

# Software Model Checking with Abstraction-Based Methods

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# INTRODUCTION

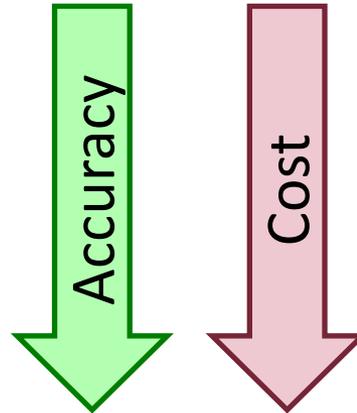
# Introduction

## ■ Motivation

- Checking the **source code directly**
- Should work by “pushing a button”
  - No deep background knowledge should be required

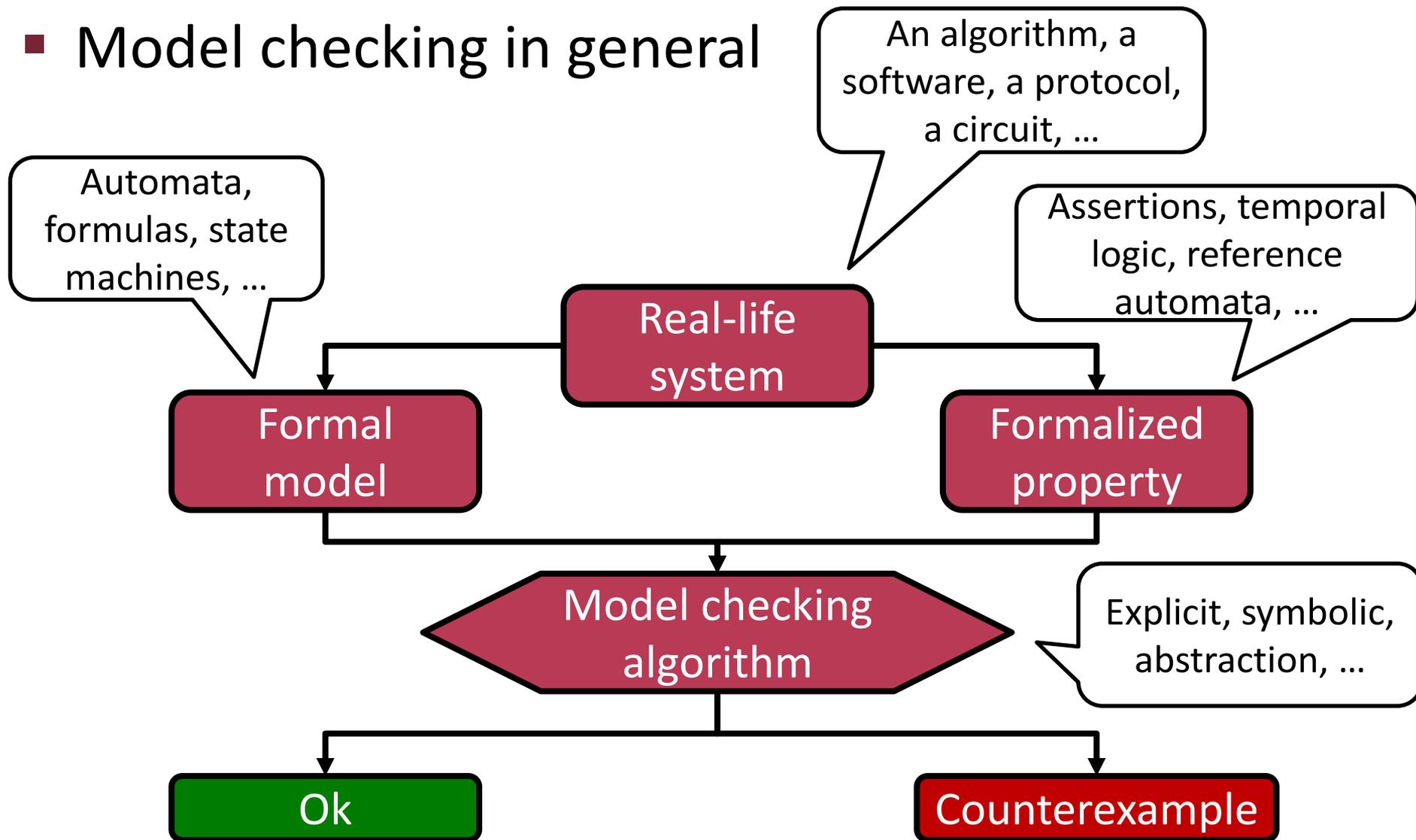
## ■ Software verification techniques

- Static analysis
  - Error patterns
  - Abstract interpretation
- **Model checking**



