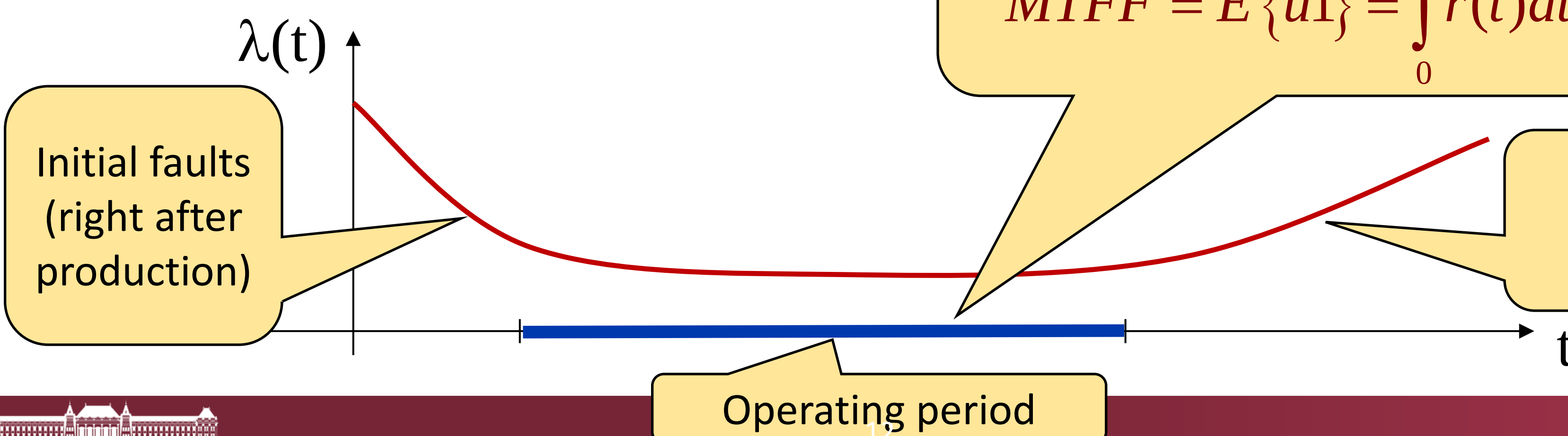
- Reliability of a component on the basis of this definition:

$$r(t) = e^{-\int_0^t \lambda(t)\,dt}$$

- For electronic components:

Here $r(t) = e^{-\lambda t}$

$$MTFF = E\{u1\} = \int_0^\infty r(t)\,dt = \frac{1}{\lambda}$$



Analysis techniques

- Qualitative analysis techniques:
  - Fault effects analysis: What are the component level failures (failure modes), that cause system level failure?
    - Identification of single points of failure
  - Techniques: Systematic causes and effects analysis
    - Fault tree analysis (FTA), Event tree analysis (ETA), Cause-consequence analysis (CCA), Failure modes and effects analysis (FMEA)
- Quantitative analysis techniques:
  - Dependability analysis: How can the system level dependability be calculated on the basis of component level fault properties?
    - System level reliability, availability, …
  - Techniques: Construction and solution of dependability models
    - Reliability block diagrams (RBD)
    - Markov-chains (MC)
    - Stochastic Petri nets (SPN)

Goals of the dependability analysis

- On the basis of component characteristics
  - fault rate (in continuous operation), measured by FIT: 1 FIT = 10^{-9} faults/hour
  - fault probability (in on-demand operation)
  - reliability function

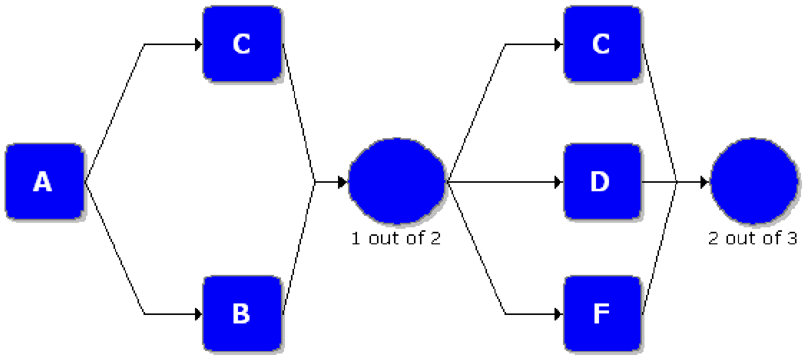
  calculation of system level characteristics
  - reliability function
  - availability function
  - asymptotic availability
  - MTFF
  - safety

Calculations are based on the system architecture and the failure modes

Using the results of the analysis

- Design: Comparison of alternative architectures
  - Having the same components, which architecture guarantees better dependability attributes?
- Design, maintenance: Sensitivity analysis
  - What are the effects of selecting another component?
  - Which components have to be changed in case of inappropriate attributes?
  - Which component characteristics have to be investigated in more detail? → Fault injection and measurements
- Handover: Justification of dependability attributes
  - Approval and startup of services
  - Certification (for safety critical systems)

Combinatorial models for dependability analysis



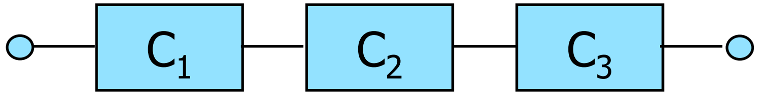
Boole models for calculating dependability

- Two states of components: Fault-free or faulty
- There are no dependences among the components
  - Neither from the point of view of fault occurrences
  - Nor from the point of view of repairs
- “Interconnection” of components from the point of view of dependability: What kind of redundancy is used?
  - Serial connection: The components are not redundant
    - If all components are necessary for the system operation
  - Parallel connection: The components are redundant
    - If the components may replace each other

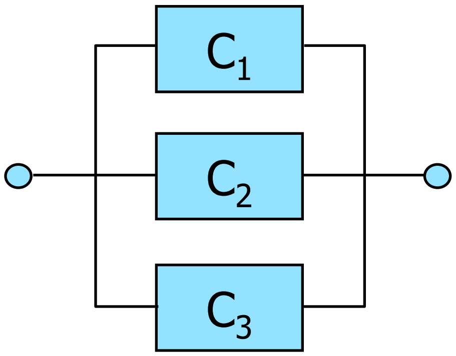
Reliability block diagram

- **Blocks:** Components (with failure modes)
- **Connection:** Serial or parallel connection
- **Paths:** System configurations
  - The system is operational (correct) if there is a path from the start point to the end point of the diagram through fault-free components

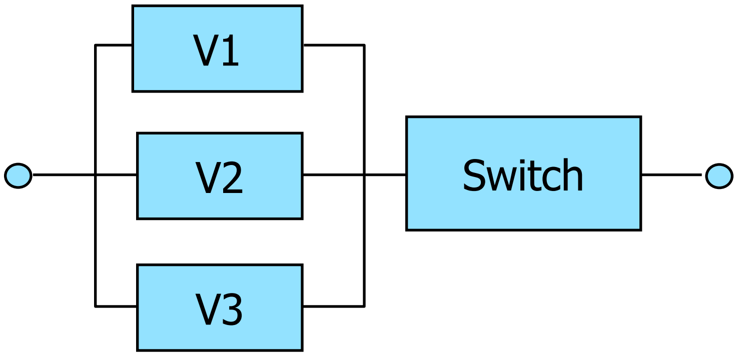
Serial:

