Colored Petri nets (CPNs)

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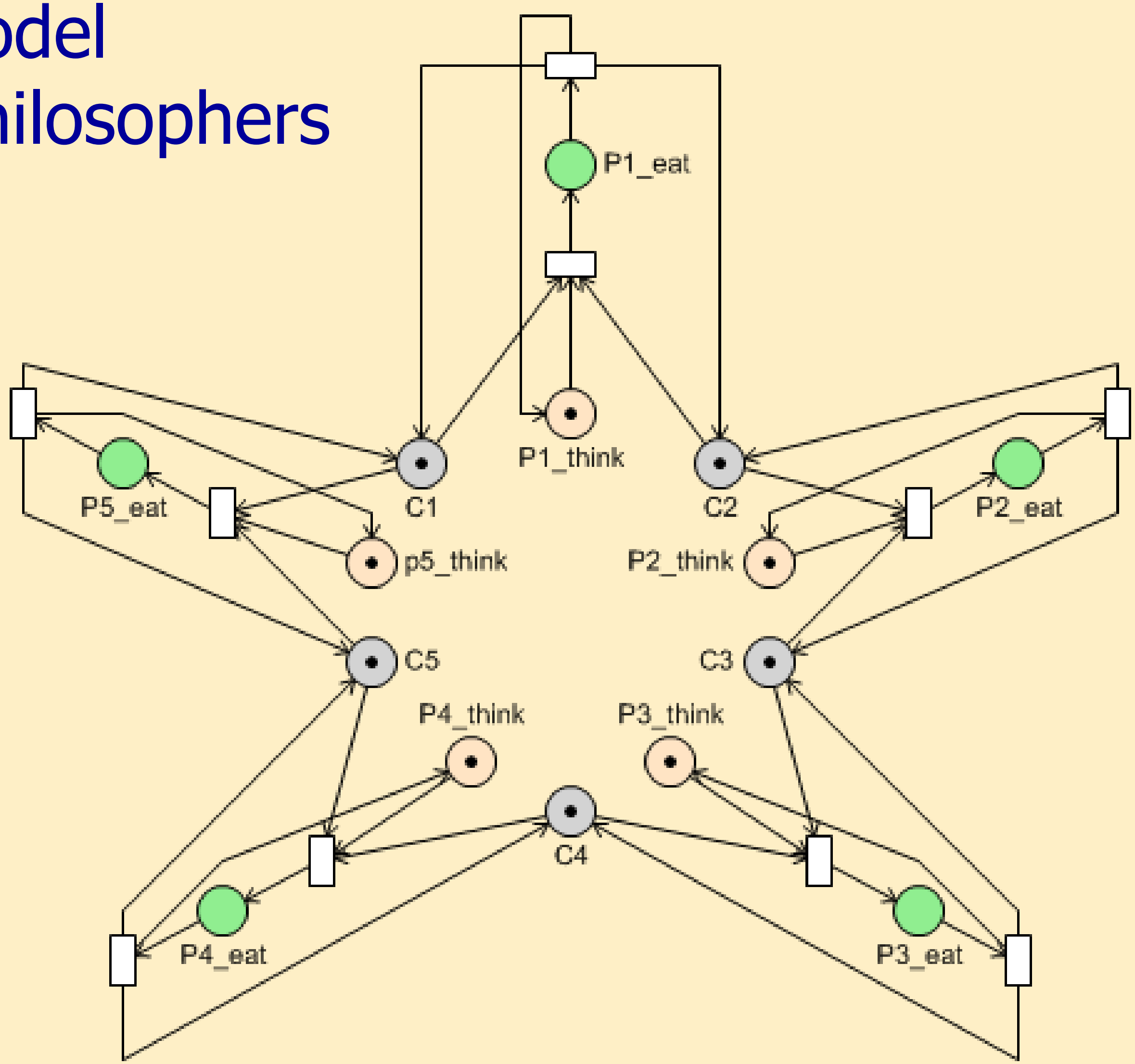
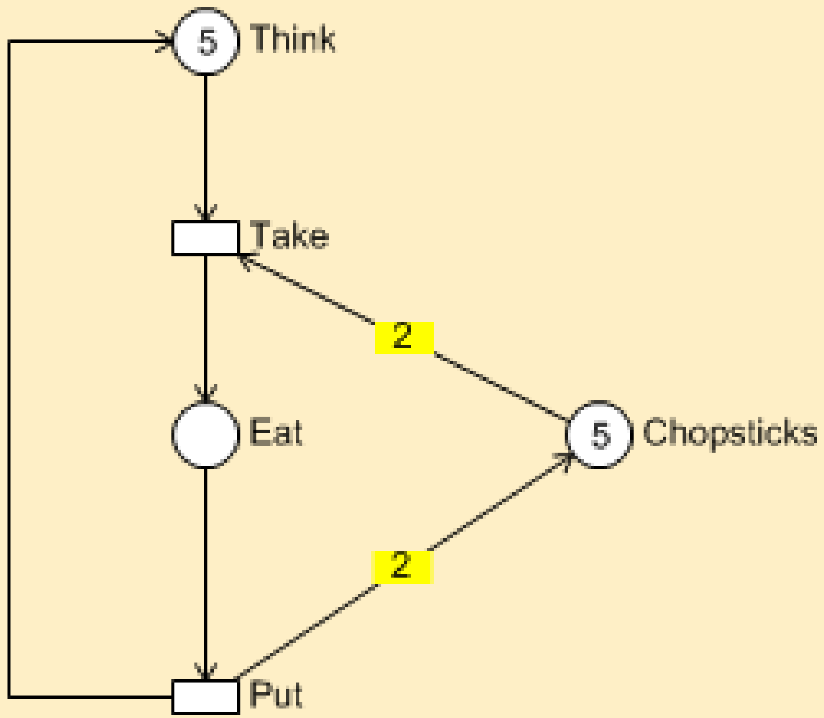
Motivation

- Petri net model
  of Dining Philosophers



Motivation

- Why not this way?