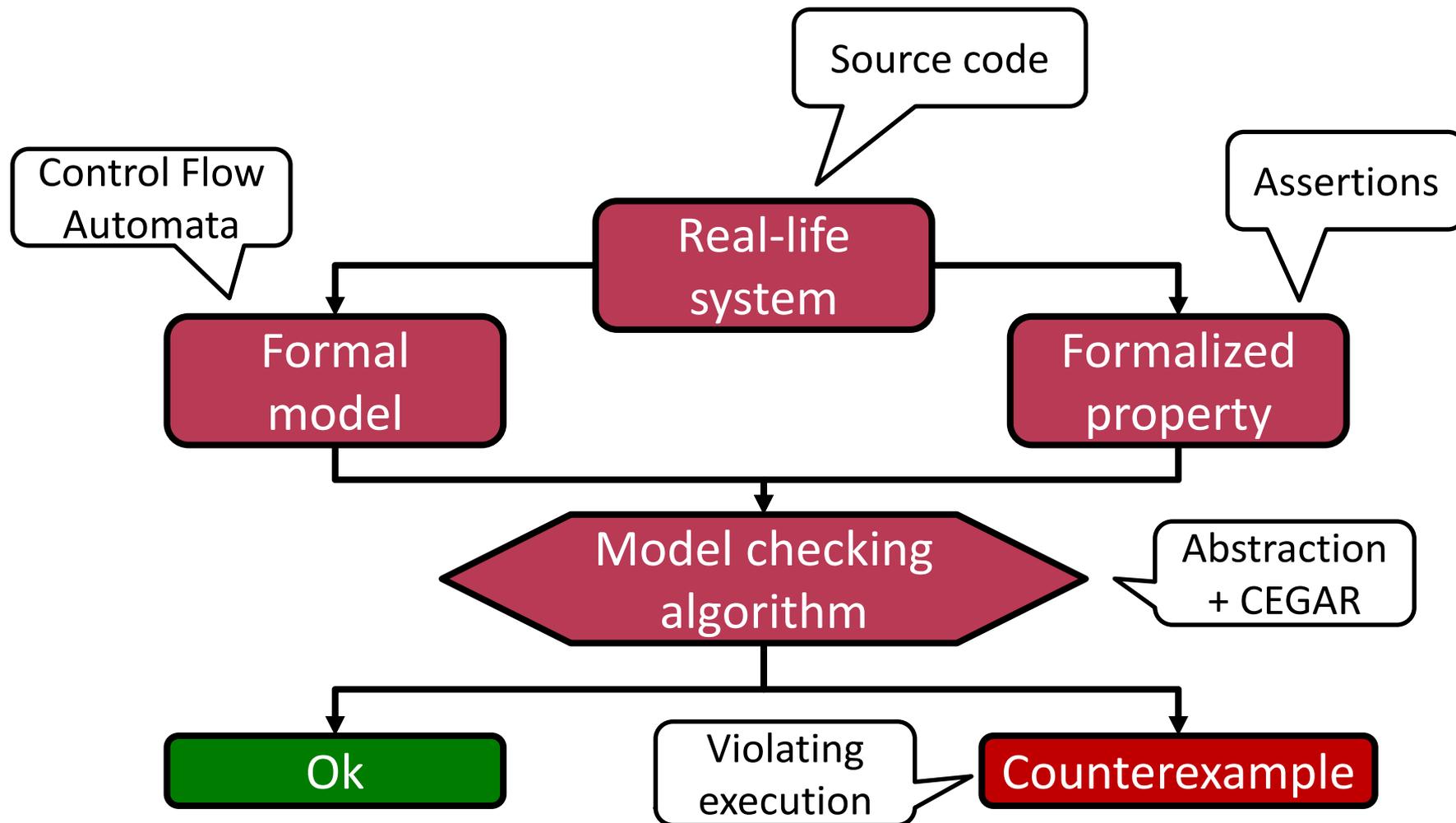
# Introduction – Model Checking

## ■ Model checking in general



# Introduction – Model Checking

- This lecture: focus on **software** and **abstraction**

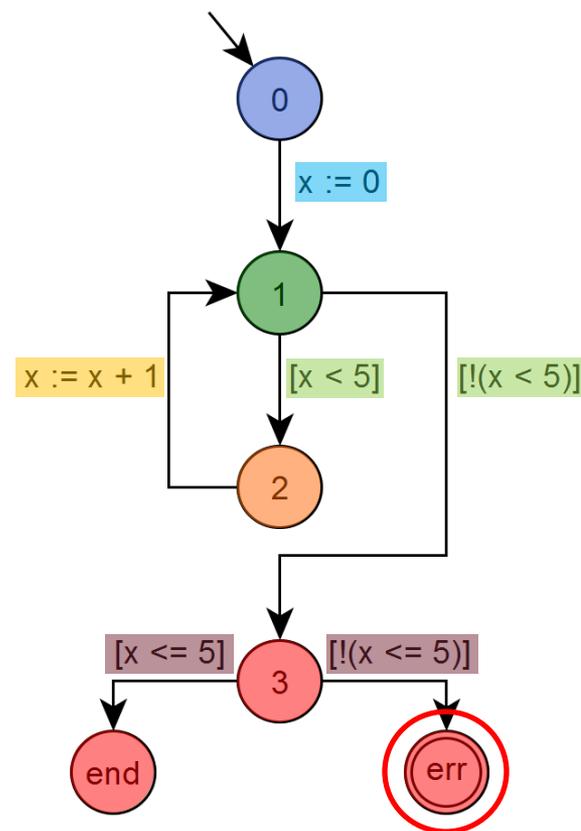
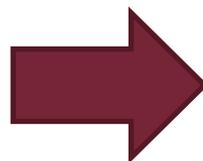


# Introduction – Model and Property

## Control-Flow Automaton

- Set of control **locations** (PC)
- Set of **edges** with **operations** over a set of **variables**
  - E.g., guard, assignment ...

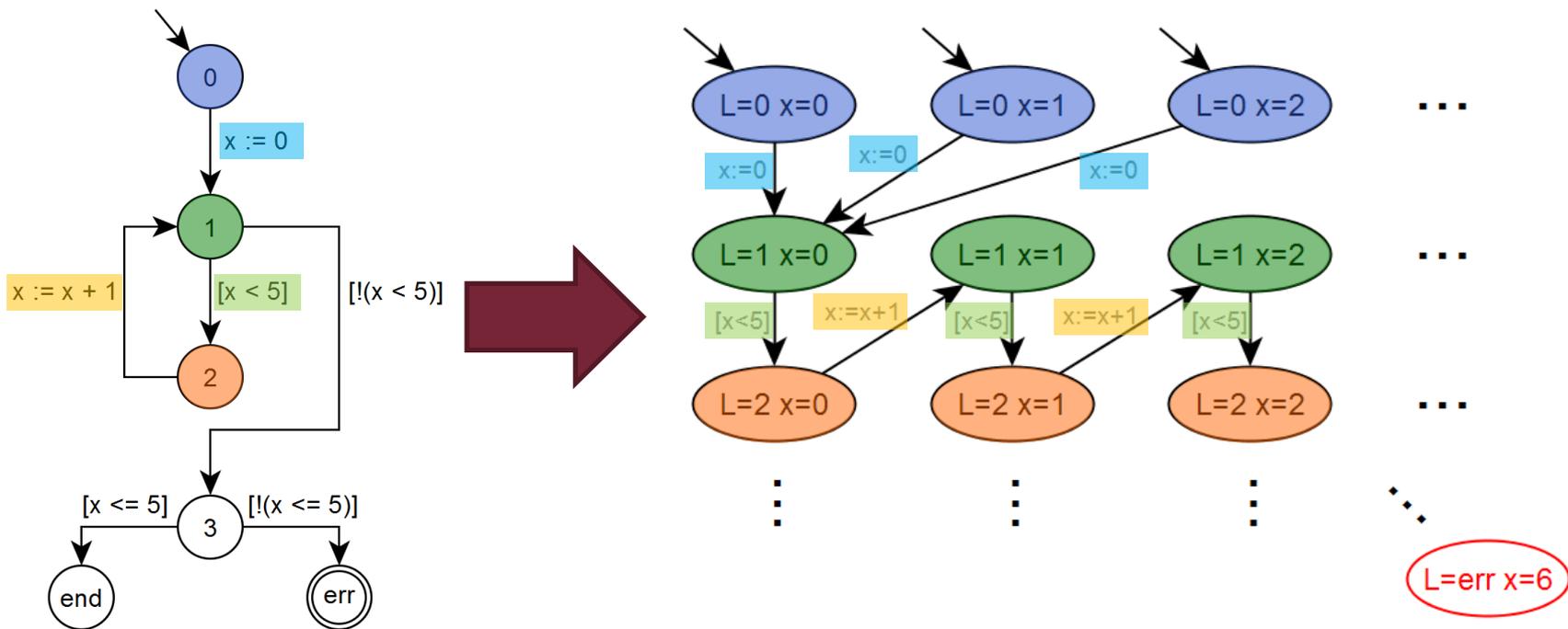
```
x : int
0: x = 0
1: while (x < 5) {
2:   x = x + 1
}
3: assert (x <= 5)
```



- Typical property: “**error**” location should not be reachable

# Introduction – States and Transitions

- **State**: location + valuation of variables ( $L, x_1, x_2, \dots, x_n$ )
- **Transition**: operations
- **Problem**: **state space explosion** caused by data variables
  - E.g., 10 locations and 2 integers:  $10 \cdot 2^{32} \cdot 2^{32}$  **possible** states
- **Goal**: **reduce** the state space representation **by abstraction**