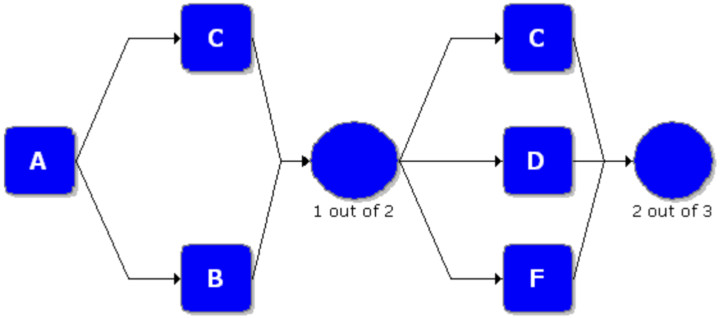


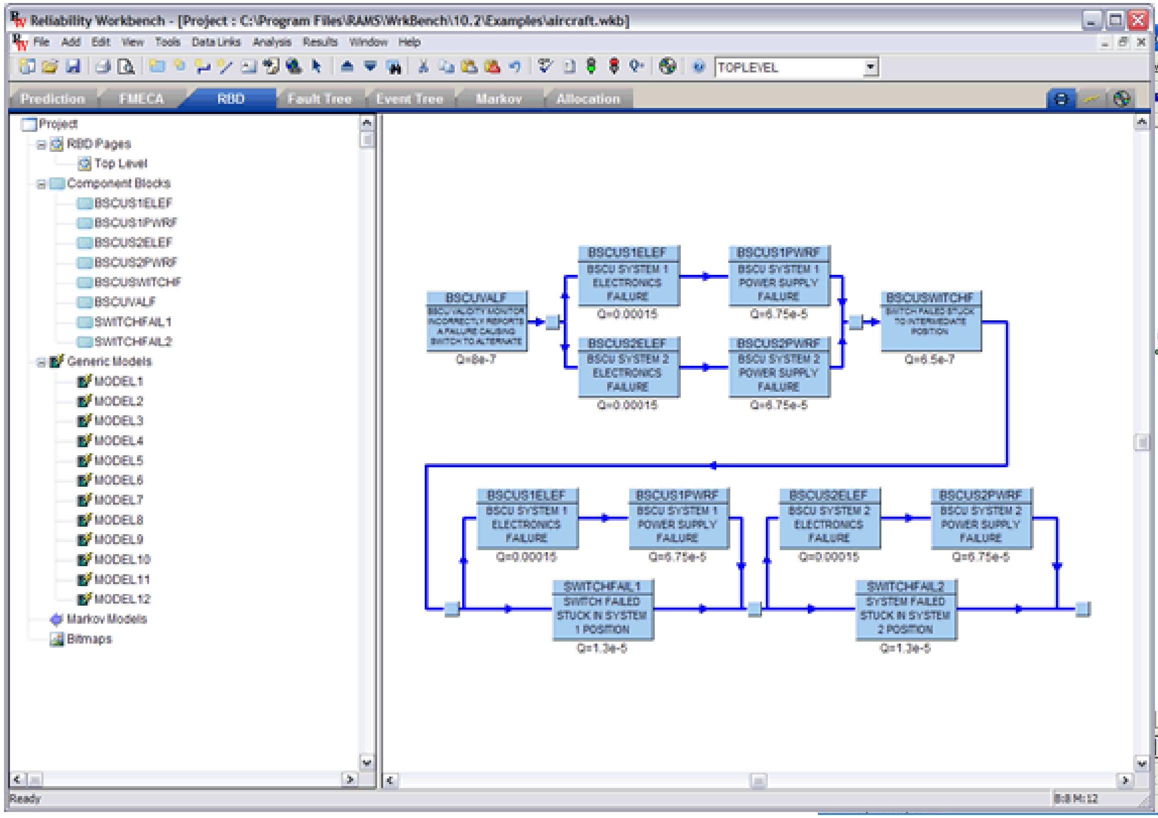
Parallel:

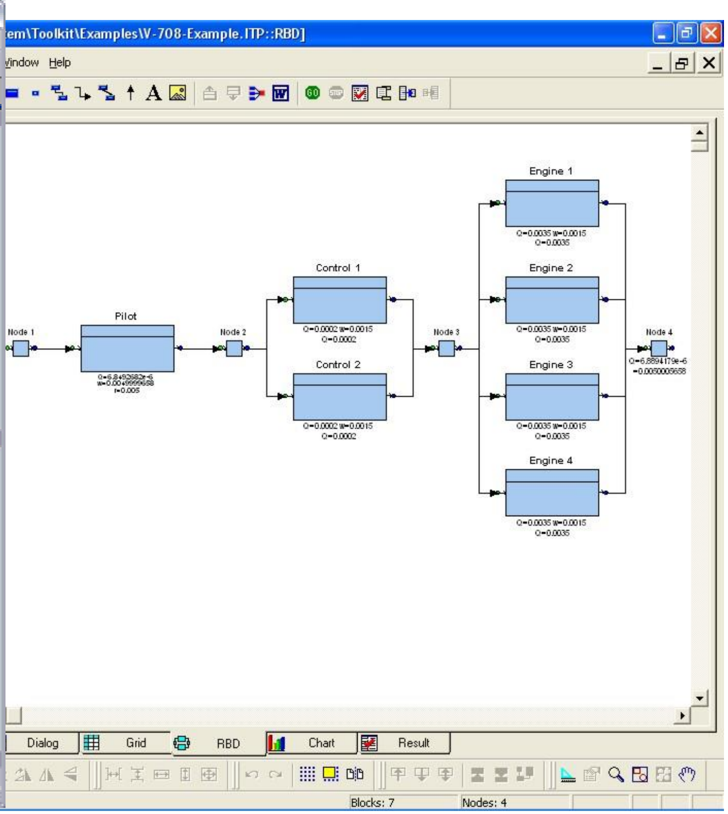


Reliability block diagram examples



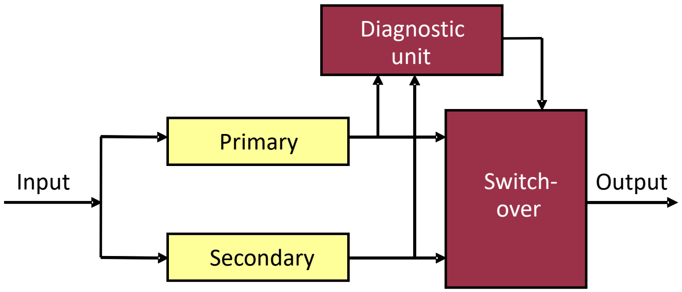


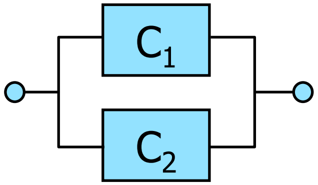




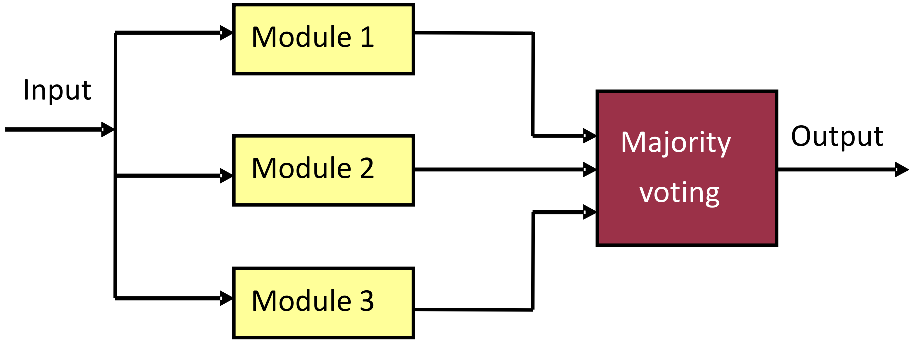
Overview: Typical system configurations

- Serial system model: No redundancy
- Parallel system model: Redundancy (replication)

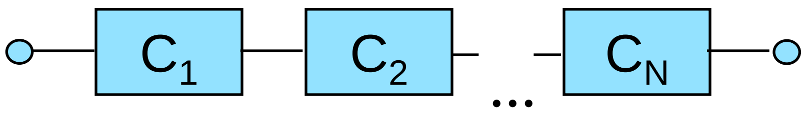


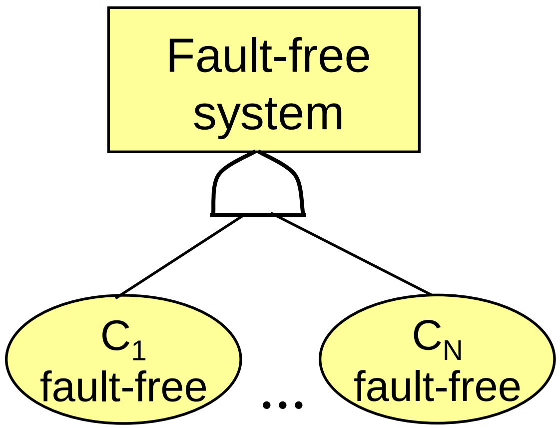


- Complex canonical system: Redundant subsystems
- M out of N components: Majority voting (TMR)