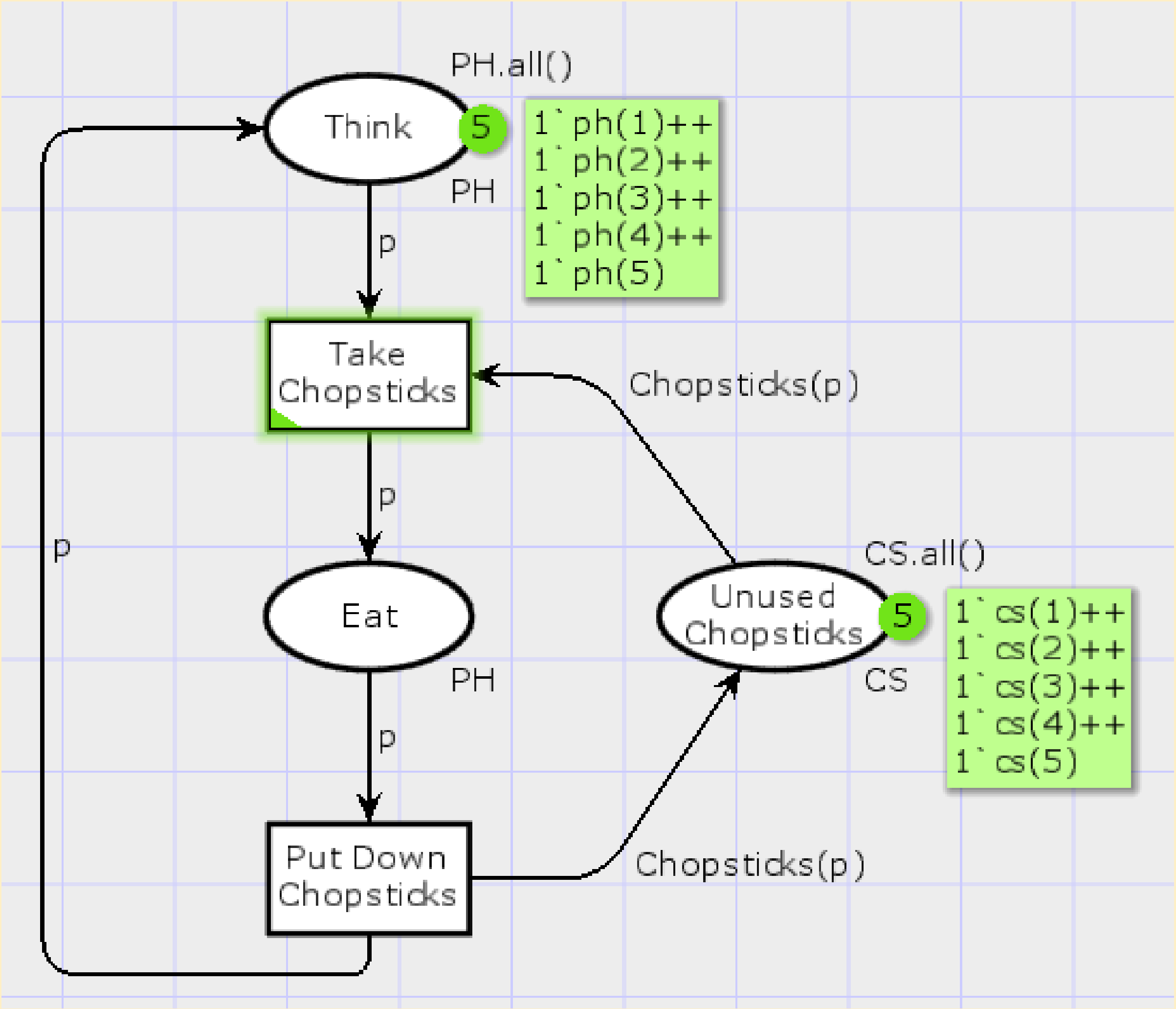


Motivation

- Distinction of tokens: colored Petri net

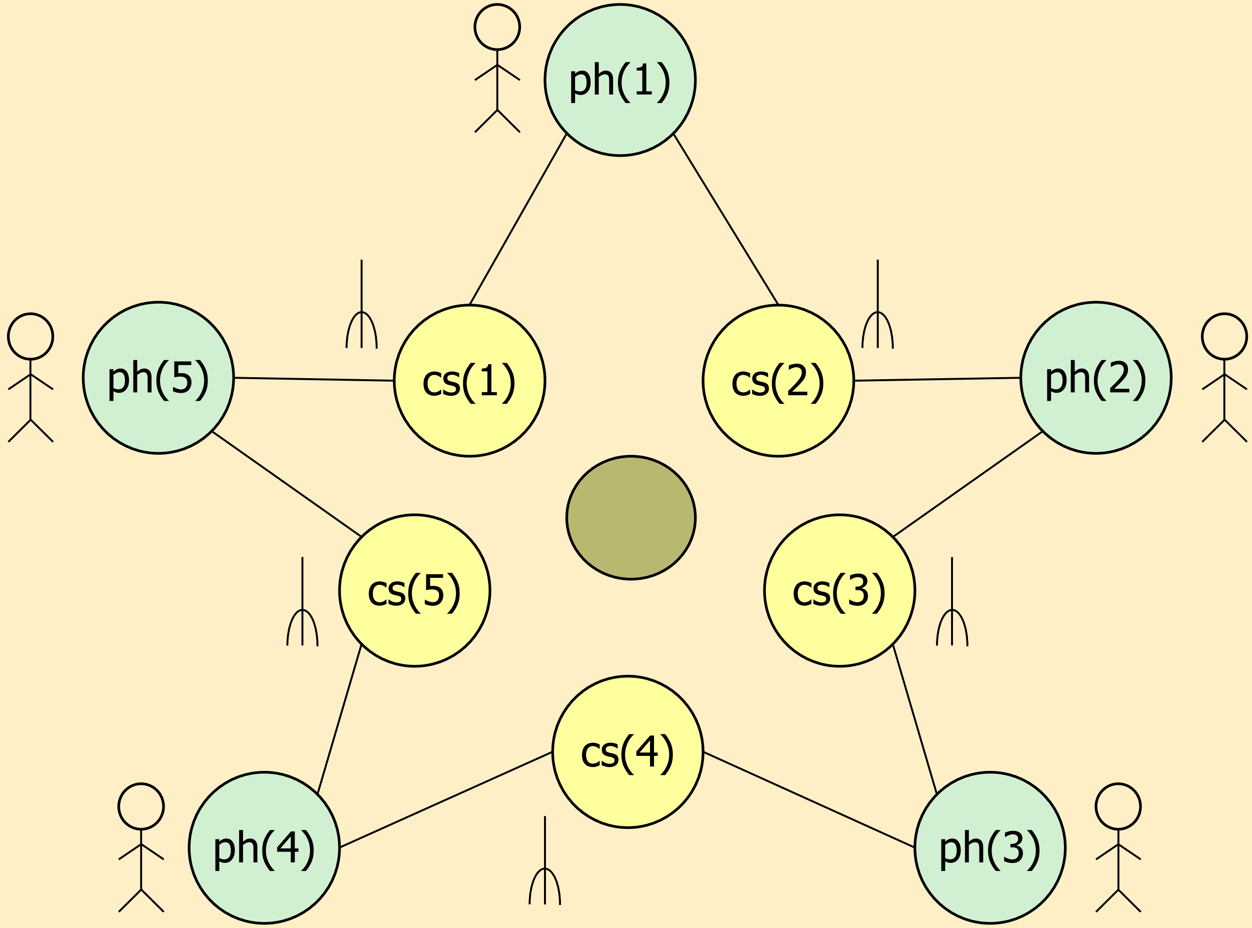
```
val n = 5;
colset PH = index ph with 1..n;
colset CS = index cs with 1..n;
var p: PH;

fun Chopsticks(ph(i)) =
  1`cs(i) ++
  1`cs(if i=n then 1 else i+1);
```