# Introduction – Mathematical Logic

## ■ Propositional logic (PL)

- Boolean variables and operators
- SAT problem: is the formula satisfiable
  - Example: bounded model checking
- Expressive power sometimes not enough

$$\neg p \wedge (p \vee q)$$

## ■ First order logic (FOL)

- Functions, predicates, quantifiers
- Satisfiability is not decidable in general

$$\forall x, y \exists z: p(f(x, y), g(z))$$

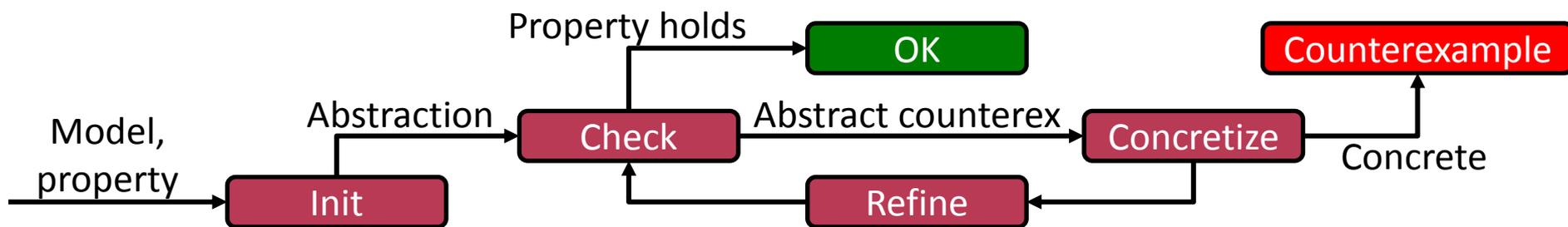
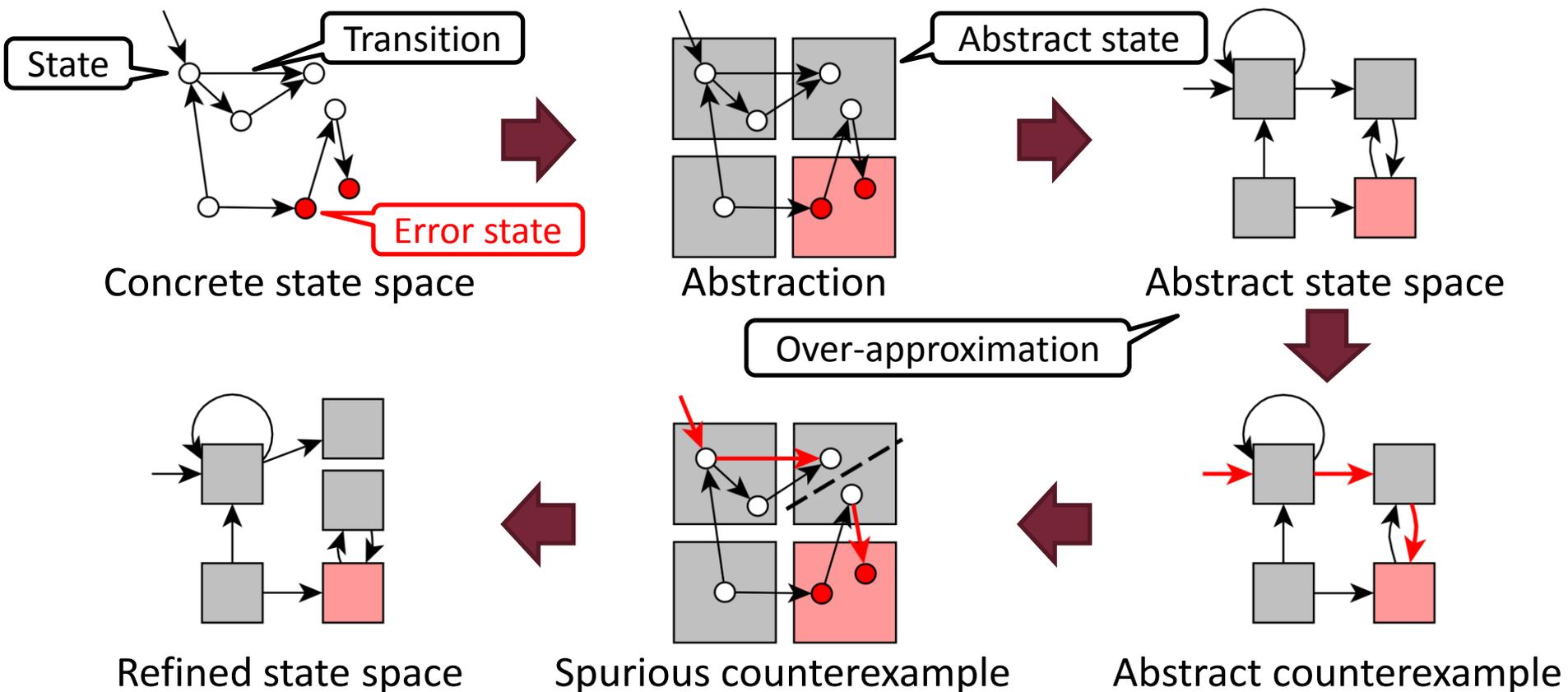
## ■ Satisfiability Modulo Theories (SMT)

- “Restricted” FOL formulas
- Only interpreted symbols (e.g., integer arithmetic)
- Satisfiability can be decided

$$(x \leq y + 1) \wedge (y \geq 3)$$

# COUNTEREXAMPLE-GUIDED ABSTRACTION REFINEMENT (CEGAR)

# CEGAR – Introduction: abstract states



# Abstraction – Introduction

## ■ Abstraction

- General mathematical concept
- Hide details
- Get an easier problem to solve

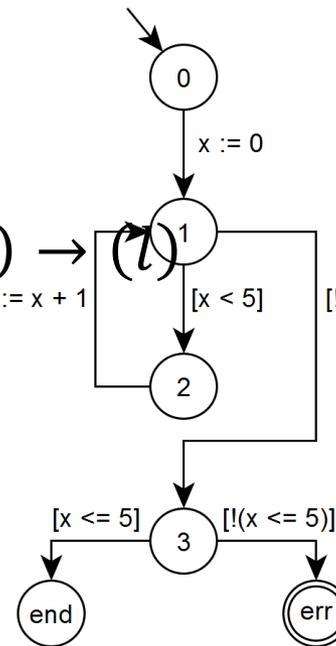
## ■ Example

- Location abstraction

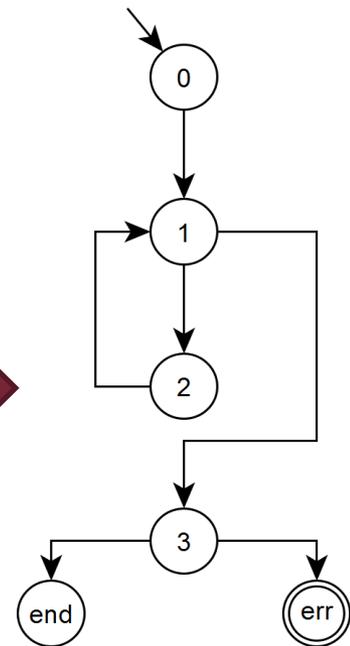
$(l, x_1, x_2, \dots, x_n) \rightarrow (l)$

- Usually not enough

- Trivial counterexamples are found (no conditions)
- Extension with predicates:  
**predicate abstraction**



CFA



Abstract state space

# Predicate Abstraction

- Predicate abstraction
  - Keep track of **predicates instead of concrete values** for variables
  - Abstract state: concrete states corresponding to the same location + satisfying the same predicates
- Performing abstraction (initial attempt)
  - Enumerate and join concrete states
  - 3x3 concrete states in the example  $\rightarrow$  5 abstract states
  - State space explosion 😞

Variables:

$$x, y; D_x = D_y = \{0,1,2\}$$

Predicates:

$$(x = y), (x < y), (y = 2)$$



$y \backslash x$	0	1	2
0	$(x = y)$		
1	$(x < y)$	$(x = y)$	
2	$(x < y)$ $(y = 2)$	$(x < y)$ $(y = 2)$	$(x = y)$ $(y = 2)$

# Predicate Abstraction

## ■ Performing abstraction (differently)

- Enumerate **abstract states only**
- Predicate set  $P \rightarrow |L| \cdot 2^{|P|}$  possible abstract states
- Feasibility of abstract states and state transitions shall be checked

## ■ Example

- 3 predicates  $\rightarrow$  8 possible abstract states (for each location)
- Some are **not feasible**
  - E.g.  $(x = y) \wedge (x < y) \wedge \neg(y = 2)$  is not feasible (not satisfiable)
  - Use SMT solver to check whether a combination of predicates is satisfiable