Serial system model





P(A∧B)=P(A)·P(B)
If independent

- Reliability
  for N components:

$$r_R(t) = \prod_{i=1}^{N} r_i(t)$$

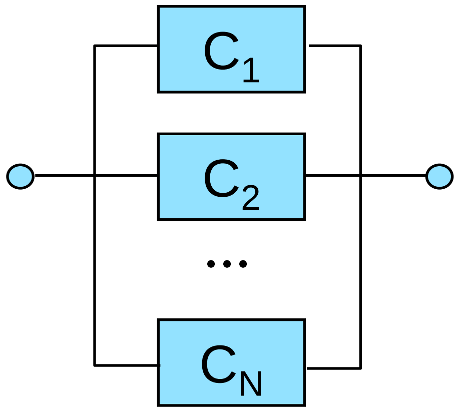
System reliability

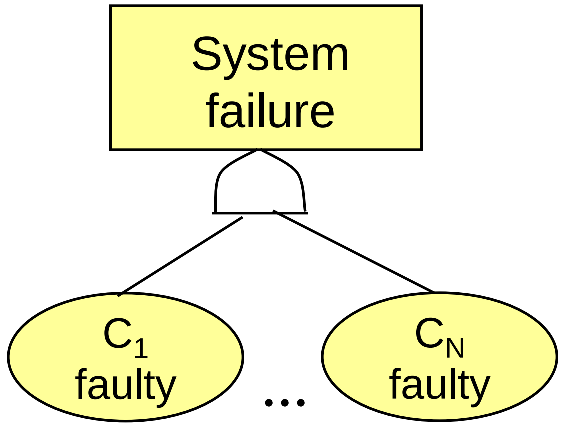
Components' reliability

- MTFF:

$$MTFF = \frac{1}{\sum_{i=1}^{N} \lambda_i}$$

Parallel system model





P(A∧B)=P(A)·P(B)
if independent

- Reliability:

$$1 - r_R(t) = \prod_{i=1}^{N} (1 - r_i(t))$$

- Identical N components:

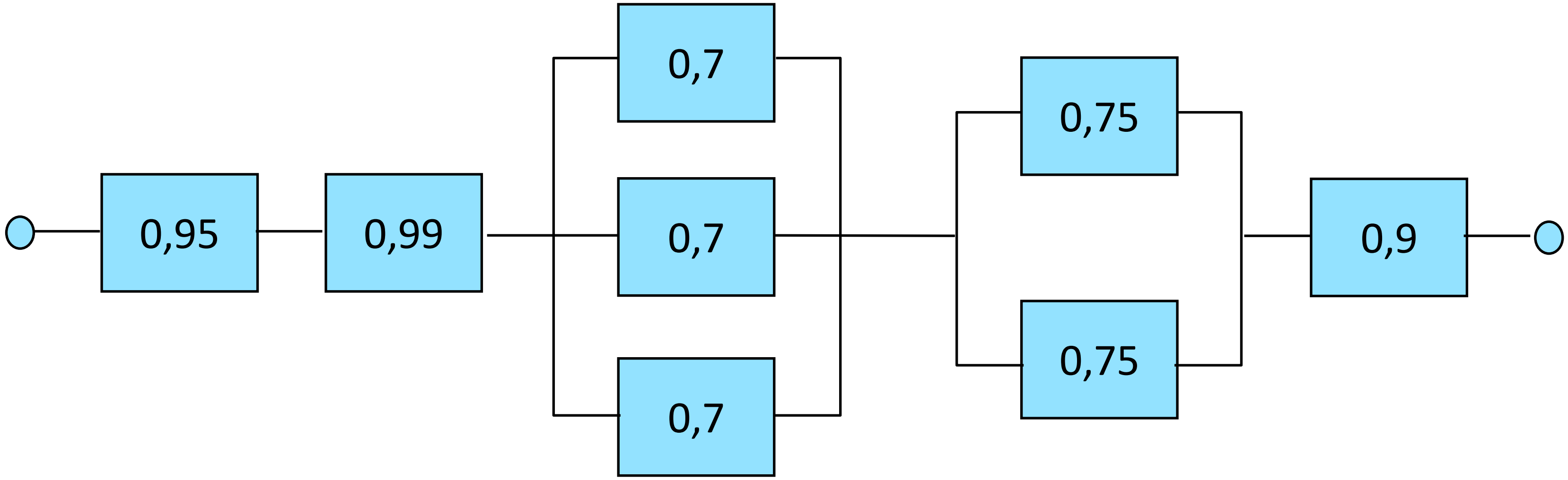
$$r_R(t) = 1 - (1 - r_C(t))^N$$

- MTFF:

$$MTFF = \frac{1}{\lambda} \sum_{i=1}^{N} \frac{1}{i}$$

Complex canonical system

- Calculation on the basis of parts with basic connections
  - Example: Calculation of asymptotic availability



$$K_R = 0,95 \cdot 0,99 \cdot \left[1-(1-0,7)^3\right] \cdot \left[1-(1-0,75)^2\right] \cdot 0,9$$

M faulty out of N components

- N replicated components;
  If M or more components are faulty: the system is faulty

$$r_R = \sum_{i=0}^{M-1} P\{\text{"there are i faulty components"}\}$$

$$r_R = \sum_{i=0}^{M-1} \binom{N}{i} (1-r)^i \cdot r^{N-i}$$

- Application: Majority voting (TMR): N=3, M=2

$$r_R = \sum_{i=0}^{1} \binom{3}{i} (1-r)^i \cdot r^{3-i} = \binom{3}{0} (1-r)^0 \cdot r^3 + \binom{3}{1} (1-r)^1 \cdot r^2 = 3r^2 - 2r^3$$

$$MTFF = \int_0^\infty r_R(t)dt = \int_0^\infty (3r^2 - 2r^3)dt = \frac{5}{6} \cdot \frac{1}{\lambda}$$

Less than in case of a single component!

Cold redundant system

- A new component is switched on to replace a faulty component:

$$MTFF = \sum_{i=1}^{N} MTFF_i$$

- In case of identical replicated components, the system reliability function:

$$r_R(t) = \sum_{i=0}^{N-1} \frac{(\lambda t)^i}{i!} e^{-\lambda t}$$

EXERCISE Reliability block diagram

A SCADA system consists of the following components:
4 data collector units, 3 control units, 2 supervisory servers, 1 logging server and the corresponding network

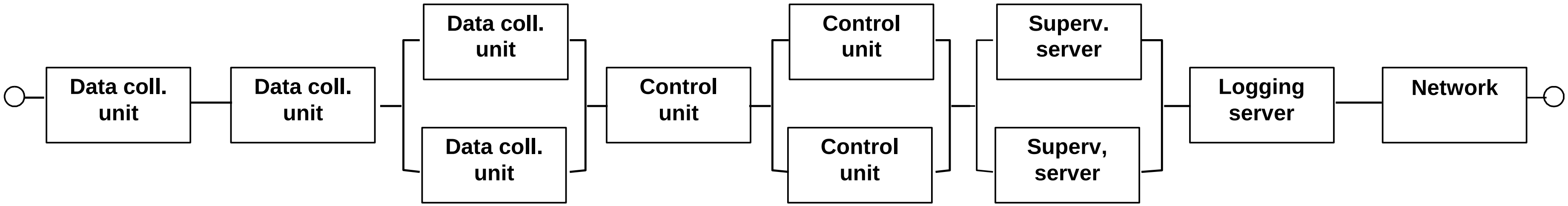
- The 2 supervisory servers are in a hot redundancy structure.
- 2 data collector units and 2 control units are hot redundant units
- The reliability data of the system components are given as follows (measured in hours, with independent repairs in case of faults):

	Data coll. unit	Control unit	Superv. server	Logging server	Network
MTTF	9000	12000	4500	2000	30000
MTTR	2	3	5	1	2

- Evaluate the system level availability using a reliability block diagram.
- Compute the asymptotic availability of the system using the above given parameters of the system components.
- In average, how many hours is the system out of service in a year?

EXERCISE Solution

Reliability block diagram:



Component level asymptotic availability: K = MTTF / (MTTF+MTTR)

	Data coll. unit (D)	Control unit (C)	Superv. server (S)	Logging server (L)	Network (N)
MTTF	9000	12000	4500	2000	30000
MTTR	2	3	5	1	2
K	KD=0.99977	KC=0.99975	KS=0.99889	KL=0.9995	KN=0.99993

System level asymptotic availability:

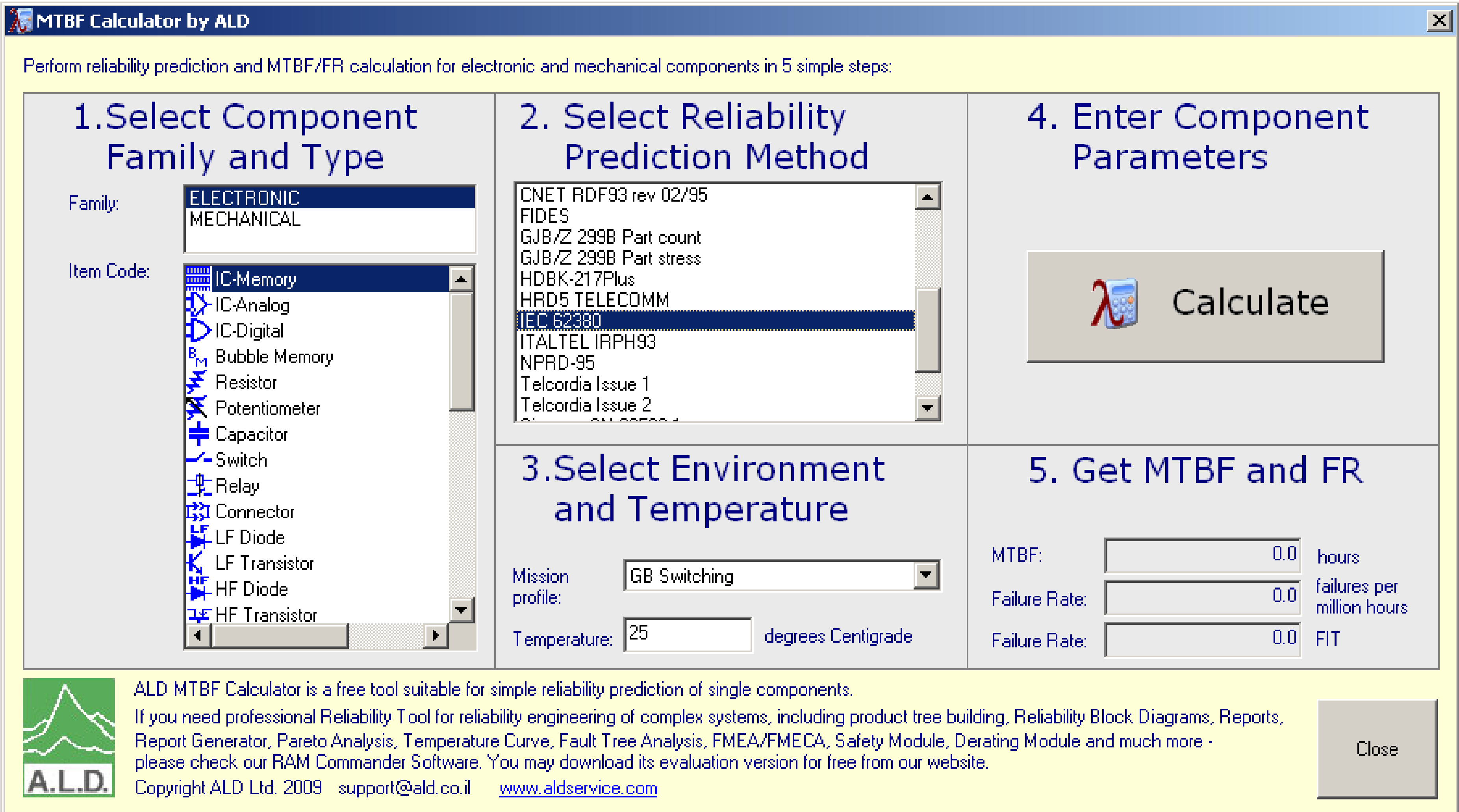
$$KD*KD*(1-(1-KD)^2) * KC*(1-(1-KC)^2) * (1-(1-KS)^2) * KL * KN = 0.9987362$$

Approx. 11 hours out of service per year

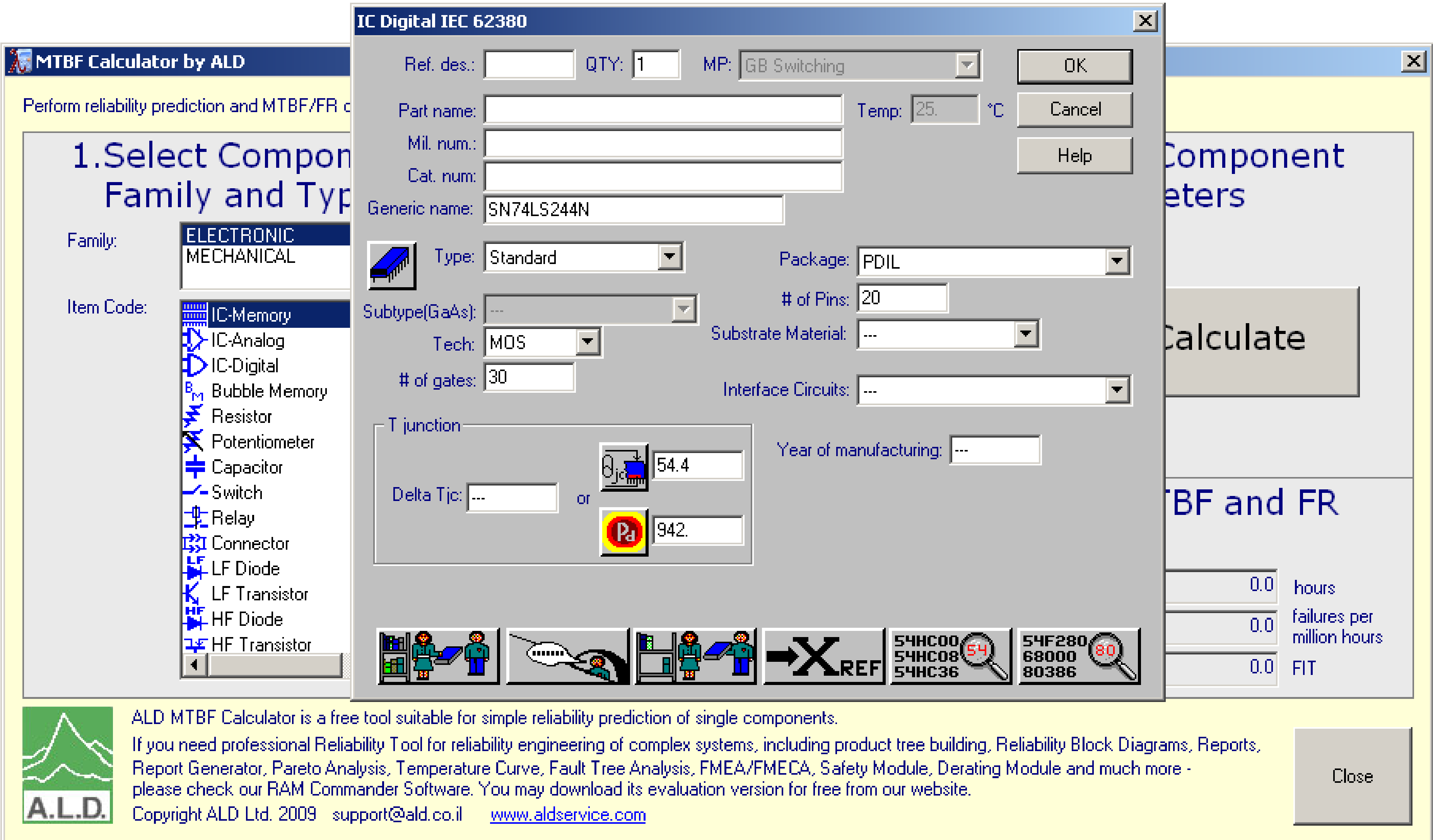
Component reliability data

- Component level reliability data are available in handbooks
  - MIL-HDBK-217: The Military Handbook Reliability Prediction of Electronic Equipment (for military applications. pessimistic)
  - Telcordia SR-332: Reliability Prediction Procedure for Electronic Equipment (for telco applications)
  - IEC TR 62380: Reliability Data Handbook - Universal Model for Reliability Prediction of Electronic Components, PCBs, and Equipment (less pessimistic, supporting new component types)
- Dependencies of component level reliability data:
  - Temperature, weather conditions, shocking (e.g., in vehicles), height, …
  - Operational profiles
    - Ground; stationary; weather protected (e.g., in rooms)
    - Ground; non stationary; moderate (e.g., in vehicles)
- Computations: hierarchic approach (with redundancy schemes)
  - Component –> Module –> Subsystem –> System