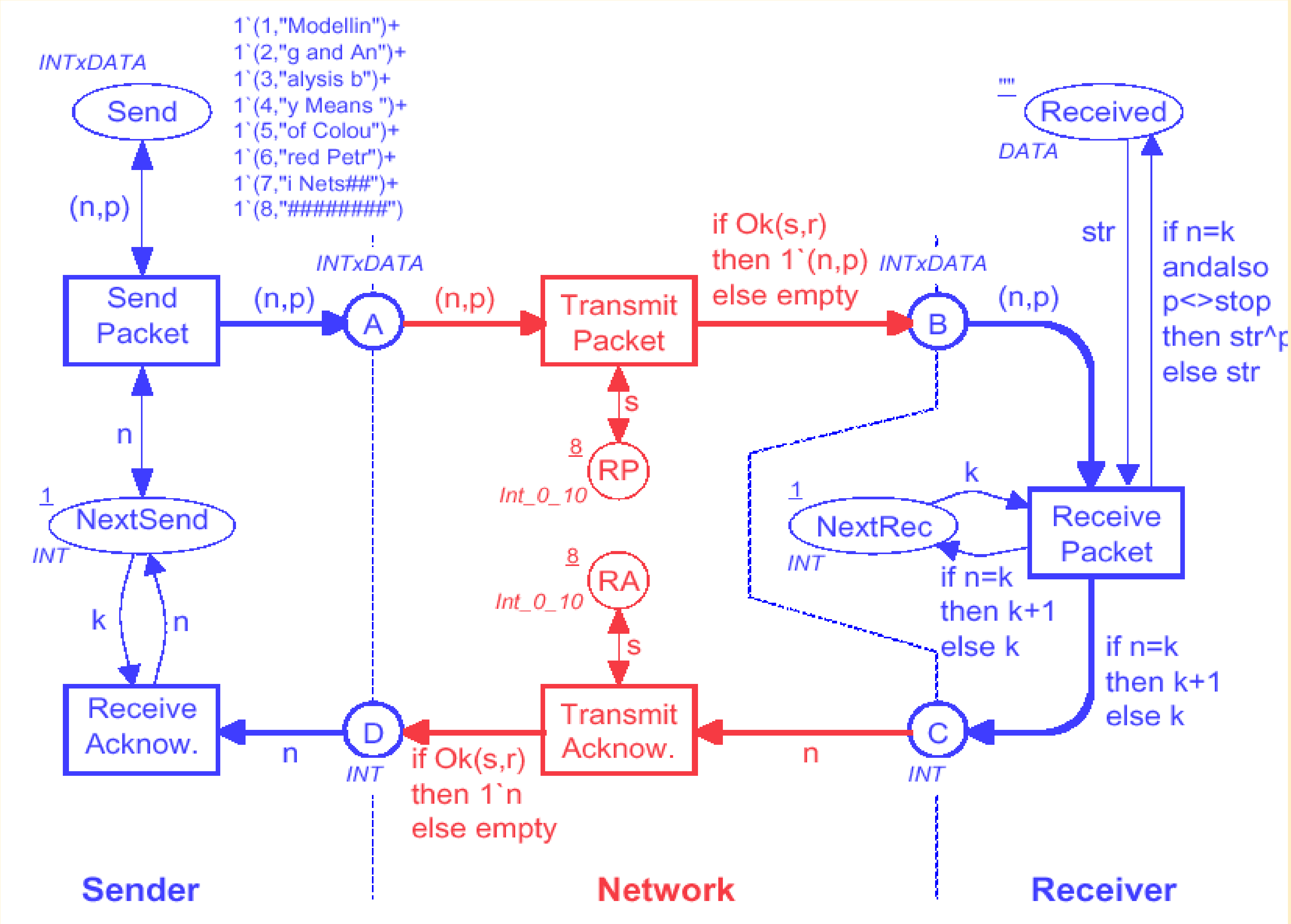


Motivation

- Meaning of colored tokens



A more complex example (see later)



Colored Petri nets

- Colored Petri net (CPN)
  - Extension of uncolored Petri nets with:
    - Flexible data structures
    - Data manipulation language
  - Colored Petri nets unite:
    - Graphical representation $\rightarrow$ clarity
    - Well-defined semantics $\rightarrow$ formal analysis
  - CPN model = net structure + declarations + net markings, expressions + initialization

Main components of CPNs (overview)

- Extensions of tokens
  - Data value: colored token
  - Data type: color set
- Extensions of places
  - Type of place: data type of accepted tokens
  - Initial marking inscription: initial tokens
  - Current marking: multiset of tokens matching the place’s type
- Extensions of arcs
  - Arc expression: tokens moved (with variables to be bound)
- Extensions of transitions
  - Guard for firing
  - To fire: arc expressions shall be bound to colored tokens

Comparison of colored and uncolored Petri nets

Uncolored (P-T) Petri nets:

- Uncolored tokens
- Set of tokens (cardinality)
- Token manipulation
- Initial marking
- Inhibitor edges
- Edge weights
- Transition can be enabled
- Conflict between different enabled transitions
- *~ assembly*

Colored Petri nets:

- Colored tokens
- Multiset of tokens
- Data manipulation
- Initial marking inscription
- Guards
- Arc expressions (+variables)
- Binding can be enabled
- Conflict between different bindings of the same transition
- *~ high-level programming lang.*

Structure of colored Petri nets

Extensions of tokens

- Colored token
  - Represents a data value
- Color set:
  - Defines the data type

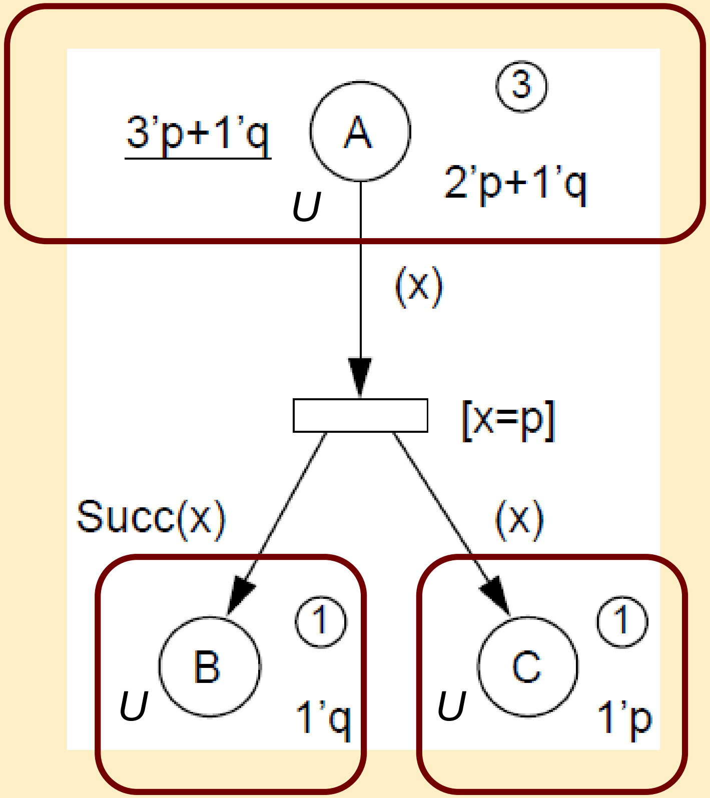
    E.g., enumeration (with),
    base type (int, bool, string, …)
  - Can be complex (compound)

    E.g., color P = product U * I
- Declaration: in formal language
  - Standard ML

```
color U = with p | q;
color I = int;
color P = product U * I;
color E = with e;
var x : U;
var i : I;
```

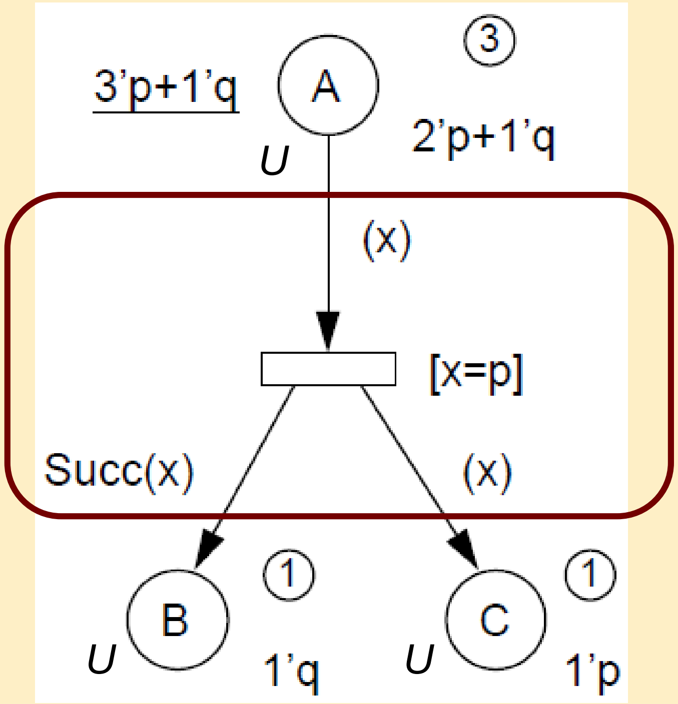
Extensions of PN places

- Color set inscription: type (color) of the place
  - Type of tokens accepted by the place (one of the declared types)
  - Visualization: written next to the place, in italic
- Initial marking inscription
  - Defines the initial marking
  - A multiset of the accepted color set (may be more than one token per color)
  - Visualization: written next to the place, underlined
- Current marking
  - Description of current tokens
  - Visualization: written next to the place, number of tokens in circle and detailed description