	$x = y$	$x < y$	$y = 2$
1	X	X	X
2	X	X	✓
3	X	✓	X
4	X	✓	✓
5	✓	X	X
6	✓	X	✓
7	✓	✓	X
8	✓	✓	✓

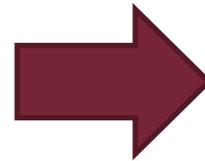
# Predicate Abstraction

- **Abstract states** with predicate abstraction

- Concrete                      Abstract
- $(l, x_1, \dots, x_n) \rightarrow (l, b_1, \dots, b_m)$
- $b_i$ : Boolean variable: its value gives if predicate  $p_i$  holds or not
- Notation:  $p(b_i) = \begin{cases} p_i & \text{if } b_i \text{ is true} \\ \neg p_i & \text{otherwise} \end{cases}$

- **Example**

Variables:  
 $x, y; D_x = D_y = \{0,1,2\}$   
Predicates:  
 $(x = y), (x < y), (y = 2)$



$l \ x \ y$   
↓ ↓ ↓  
 $(0,0,0) \rightarrow (0, T, F, F)$   
 $(6,1,2) \rightarrow (6, F, T, T)$   
↑ ↑ ↑ ↑  
 $l \ (x = y) \ (x < y) \ (y = 2)$

# Predicate Abstraction

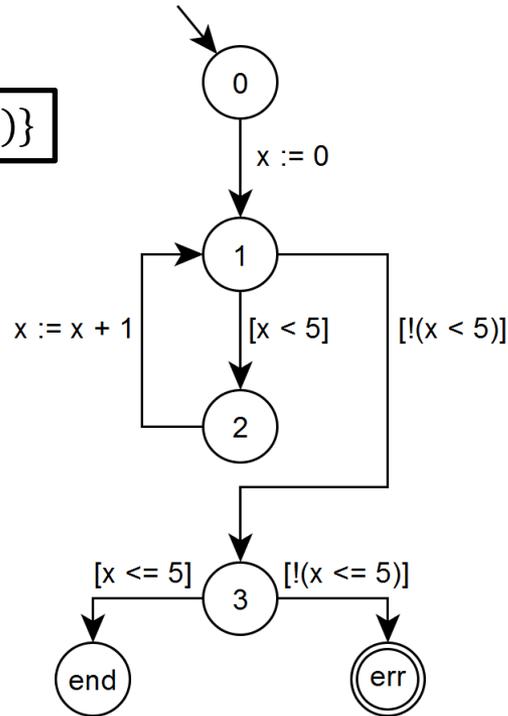
- Abstract initial states, error states, transitions
  - **Abstract initial** state:  $(l, b_1, \dots, b_m)$ , where  $l = l_0$
  - **Abstract error** state:  $(l, b_1, \dots, b_m)$ , where  $l = l_E$
  - **Abstract transition**: at **least one concrete** transition exists between contained concrete states
    - Calculate with SMT solver (without enumerating concrete states)
    - For  $(l, b_1, \dots, b_m)$  and  $(l', b'_1, \dots, b'_m)$ :
      - $\exists op: (l, op, l') \in G$  (there is an edge between locations in the CFA)
      - $p(b_1) \wedge \dots \wedge p(b_m) \wedge op \wedge p(b'_1) \wedge \dots \wedge p(b'_m)$  is satisfiable

Existential  
abstraction

# Predicate Abstraction

- Example:

$$P = \{(x \leq 5)\}$$



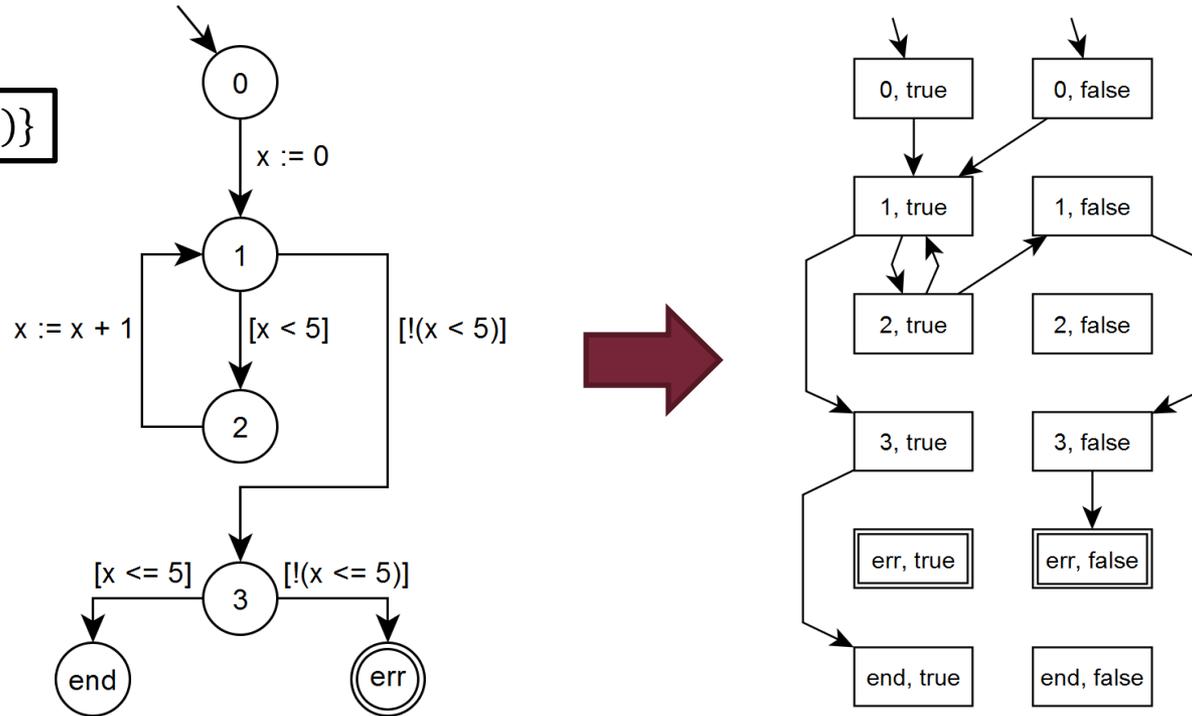
0, true	0, false
1, true	1, false
2, true	2, false
3, true	3, false
err, true	err, false
end, true	end, false

- Here 6 locations, 1 predicate  $\rightarrow$  12 abstract states

# Predicate Abstraction

- Example:

$$P = \{(x \leq 5)\}$$

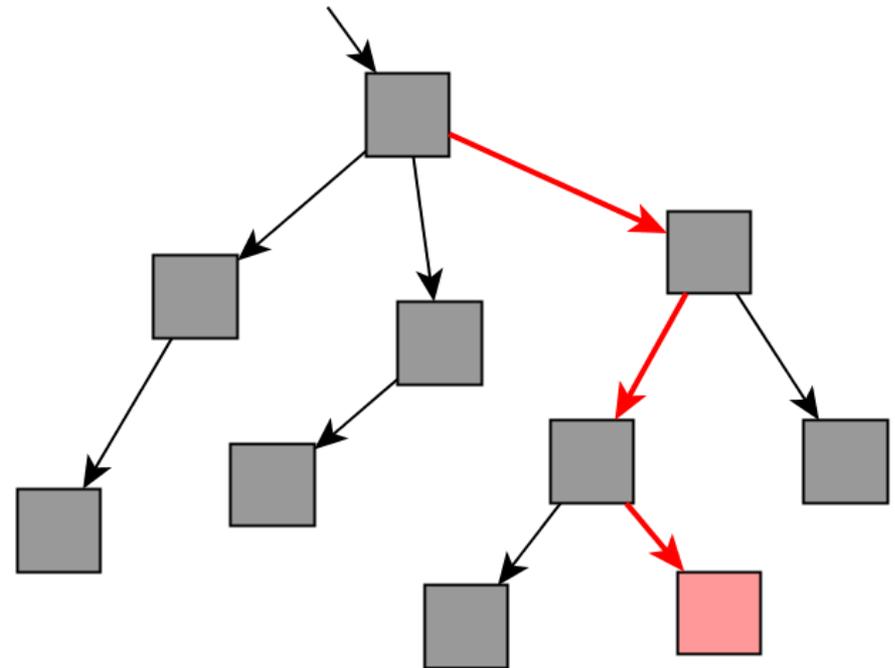


- Transitions: checking general feasibility with SMT solver

- E.g.,  $(2, \text{true}) \rightarrow (1, \text{true})$  is feasible
  - $(2, x := x + 1, 1) \in G$  and  $(x \leq 5) \wedge (x' = x + 1) \wedge (x' \leq 5)$  is satisfiable:  $x = 0, x' = 1$
- E.g.,  $(2, \text{true}) \rightarrow (1, \text{false})$  is feasible
  - $(2, x := x + 1, 1) \in G$  and  $(x \leq 5) \wedge (x' = x + 1) \wedge \neg(x' \leq 5)$  is satisfiable:  $x = 5, x' = 6$

# Model Checking

- **Traverse** abstract state space
  - Search for **error state**
  - With some **search strategy**, e.g., DFS, BFS
- **Optimizations**
  - **On-the-fly**
    - Calculate abstract states during the search
  - **Incremental**
    - Do not explore unchanged parts after refinement



# Model Checking

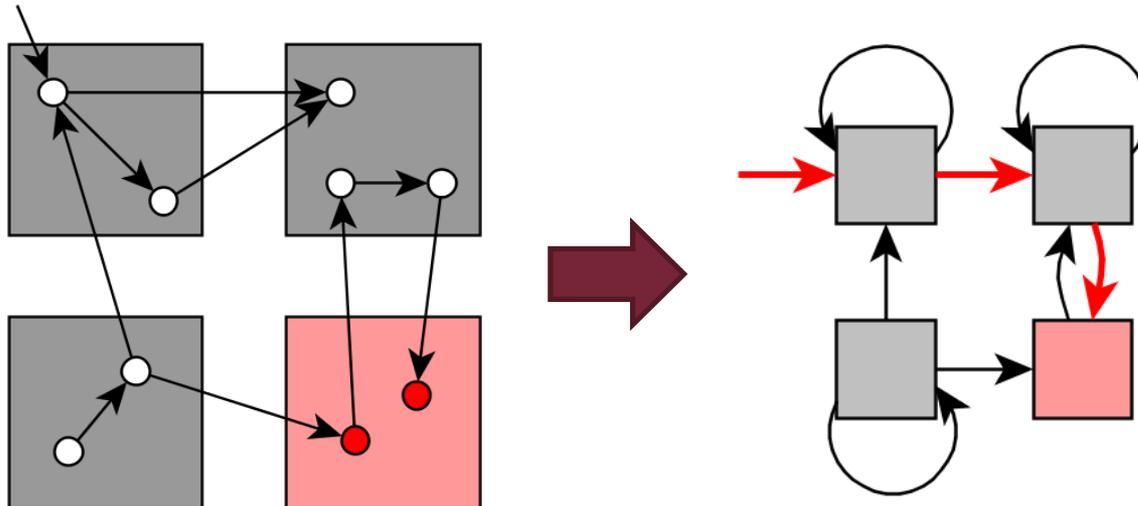
## ■ Properties of existential abstraction

### ○ Over-approximates the original model

- There is a corresponding **abstract path** for each **concrete path**
- Universally quantified property holds  $\rightarrow$  holds in the original model
  - Error state is not reachable ( $AG \neg \text{Error}$ )  $\rightarrow$  not reachable in original

### ○ What about **abstract counterexamples**?

- Not all abstract paths have corresponding concrete paths!

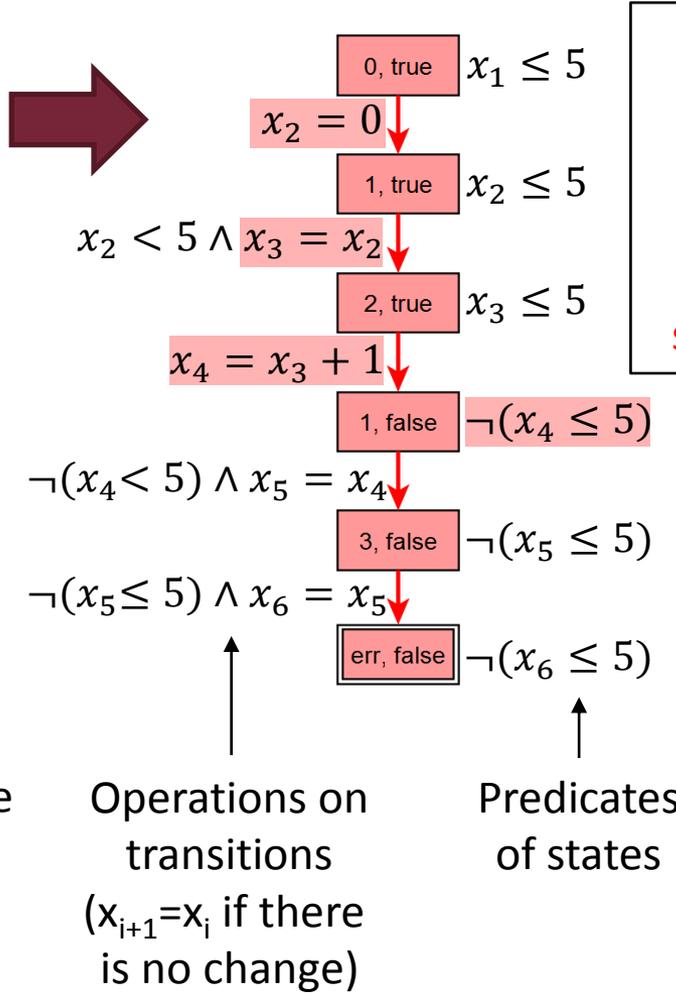
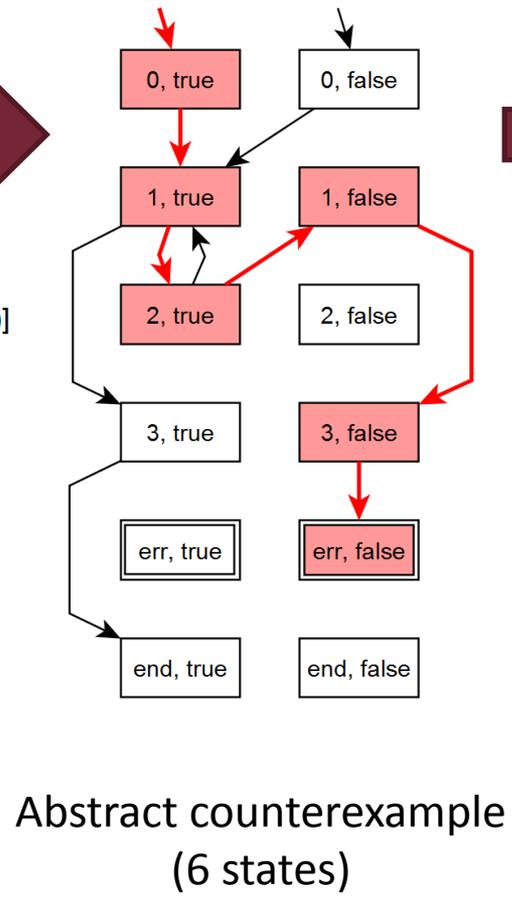
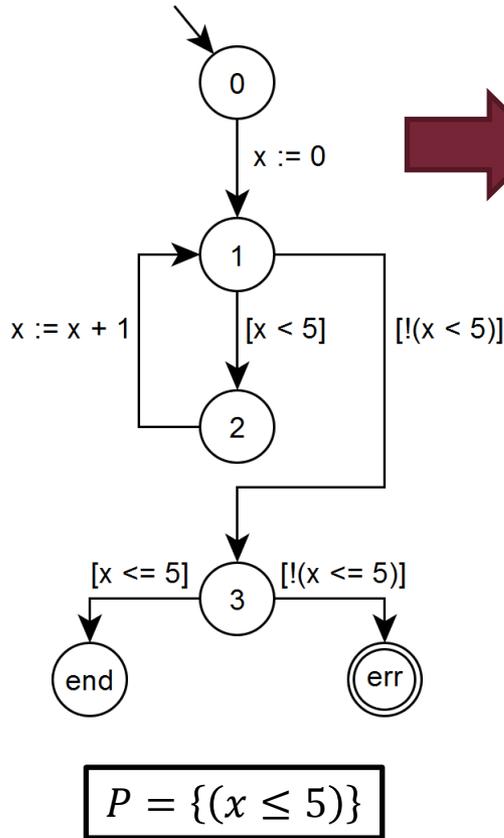