Tool example: The ALD MTBF Calculator



Tool example: The ALD MTBF Calculator

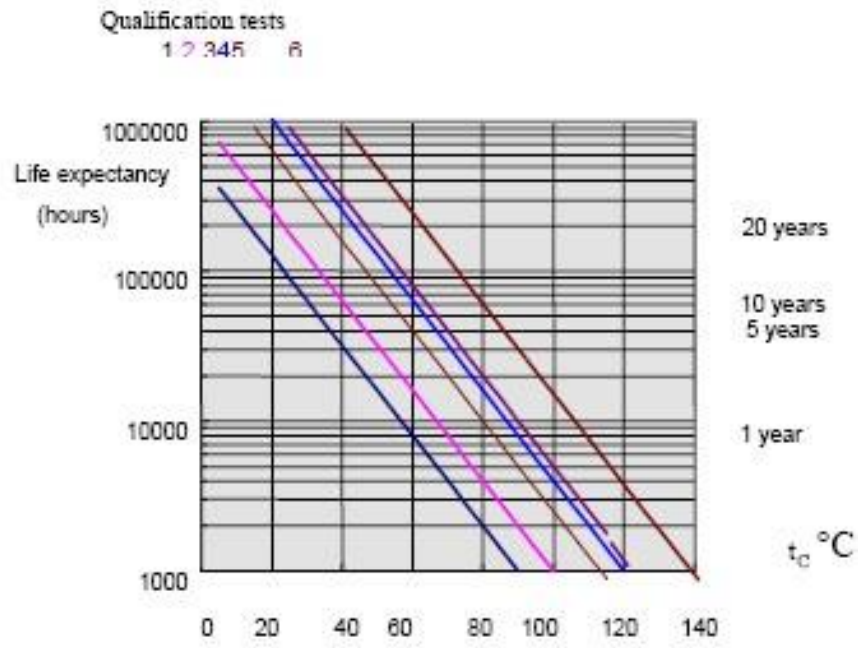


Example: Reliability of a module (serial system)

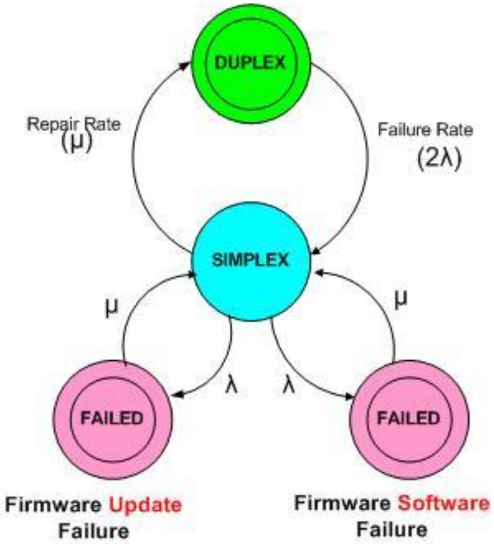
Component name	Type	Additional data	IEC 62380 reference	Failure rate	Quantity
Panduit D461612	Connector	Rectangular	Default value	0,003625	1
Panduit D461612	Connector	Rectangular	Default value	0,007200	1
74AHCT14	IC-Digital	Standard	Substituted with - SN74AHCT14D	0,014200	3
74HC/HCT540	IC-Digital	Standard	Substituted with - CD74HC540E	0,019000	2
74HC/HCT541	IC-Digital	Standard	Substituted with - SN74AHCT541DW	0,014000	3
PALCE16V8	IC-Digital	PAL	Exact matching	0,036000	1
HMA124	Optoelectronic	Optocoupler	Default value	0,011600	16
MB6S	IC-Digital	Standard	Default value	0,012700	16
Resistor	Resistor	General purpose	Default value	0,000232	32
Resistor	Resistor	Fixed, high dissipation film	Default value	0,001047	32
Capacitor	Capacitor	Tantalum - solid electrolyte	Default value	0,000725	17
Capacitor	Capacitor	Ceramic class II.	Default value	0,000223	41
SMD led	Optoelectronic	Solid State Lamp	Default value	0,002000	16
U22-DI016-C3	PWB		Default value	0,003403	1
SOD80 BZV55C	LF Diode	Zener	Default value	0,011500	64
Module:	1,392021 failure per million hours				

Estimation of life expectancy

- What is the lifetime of electronic components?
  - When does the fault rate start increasing?
  - At this time scheduled maintenance (replacement) is required
- IEC 62380: „Life expectancy”
- Especially limited: In case of electrolyte capacitors
  - Depends on temperature
  - Depends on qualification
  - Example: at 25°C,
    ~ 100 000 hours (~ 11 years)



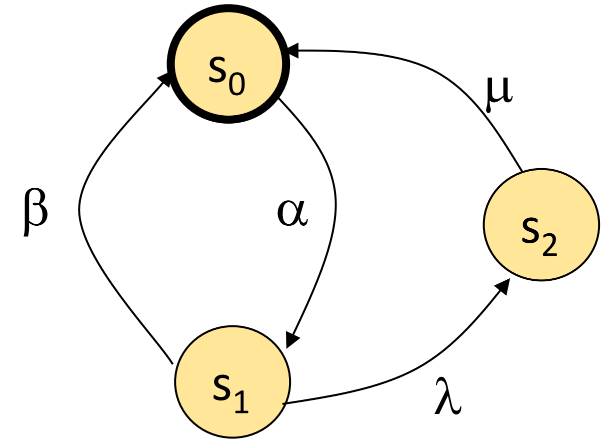
Markov models for dependability analysis



Model: Continuous Time Markov Chain

- Definition: CTMC = (S, $\underline{\underline{R}}$)
  - S set of discrete states:

    $s_0, s_1, \ldots, s_n$
  - $\underline{\underline{R}}: S \times S \to R_{\geq 0}$ state transition rates



- Notation:
  - Rate of leaving a state: $E(s) = \sum_{s' \in S, s \neq s'} R_{s,s'}$
  - $\underline{\underline{Q}} = \underline{\underline{R}} - \text{diag}(\underline{E})$ infinitesimal generator matrix
  - $\sigma = s_0, t_0, s_1, t_1, \ldots$ path (s_i is left at t_i)
  - $\sigma@t$ the state at time t
  - Path(s) set of paths from s

Solution of a CTMC

- Transient state probabilities:
  - $\pi(s_0, s, t) = P\{\sigma \in Path(s_0) \mid \sigma@t=s\}$ probability that starting from s_0 the system is in state s at time t
  - $\underline{\pi}(s_0, t)$ starting from s_0, the probabilities of the states at t
  - CTMC transient solution:

$$\frac{d\underline{\pi}(s_0,t)}{dt} = \underline{\pi}(s_0,t)\underline{\underline{Q}}$$

$$P\{\text{being in } s \text{ for } t\} = e^{-E(s)t}$$

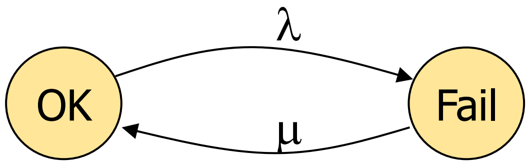
$$E\{\text{time spent in } s\} = \frac{1}{E(s)}$$

- Steady state probabilities:
  - $\pi(s_0, s) = \lim_{t\to\infty} \pi(s_0,s,t)$ state probabilities, starting from s_0
  - $\underline{\pi}(s_0)$ state probabilities (vector)
  - CTMC steady state solution:

$$\underline{\pi}(s_0)\underline{\underline{Q}} = 0 \quad \text{where} \quad \sum_{s} \pi(s_0, s) = 1$$

CTMC dependability model

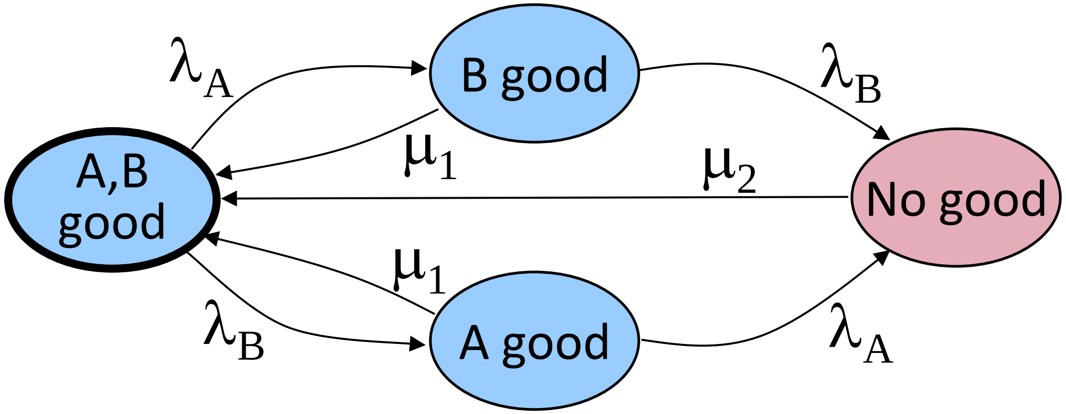
- CTMC states
  - System level states: Combination of component states (fault-free, or faulty according to a failure mode)
- CTMC transitions
  - Component level fault occurrence: Rate of the transition is the component fault rate λ
  - Component level repair: Rate of the transition is the component repair rate μ, which is the reciprocal of the repair time



  - System level repair: Rate of the transition is the system repair rate (which is the reciprocal of the system repair time)