Extensions of PN transitions

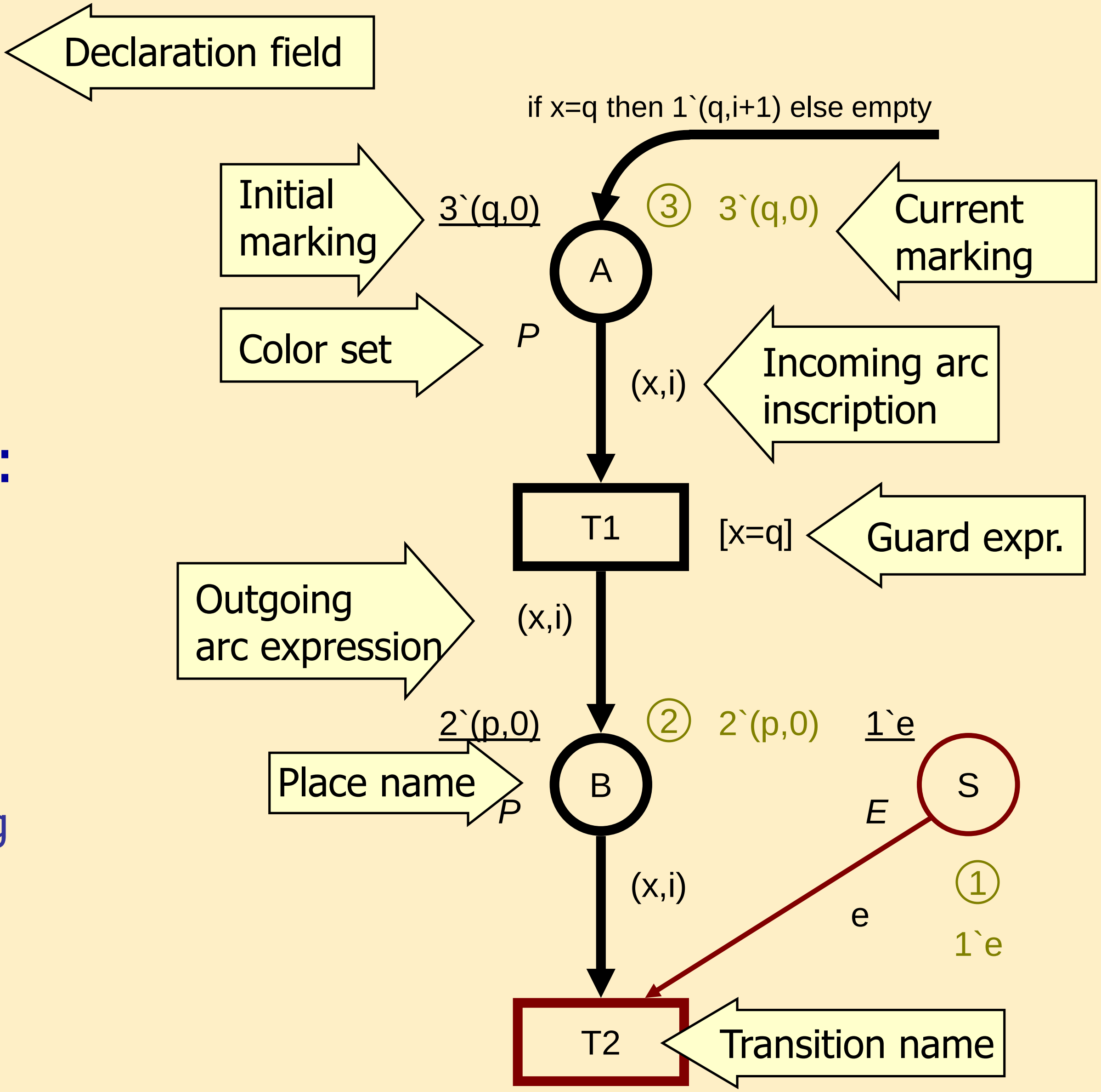
- Arc expression
  - Precondition of enablement (removed tokens) and the result of firing (placed tokens)
  - Type: type of the place connected to the arc (one transition have arcs with different types)
  - Visualization: next to the arc
- Variable can be used in the expression
  - Can be bound to data values (colored tokens)
  - Shall have a type (the color set of tokens that can be bound to it)
- Guard
  - Boolean expression, needs to be true to enable the transition
  - Visualization: next to the transition, within [ ]



Structure of colored Petri nets: Summary

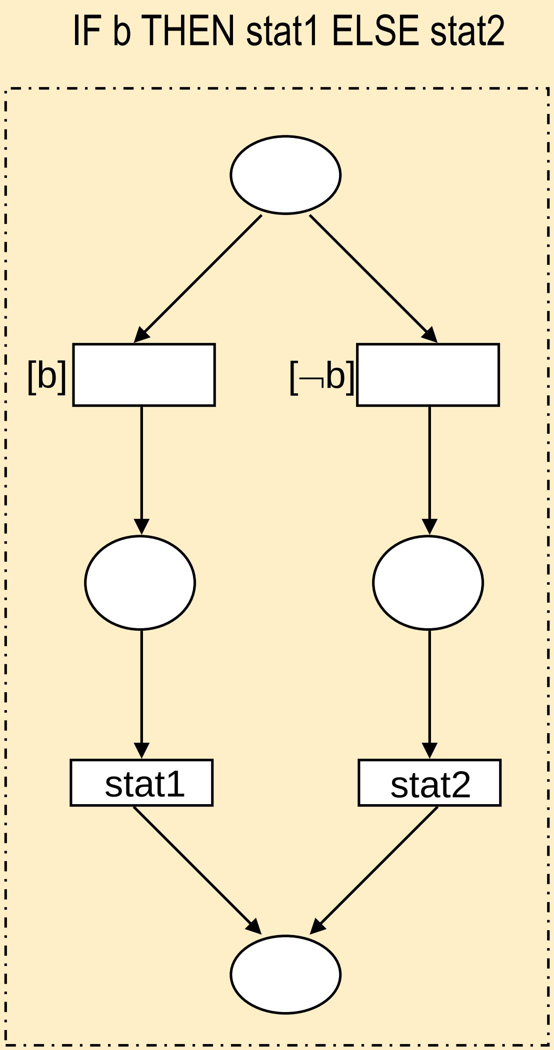
- Net structure:
  - Represents the control and data flow structure of the system
  - Places, transitions, arcs
- Declarations:
  - Define the data structures and used functions
  - Color sets, variables, arc expressions
- Markings, naming:
  - Define the syntactic and data manipulation items
  - Names, color sets, in/out arc expressions, guards, current state
- Initializing expression:
  - Defines the initial state of the model (constants)