# Abstract Counterexample

- Form of abstract counterexample
  - Sequence of **locations and predicates**
  - $(l_1, b_{1,1}, \dots, b_{1,m}), (l_2, b_{2,1}, \dots, b_{2,m}), \dots, (l_n, b_{n,1}, \dots, b_{n,m})$
- Finding a **concrete path**: trying to traverse a part of the concrete state space
  - Guided by the abstract counterexample
  - Using SMT solver
    - Starting from the initial state
    - Traversing: Similarly to bounded model checking (BMC)
    - Generalize the method presented at existential abstraction for  $n$  steps
- Concrete path **exists**  $\rightarrow$  concrete model is **faulty**
- Concrete path **does not exist**  $\rightarrow$  **spurious** counterexample

# Abstract Counterexample

## Example

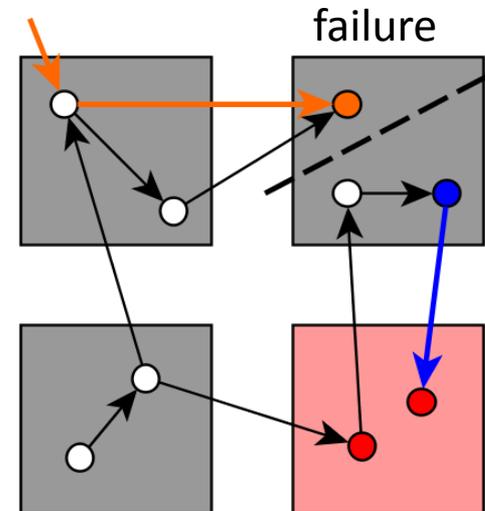


$(x_1 = 3)$   
 $(x_2 = 0)$   
 $(x_3 = 0)$   
 $(x_4 = ?)$

**Not satisfiable**

# Spurious Counterexample

- A concrete path exists until a state and after, but it is “broken” in a so-called “failure” state
- Grouping concrete states mapped to the “failure” state
  - D = “Dead-end”: reachable
  - B = “Bad”: transition to next state
  - IR = “Irrelevant”: others
- Reason for spurious counterexample
  - Set of predicates does not distinguish D and B



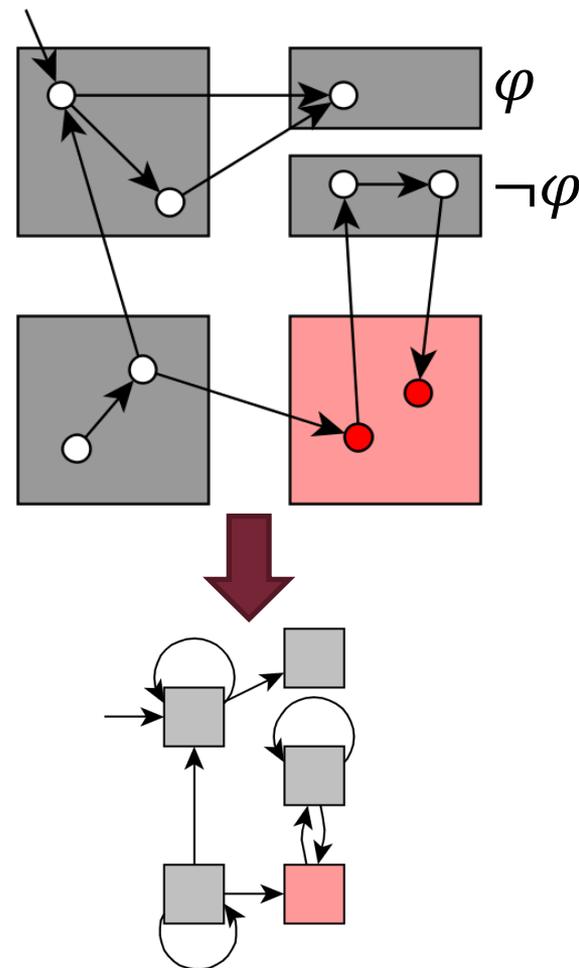
# Abstraction Refinement

## ■ Eliminating the spurious counterexample

- More predicates (finer abstraction)
- Separate **D** and **B**
  - Without enumerating concrete states
  - Describe **D** and **B** with formulas
  - SMT solver can generate a formula  $\varphi$  that separates **D** and **B** (*interpolation*)
- The set  $P \cup \{\varphi\}$  will eliminate this spurious counterexample
  - Moreover it is enough to split only the failure state (*lazy abstraction*)

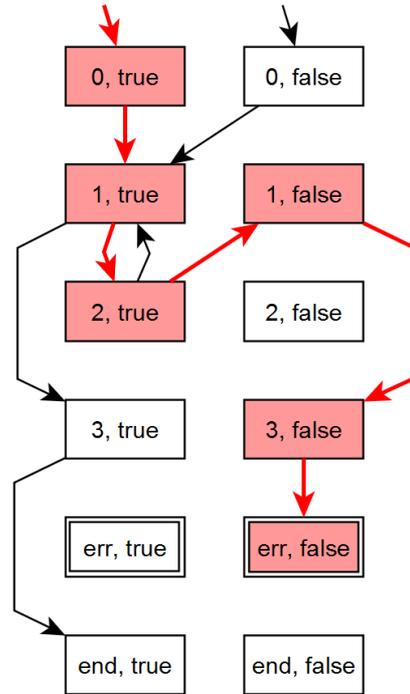
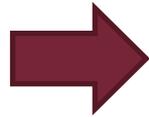
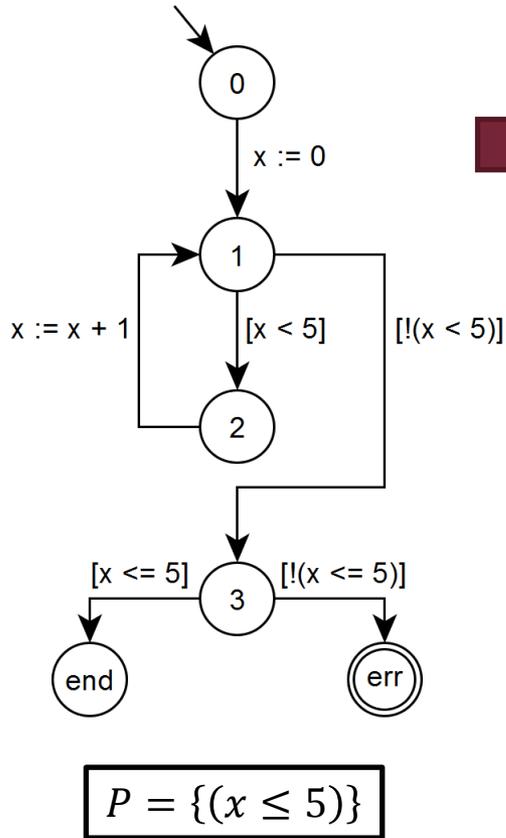
## ■ Additional spurious counterexamples