Example: CTMC dependability model

- System consisting of two servers, A and B:
  - The servers may independently fail
  - The servers can be repaired independently or together
- System states: Combination of the server states (good/faulty)
- Transition rates:
  - Fault of server A: λ_A failure rate
  - Fault of server B: λ_B failure rate
  - Repair of a server: μ_1 repair rate
  - Repair of both servers: μ_2 repair rate



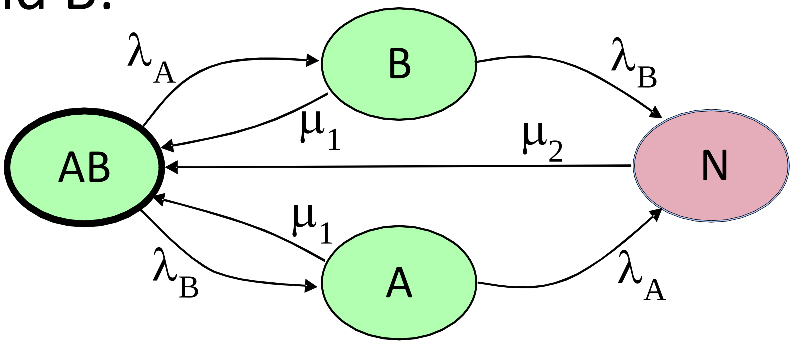
Computation of system level attributes

- Identifying state partitions
  - System level "up" state partition U and "down" partition D
- Solution of the CTMC model:
  - Transient solution: $\pi(s_0, s, t)$ time functions
  - Steady state solution: $\pi(s_0, s)$ probabilities
- Availability: $a(t) = \sum_{s_i \in U} \pi(s_0, s_i, t)$
- Asymptotic availability: $A = \sum_{s_i \in U} \pi(s_0, s_i)$
- Reliability: $r(t) = \sum_{s_i \in U} \pi(s_0, s_i, t)$

  Here: Before the solution the model shall be modified: transitions from partition D to U shall be deleted

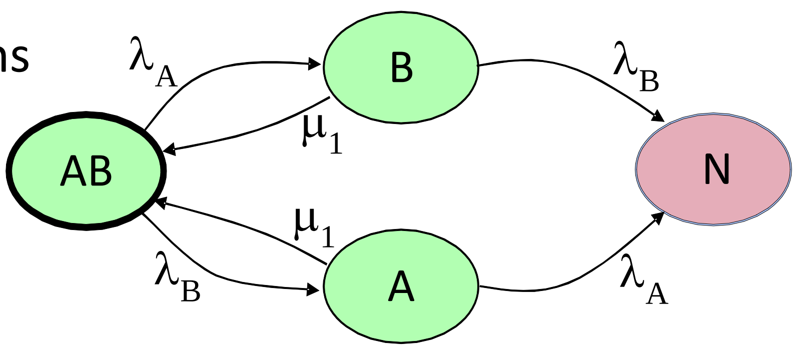
Example: CTMC dependability model

- System consisting of two servers, A and B:
  - The servers may independently fail
  - The servers can be repaired independently of together



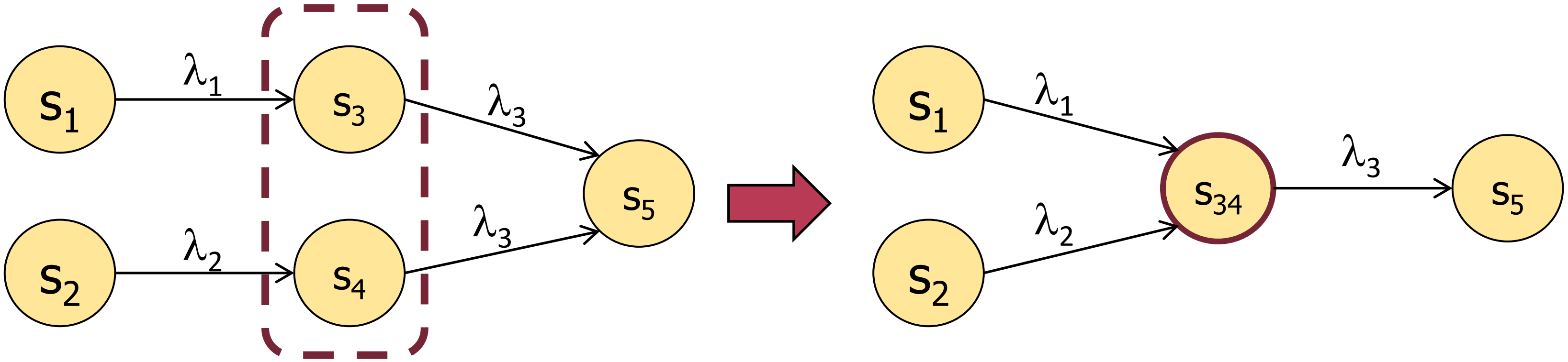
- State partitions:
  - $U = \{s_{AB}, s_A, s_B\}$, $s_0 = s_{AB}$
  - $D = \{s_N\}$
- Availability: $a(t) = \pi(s_0, s_{AB}, t) + \pi(s_0, s_A, t) + \pi(s_0, s_B, t)$
- Asymptotic availability: $K = A = \pi(s_0, s_{AB}) + \pi(s_0, s_A) + \pi(s_0, s_B)$
- Reliability:
  - Modifying the model: Deleting transitions from $D = \{s_N\}$ partition to U
  - Solution of the modified model:

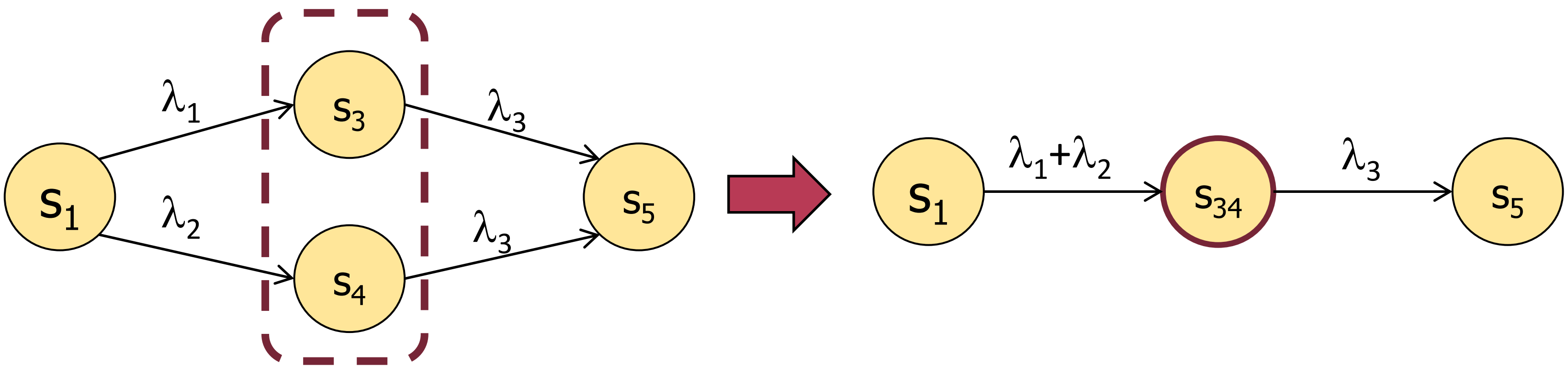
  $r(t) = \pi(s_0, s_{AB}, t) + \pi(s_0, s_A, t) + \pi(s_0, s_B, t)$