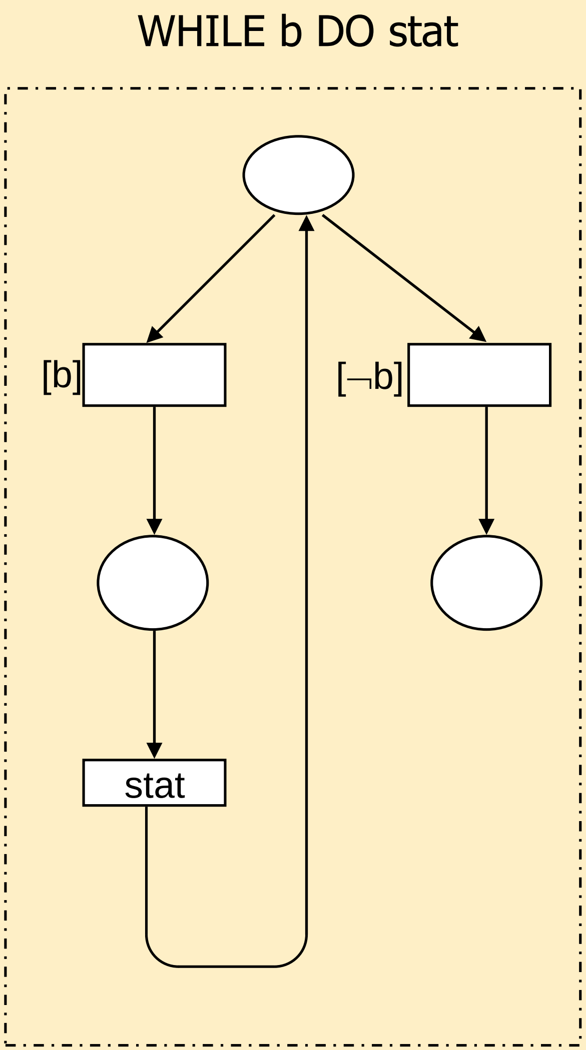
```
color U = with p | q;
color I = int;
color P = product U * I;
color E = with e;
var x : U;
var i : I;
```



- Elements of CPNs:
  - Places
    - Name
    - Color set
    - Initial marking
    - Current marking
  - Transitions
    - Name
    - Guard
  - Arcs
    - Arc expressions (incoming, outgoing)

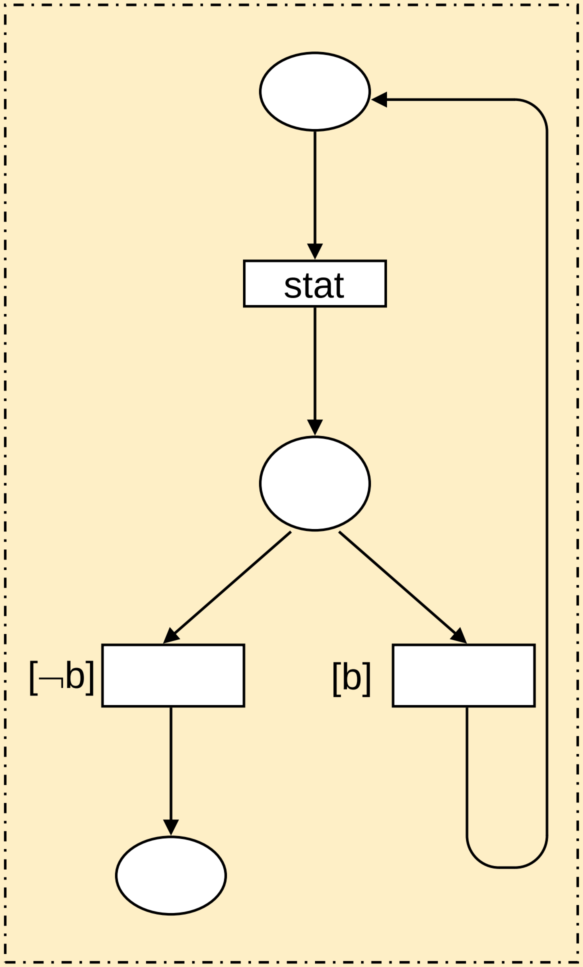
Example: Control structures 1



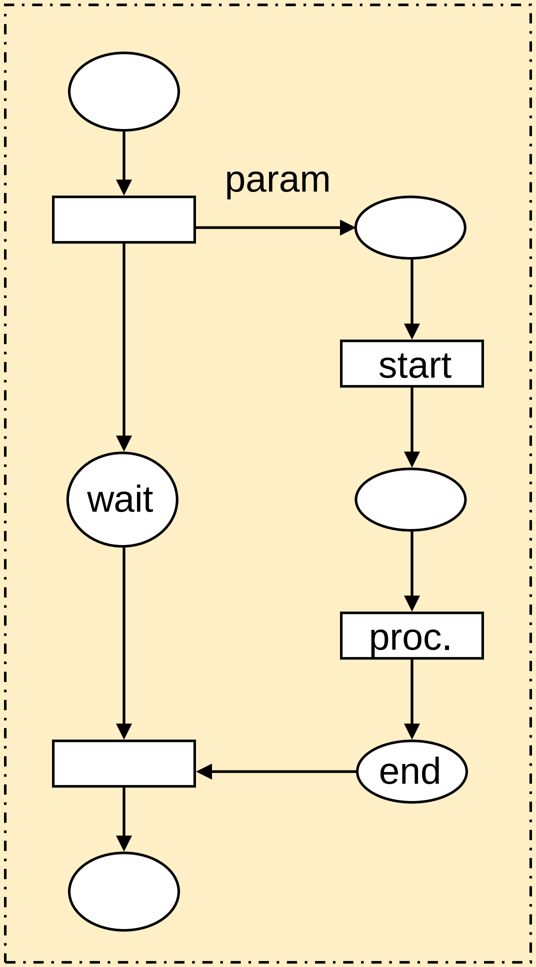


Example: Control structures 2

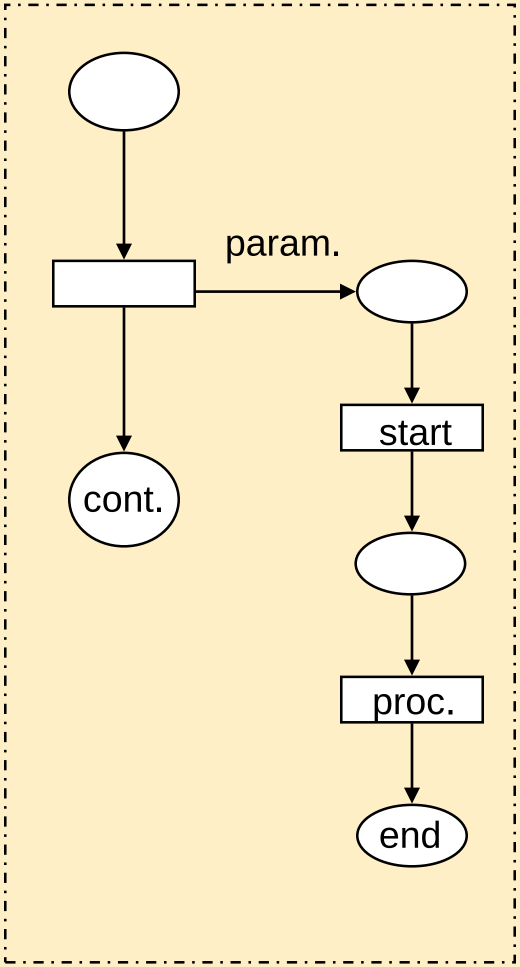
REPEAT stat UNTIL b



Subroutine call



Start of a process



Toolset of colored Petri nets

CPN: Definition of color sets

- Simple color sets
  - Uncolored tokens:
    `unit`
  - Base types:
    `int, bool, real, string`
  - Subset:
    `with 1..4;`
  - Enumeration:
    `with true | false;`
  - Indexing (vector):
    `index d with 1..4;`
- Can be used in the definitions of the following:
  - Compound color sets
  - Variables, constants
  - Functions, operators

Compound color sets

- Ways to create compound color sets:
  - Union:

    ```
    union S + T;
    ```

  - Cross product (construction of tuples):

    ```
    product P * Q * R;
    ```

  - Record (labelled tuples):

    ```
    record p:P * q:Q * r:R;
    ```

  - List:

    ```
    list int with 2..6;
    ```

Additional CPN elements: Variables

- Variables

  Symbolic names of tokens

  – Variable declaration:

  ```
  var proc : P;
  ```

- Constants

  With fixed values

  – Constant declaration:

  ```
  val n = 10;
  val d1 = d(1):D;
  ```

- In the following expr.'s:
  – Arc expressions
  – Guards

- In the following decl.'s:
  – Color sets
  – Functions, operators
  – Arc expressions, guards, initialization expressions