- More predicates may be needed

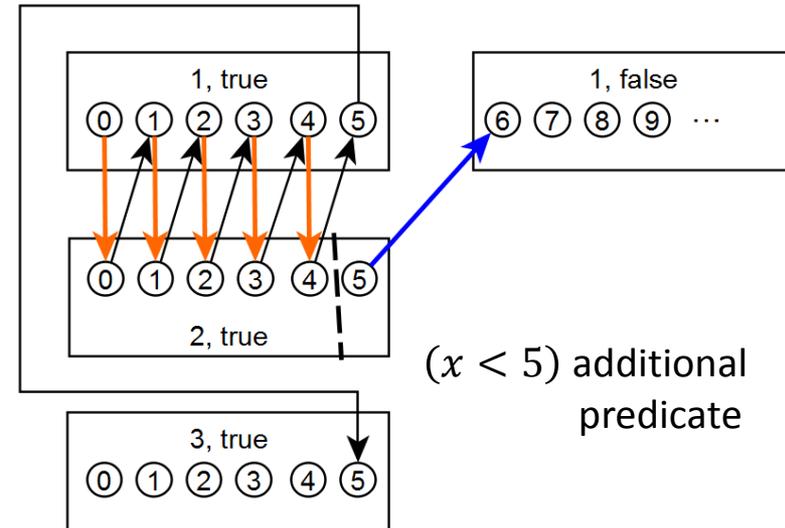


# Abstraction Refinement

## Example

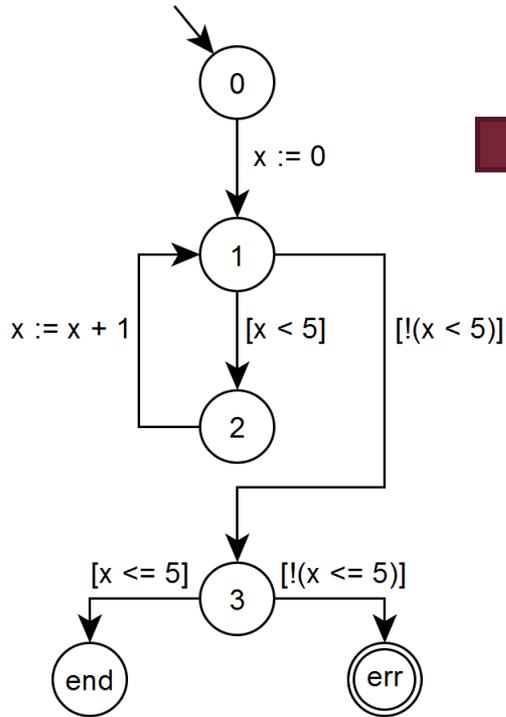


$(x_1 = 3)$   
 $(x_2 = 0)$   
 $(x_3 = 0)$   
 $(x_4 = ?)$

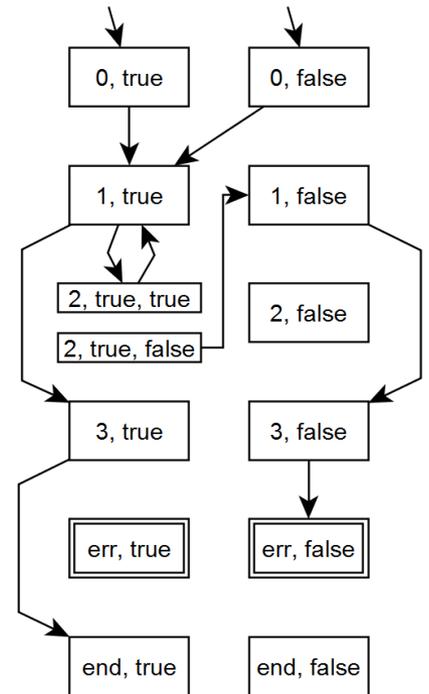
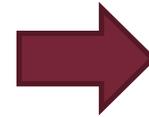
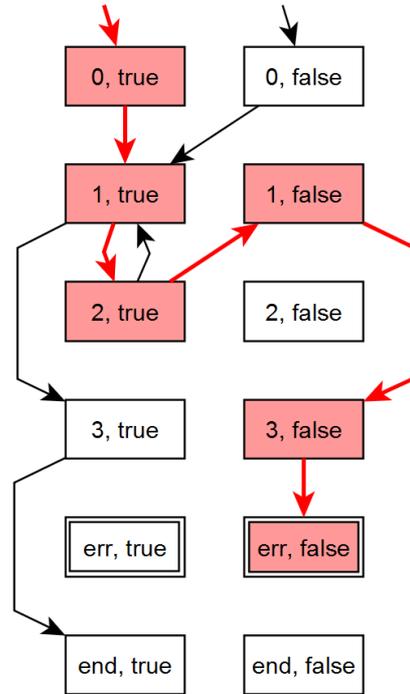
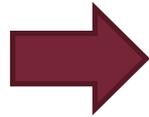