Reducing CTMC models

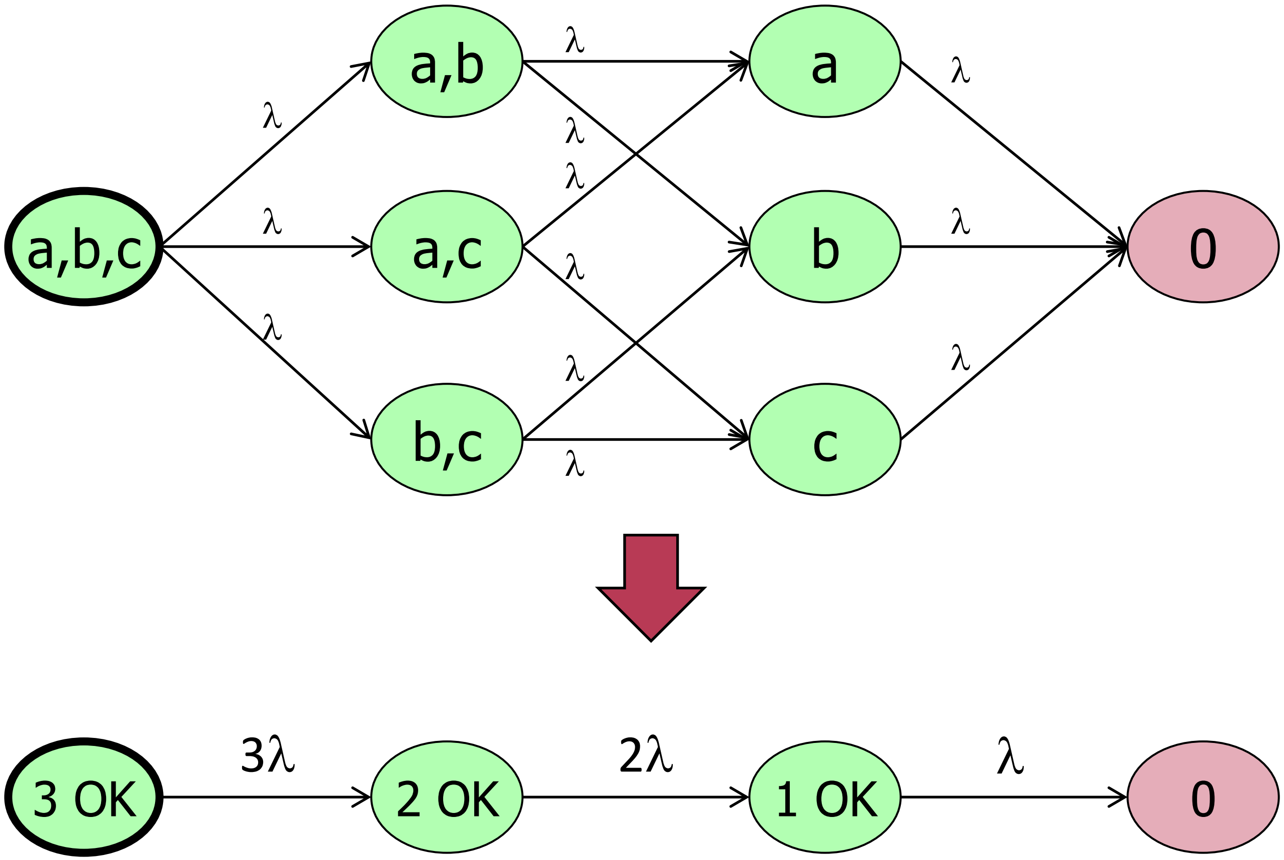
- Merging states
  - Condition: Have transitions to the same states with the same rates (outgoing transitions and rates do not distinguish these states)
  - After merging, the outgoing rate and the incoming rates remain the same (incoming transitions from the same state: rates are summarized)





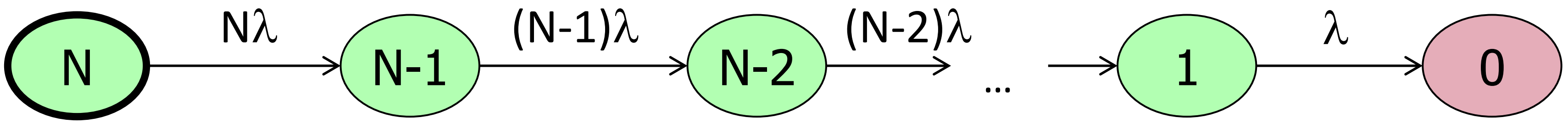
Example: Merging states

- Model: 3 redundant (replicated) components
- The components (a, b, c) have the same fault rate λ

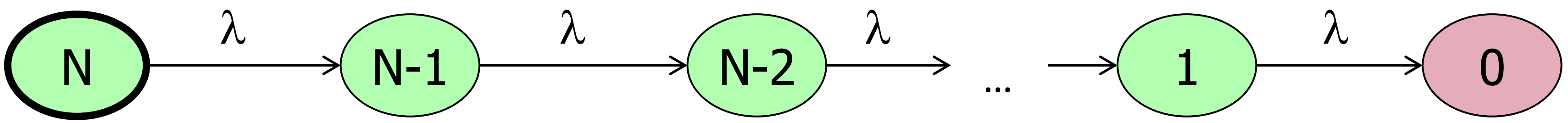


CTMC dependability models (1)

- Hot redundancy, N components:

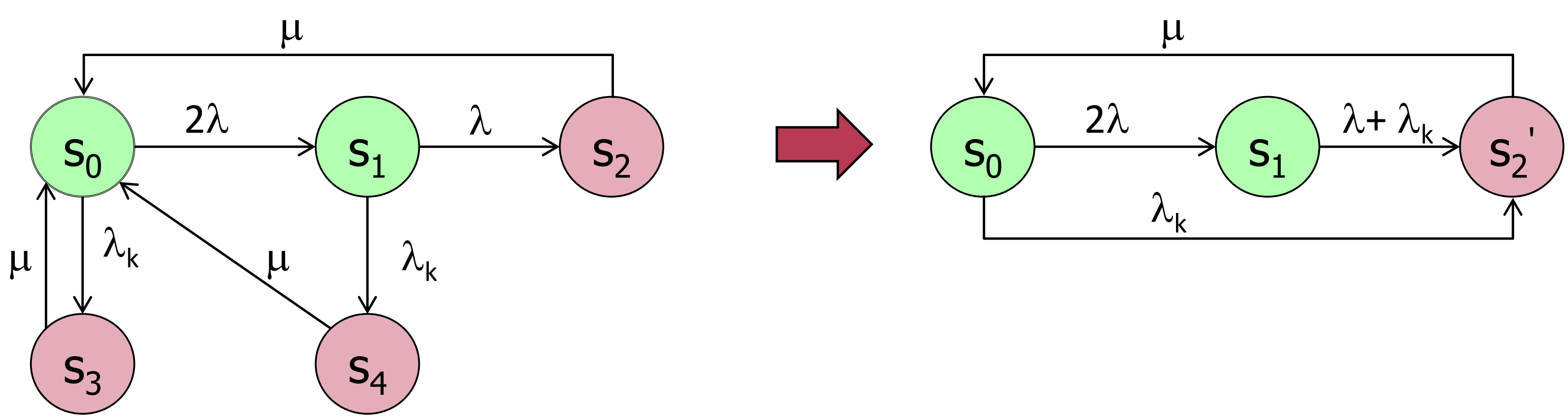


- Computing MTTF in case of hot redundancy
  - Time spent in state where k components are good: $\frac{1}{k\lambda}$
- Cold redundancy, N components:



CTMC dependability models (2)

- Active redundancy scheme
  - 2 components, each with λ failure rate
  - Switch between components, with λ_k failure rate
  - In case of a fault: complete repair, with μ repair rate



Tools for dependability analysis

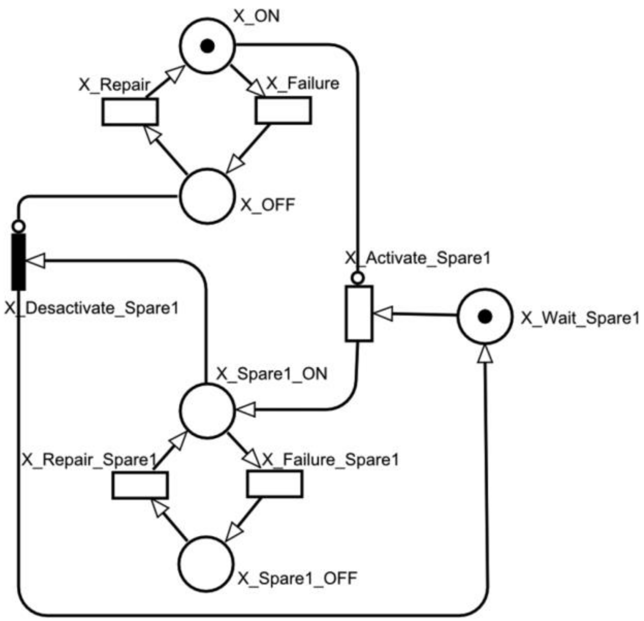
For both combinational dependability models

- Fault tree,
- Event tree,
- Reliability block diagram,
- FME(C)A, …

and Markov chains:

- Relex (www.relex.com)
- Item Toolkit (www.itemuk.com)
- RAM Commander (www.aldservice.com)
- Functional Safety Suite

Stochastic Petri nets for dependability analysis

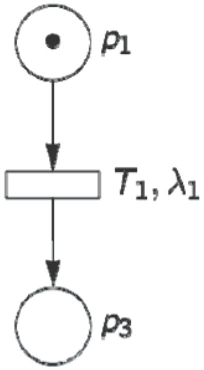


Model: Stochastic Petri-nets (SPN)