Additional CPN elements: Functions

- Functions

  Side effect-free functions in SML language

  - Example:

```
fun Chopsticks(ph(i)) =
   1`cs(i) ++
   1`cs(if i=n then 1 else i+1);
```

- Operations, operators

  Infix notation

- In the following decl.'s:
  - Color sets
  - Functions, operators, constants
  - Arc expressions, guards, initialization expressions

Additional CPN elements: Expressions

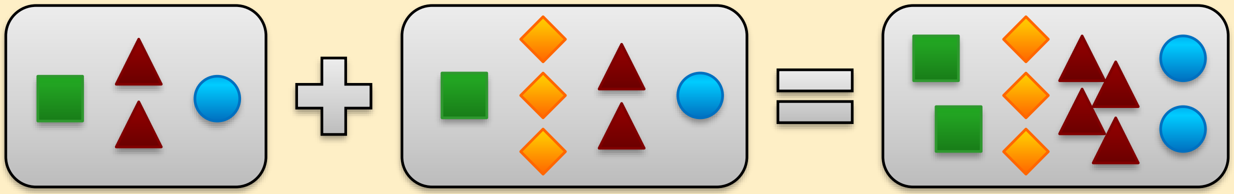
- Net expressions
  - Value: evaluated with a specific binding of the variables
  - Type: set of all possible evaluations
  - Examples:

```
x=q
2`(x,i)
if x=q then 2`i else empty
Mes(s)
```

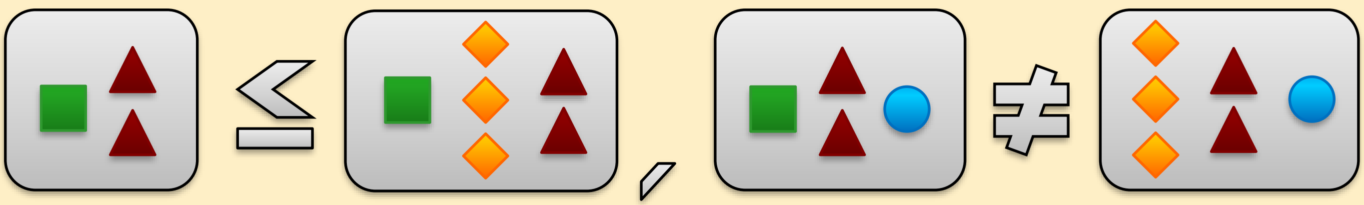
- Usage in:
  - Arc expressions, guards, initialization expressions

Expressions: Operations with multisets

Addition: $a_1 + a_2$

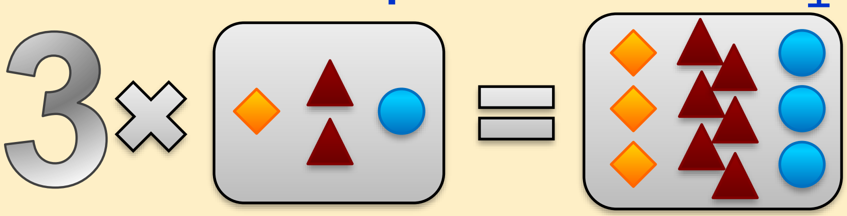


Comparison: $a_1 \leq a_2$, $a_1 \neq a_2$

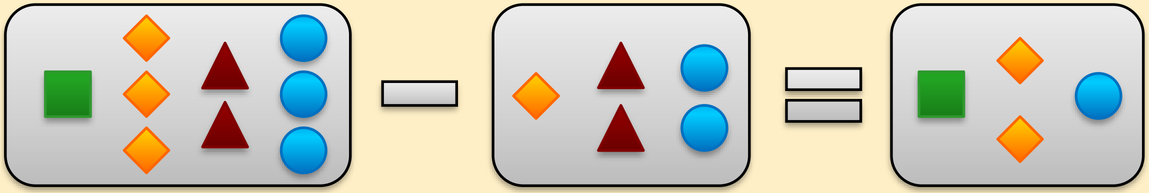


Size: $|a_1|$

Scalar multiplication: $n{\cdot}a_1$



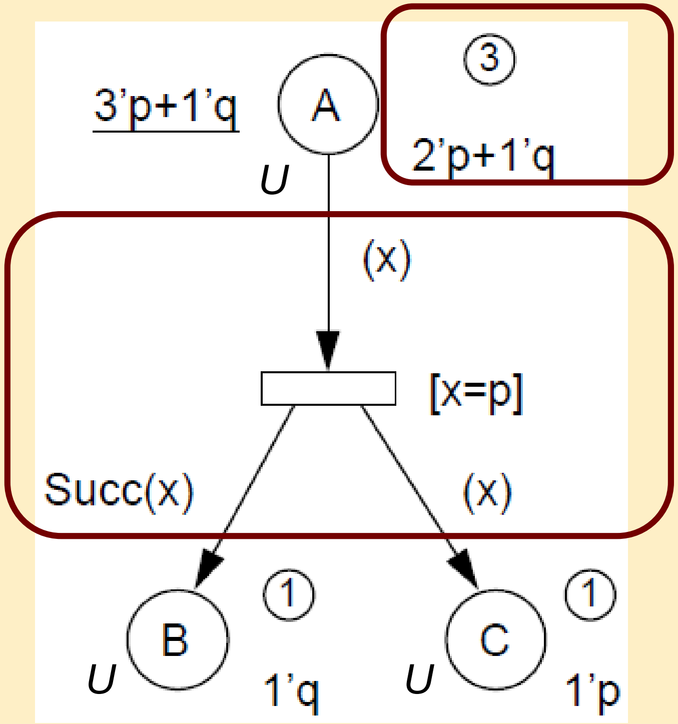
Subtraction: a_1 - a_2 (only if $a_2 \leq a_1$)



Behavior of colored Petri nets (informal semantic)

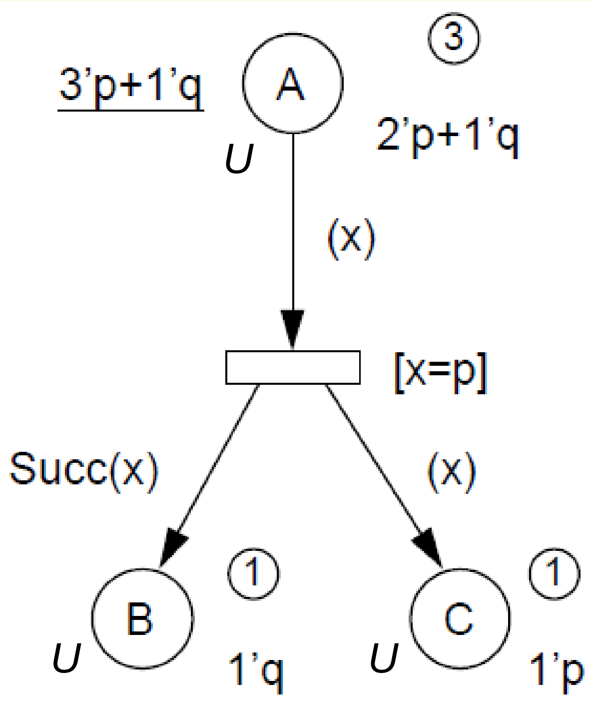
Marking and binding

- Marking:
  - Distribution of tokens (count, by color) on the places
- Binding the arc expressions of a transition:
  - The variables are bound to data values (colored tokens)
  - For a given transition each occurrence of a variable will be bound to the same value
  - Unbound variable on outgoing arc: Can be bound to any value of its type
  - The bindings of different transitions are independent