# Abstraction Refinement

## Example



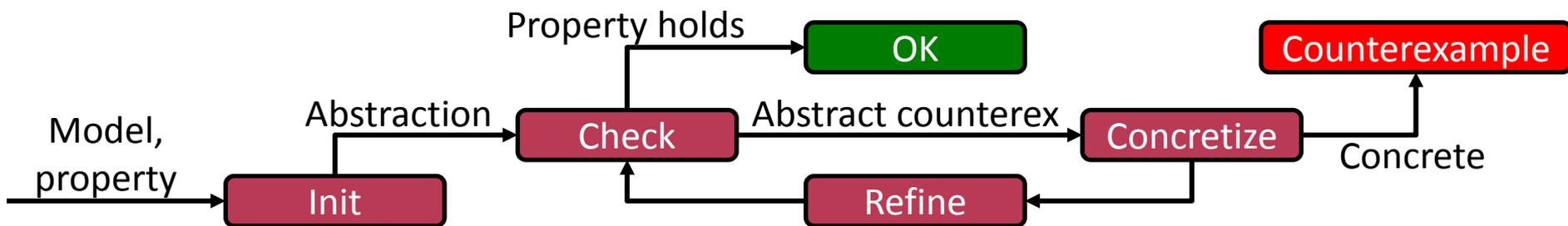
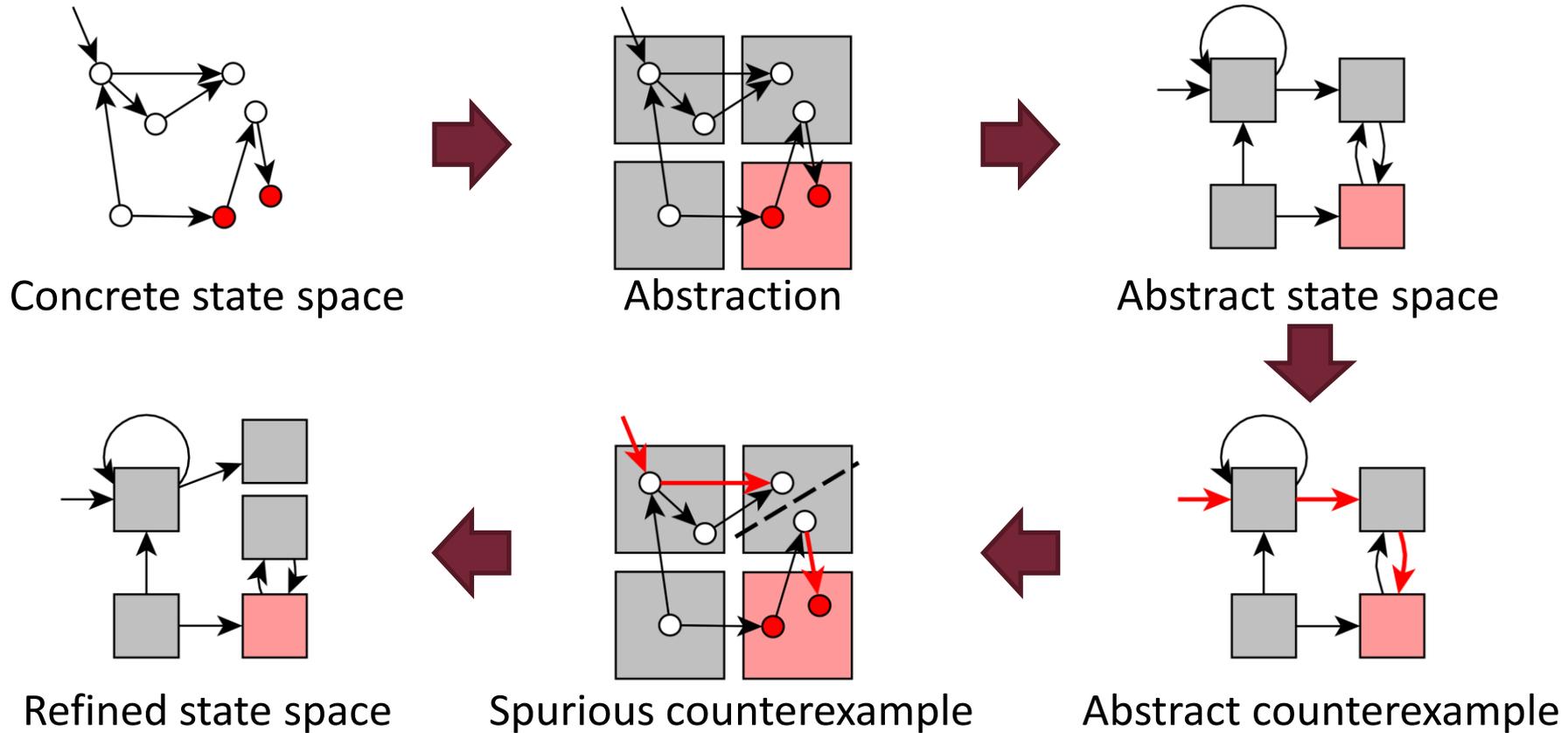
$$P = \{(x \leq 5)\}$$



$$P = \{(x \leq 5), (x < 5)\}$$

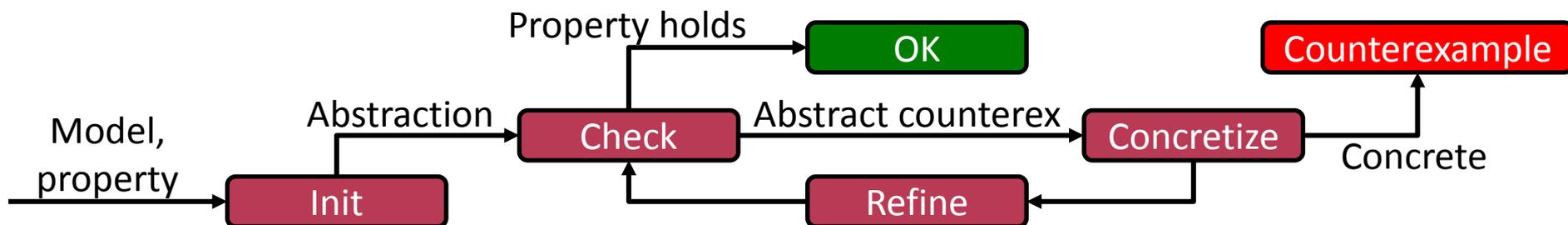
Error state is not reachable

# CEGAR – Summary



# The algorithm

- Counterexample-Guided Abstraction Refinement (CEGAR)
  - Automatic method
    - Each step is automatic
    - Deep knowledge of formal methods is not required
    - Hidden steps: checking feasibility of formulas (SMT solver)
  - How about the **initial** set of **predicates**?
    - It can be an empty set
    - It can come from conditional statements in the software
    - Other heuristics may also be used



# TOOLS

# Tools

## ■ SLAM2

- Part of Static Driver Verifier Research Platform (SDVRP)
- Structure
  - Driver **C code**: analyzed component
  - Platform model: describe environment
  - Analysis: adherence to API usage rules
- Algorithms
  - Create Boolean program with **predicate abstraction**
  - Symbolic model checking: BEBOP tool
  - **CEGAR loop**
- [research.microsoft.com/en-us/projects/slam/](https://research.microsoft.com/en-us/projects/slam/)

# Tools

## ■ BLAST

- Berkeley Lazy Abstraction Software Verification Tool
- Input: **C program** + requirement (BLAST Query Language)
- **Predicate abstraction**
  - Building abstract reachability tree (ART)
- Refinement: new predicate with interpolation
  - **Lazy abstraction**: apply new predicate locally
- Limitations: multiplication, bit operations, overflow
- [mtc.epfl.ch/software-tools/blast/index-epfl.php](http://mtc.epfl.ch/software-tools/blast/index-epfl.php)

# Tools

## ■ CPAchecker

- (Continuation of BLAST)
- The Configurable Software-Verification Platform
- Input: **C program** + specification
  - Assertion, error label, deadlock, null dereference, ...
- Highly **configurable**
  - Different kinds of abstractions (not only predicate)
  - Can consider multiple prefixes of a counterexample
    - Chooses from different refinements (refinement strategy)
- [cpachecker.sosy-lab.org/](http://cpachecker.sosy-lab.org/)

## ■ Theta

- Generic, modular, configurable model checking framework
- Developed at BME MIT
- **Generic**: various kinds of formal models
  - Transition systems, control flow automata, timed automata
- **Modular**: reusable and combinable modules
- **Configurable**: different algorithms and strategies
- [github.com/FTSRG/theta](https://github.com/FTSRG/theta)

# Tools

- **Competition** on Software Verification 2017 (SV-COMP)
  - [sv-comp.sosy-lab.org/2017/](http://sv-comp.sosy-lab.org/2017/)
  - **32 tools, 8908 input tasks** (program + requirement)
  - Categories: Help to find the best tool in a given category
    - Arrays (ArraysReach, ArraysMemSafety)
    - Bit Vectors (BitVectorsReach, Overflows)
    - Heap Data Structures (HeapReach, HeapMemSafety)
    - Floats
    - Integers and Control Flow (ControlFlow, Simple, ECA, Loops, Recursive, ProductLines, Sequentialized)
    - Termination
    - Concurrency
    - Software Systems (DeviceDriversLinux64, BusyBox)

# SUMMARY

# Summary

- Software model checking
  - Common problem: state space explosion
  - Solution: **abstraction**
    - Location + predicates
    - Properties of existential abstraction
  - **CEGAR**: automatically obtain proper abstraction
    1. Initial abstraction
    2. Model checking
    3. Examining the counterexample
    4. Refining the abstraction
  - **Tools**