- SPN: Stochastic Petri Net
- Extension of simple Petri-nets
  - Transitions have random firing delay
  - Firing delay d_i is sampled from a negative exponential probability distribution function with parameter λ_i

$$P\left\{d_i \leq t\right\} = 1 - e^{-\lambda_i t} \qquad P\left\{d_i > t\right\} = e^{-\lambda_i t}$$

  - A transition may fire if enabled for the time period of the firing delay
- Graphical notation of transitions:
  - Empty rectangle with λ_i parameter



SPN characteristics

- The time needed to reach a new marking has negative exponential distribution
  - Even in the case of concurrent or conflicting transitions
  - This allows the modeling of healthy and faulty states of components
- The timed reachability graph is a CTMC
  - Its structure is independent from the values of firing rates
  - The solutions for CTMC can be used for SPN analysis
- Results of the analysis:
  - Steady state solution (existing if the SPN is bounded and reversible):
    - Probabilities of markings (time functions or asymptotic)
    - Throughput of transitions
  - Transient solution:
    - Probability time functions of markings

Other models used for dependability modeling

- GSPN: Generalized Stochastic Petri-net
  - Immediate transitions: Used for modeling dependencies
  - Timed transitions: Used for timed events, with exponential distribution
  - Inhibitor arcs and guards: Predicates for enabling transitions
  - The reachability graph is still a CTMC
- DSPN: Deterministic and Stochastic Petri Net
  - Deterministic firing delay (constant firing time) of transitions is also possible
    - Useful for modeling repair time
  - Analytic solution: if in each marking only a single deterministic transition is enabled
- TPN: General timed Petri-nets
  - General distribution can be used to sample the firing delay of transitions
  - In general case the reachability graph is not a CTMC (solution by simulation or approximation)

Model: Assignment of rewards

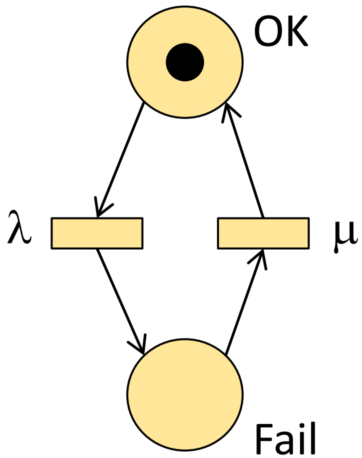
- Reward: “Profit” or “cost” functions can be assigned to markings or firings
- Rate reward:
  - Assigned to markings, reward/time value is given by the function
  - Example: If the server is healthy then the profit is 300 Ft/hour, otherwise the penalty is 200 Ft/hour:

    ```
    if (m(Healthy)>0) then ra=300 otherwise ra=-200
    ```
  - Computed: Accumulated reward (e.g., profit or penalty) for a time interval
- Impulse reward:
  - Assigned to transitions, reward/firing value is given by the function
  - Example: The cost of a repair is 500 Ft:

    ```
    if (fire(Repair)) then ri=500
    ```
  - Computed: Sum of rewards for a time interval (counting the firings)

SPN (GSPN) dependability model

- Advantages in comparison with CTMC:
  - Modeling concurrent fault occurrences and repair activities
  - It is not necessary to represent system level states
- SPN places
  - Component level states: Healthy, faulty, or according to failure modes; separately for each component
- SPN transitions
  - Component level fault occurrence: The parameter is the λ fault rate
  - Component level repair: The parameter is the μ repair rate (reciprocal of the repair time)
  - System level repair (transition between multiple places): The parameter is the repair rate of the system state



Computation of system attributes

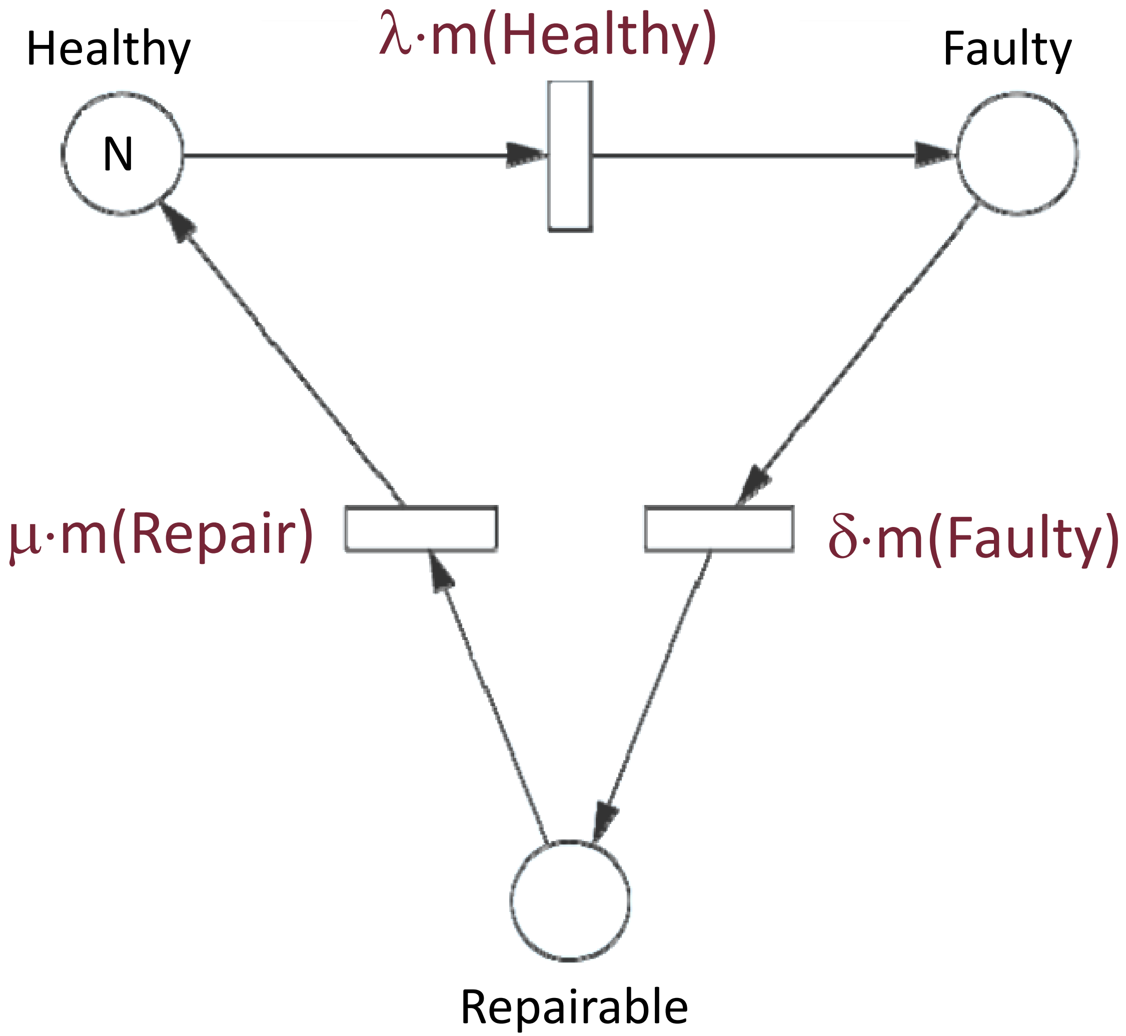
- Definition of state partitions:
  - Normal (”up”, U) and failure (“down”, D) partitions
  - Partitions are defined on the basis of markings
- Computation of availability
  - Direct:
    - Probability of being in the state partition U
    - Sum of the probability functions of the markings in U
  - Reward based:
    - if ($m \in U$) then ra=1 else ra=0
    - ra time function: availability function
    - ra expected value: asymptotic availability A=E{ra}

Example: Redundant servers

- Server cluster consisting of N servers with identical fault rates
  - The cluster is "up" is at least one server is healthy
- Fault occurrence:
  - Fault rate of a server is λ
  - The faults of servers are independent
- Fault detection and repair:
  - The detection delay of a fault is characterized by the detection rate δ (parameter of a negative exponential distribution)
  - In case of a detected fault the repair is characterized by the repair rate μ (parameter of a negative exponential distribution)
  - It is possible to detect the faults and repair more servers at a time
- Model:
  - Places: Healthy, Faulty, Repairable (marking: number of servers)
  - Transitions: Fault occurrence, detection, repair (marking dependent rates)
  - U state partition: m(Healthy)>0
  - Availability: Probability of being in state partition U

Example: Redundant servers

- Compact model with marking dependent rates:



Summary

- Attributes of dependability
  - Reliability, availability:
    Probability functions (in time)
- Combinational modeling: Reliability block diagram
  - Serial, parallel, majority voting structures
- State based models: Markov chains
  - Computation: Probability of state partitions
- Concurrency in models: Stochastic Petri-nets
  - Computation: Probability of markings