Enabling of transitions

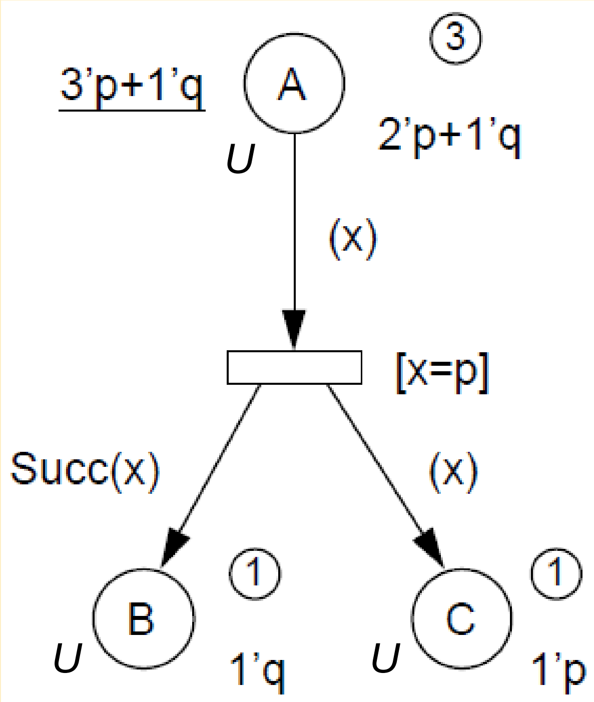
- Transition enabled with a given marking and binding:
  - Each input arc's expression evaluates to a multiset of tokens that is present on the corresponding input place
  - The guard is true
  - If a transition is enabled with a binding, it can fire



- Binding item for firing:
  - A pair (transition, binding), e.g., (T1, <x=p>)
  - Can be enabled with a marking → can fire
  - In case of one transition: many bindings, many enabled binding items may be constructed; they can fire

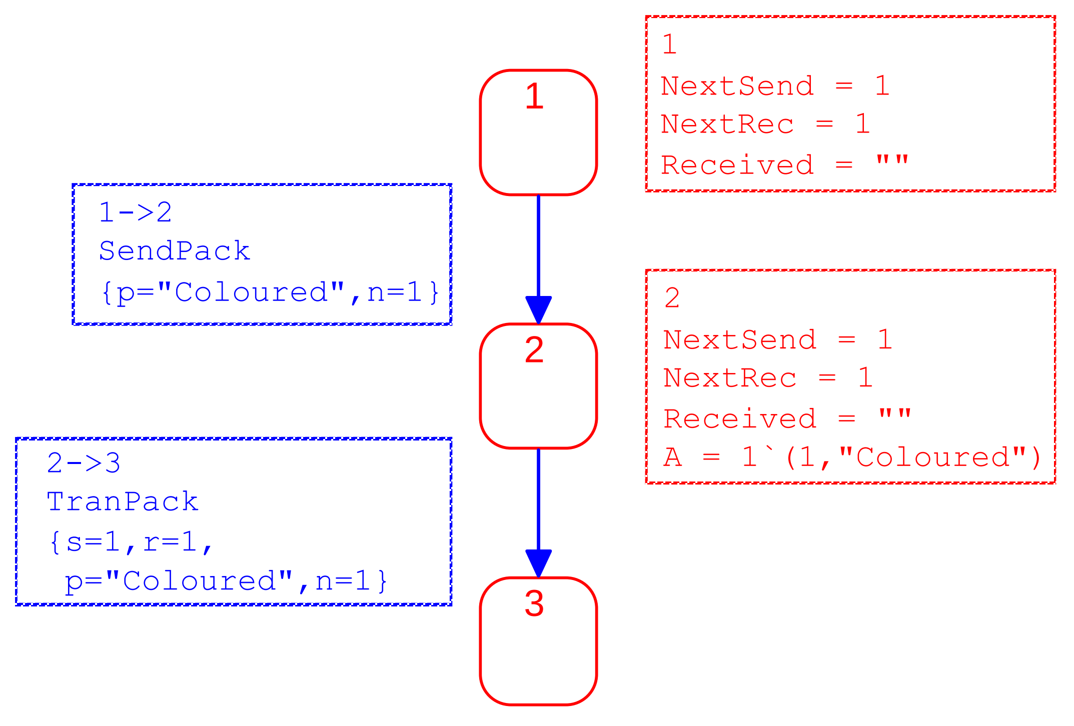
Firing

- Transition fires with a binding (i.e., a binding item fires):
  - Removes tokens from the input places according to the arc expressions and the firing binding
  - Adds tokens from the output places according to the arc expressions and the firing binding
- Step (effect of firing on the state space):
  - The marking of the CPN changes



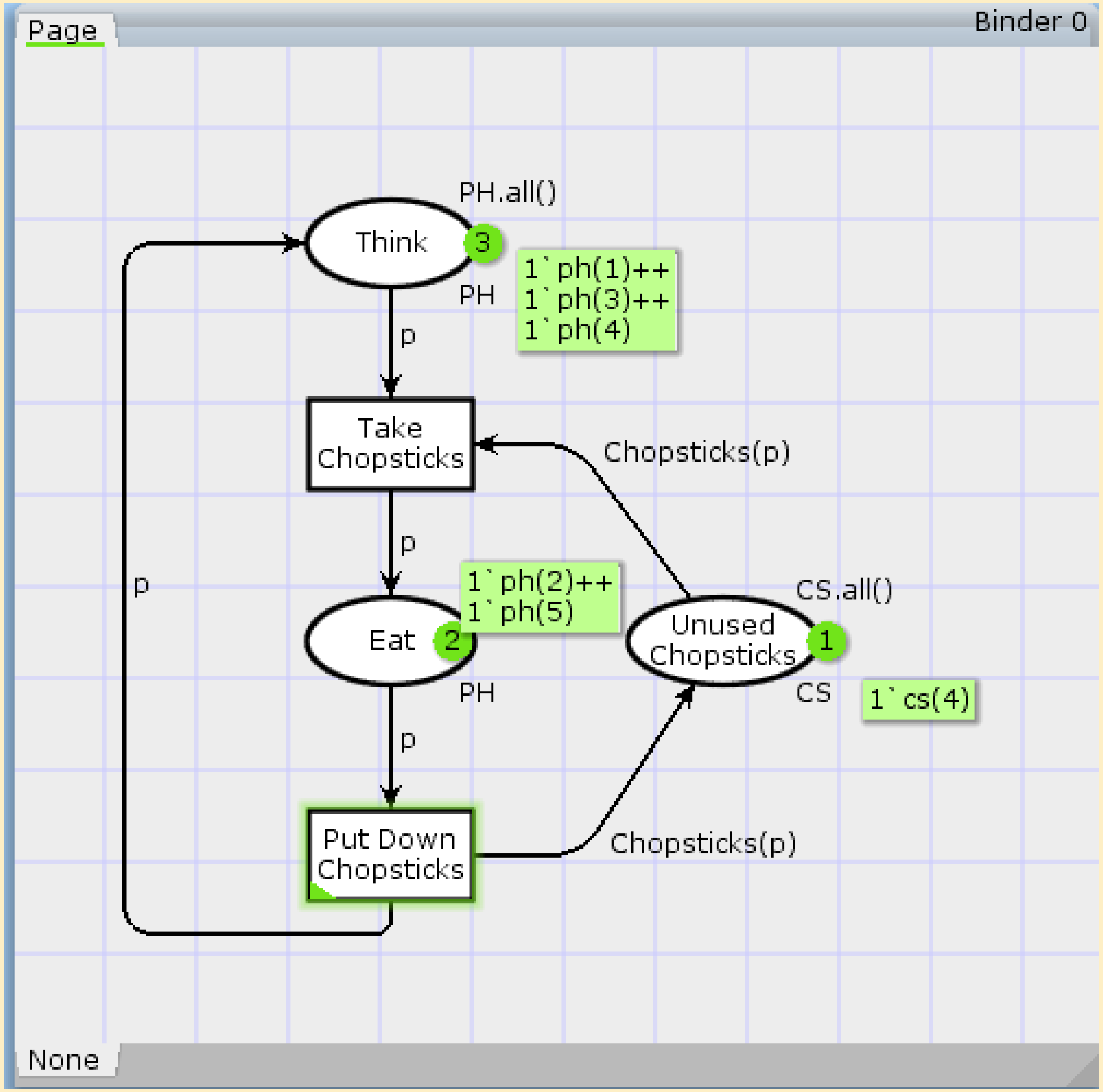
Reachability graph

- Node:
  - A marking: count and color of tokens for each place
  - May have an ID, predecessor node and successor node
- Edge:
  - The firing binding item: the transition and the binding
  - By definition only one firing binding item is shown in the reachability graph



CPN Tools demo

- Model of dining philosophers
- Simulation
- Reachability graph



Formal definition and semantics of colored Petri nets

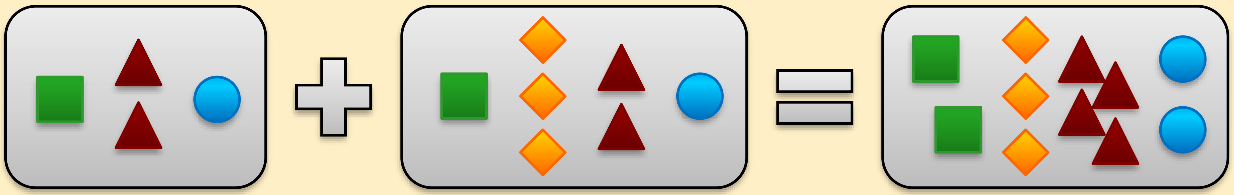
Multisets

- Multiset: may contain several of the same element
  - Mapping: $Bag(A)$, to the domain of A, $a \in [A \to \mathbf{N}]$
  - Formally: $a = \sum_{x \in A} a(x) \cdot x$, alternative notation: $a = \sum_{x \in A} a(x)'x$
- Operations on multisets:
  - Comparison: $a_2 \neq a_1$ if $\exists x \in A, a_2(x) \neq a_1(x)$

    $a_2 \leq a_1$ if $\forall x \in A, a_2(x) \leq a_1(x)$
  - Size: $|a| = \sum_{x \in A} a(x)$
  - Addition: $a_1 + a_2 = \sum_{x \in A} \big(a_1(x) + a_2(x)\big) \cdot x$
  - Subtraction: $a_1 - a_2 = \sum_{x \in A} \big(a_1(x) - a_2(x)\big) \cdot x$ if $a_2 \leq a_1$
  - Scalar multiplication: $n \cdot a = \sum_{x \in A} \big(n \cdot a(x)\big) \cdot x$

Operations with multisets

Addition: $a_1 + a_2$



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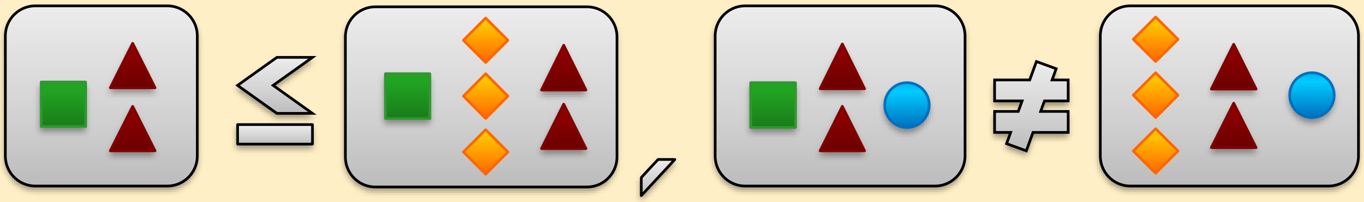
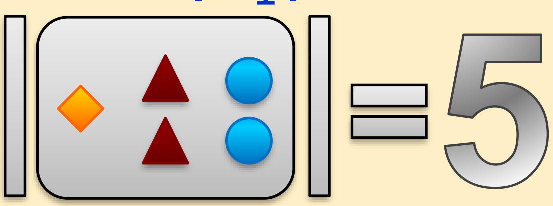


Size: $|a_1|$



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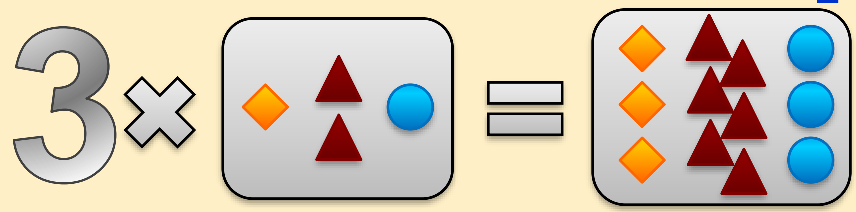

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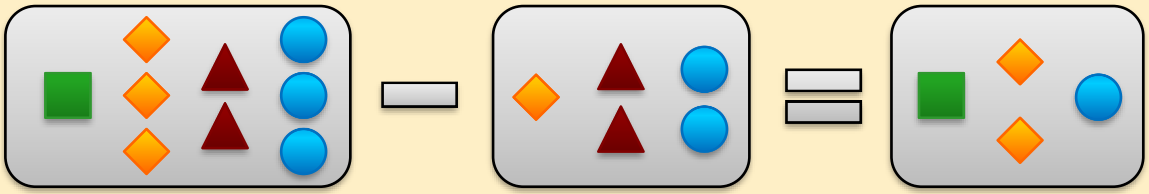


Multisets (continued)

- Union of multisets: $a_1 \cup a_2 \cup \ldots \cup a_m$
  - Domain: $A_1 \cup A_2 \cup \ldots \cup A_m$
  - Item: $e_i \in \bigcup_1^m A_k$ if $\exists A_j, e_i \in A_j$
- Construction of tuples: $\langle A_1, A_2, \ldots, A_n \rangle$
  - Domain: $A_1 \times A_2 \times \ldots \times A_2$
  - Item: $\left\langle e_1, e_2, \ldots, e_n \right\rangle \in \Diamond_1^n A_j$ if $\forall e_i \in A_i$
  - Generalization: $\langle a_1, a_2, \ldots, a_n \rangle$

Formal definition of CPNs

$$\text{CPN} = (\Sigma, P, T, A, C, G, E, M_0)$$

Color sets: $\Sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_\kappa\}$

Places: $P = \{p_1, p_2, \ldots, p_\pi\}$

Transitions: $T = \{t_1, t_2, \ldots, t_\tau\}$

$P \cap T = \varnothing$

Arcs: $A \subseteq (P \times T) \cup (T \times P)$

Color set func.: $C : P \mapsto \Sigma$

Guards: $G : \forall t \in T, \left[\text{Type}\big(G(t)\big) = \text{B} \wedge \text{Type}\Big(\text{Var}\big(G(t)\big)\Big) \subseteq \Sigma \right]$

Arc expressions: $E : \forall a \in A, \left[\text{Type}\big(E(a)\big) = C(p)_{\text{MS}} \wedge \text{Type}\Big(\text{Var}\big(E(a)\big)\Big) \subseteq \Sigma \right]$

Initial marking: $M_0 : \forall p \in P, \left[\text{Type}\big(M_0(p)\big) = C(p)_{\text{MS}} \right]$

Notations used in the formal definition

- The type (color set) of variable v: $\mathrm{Type}(v)$
- The type of expression $expr$: $\mathrm{Type}(expr)$
- The set of variables in expression $expr$: $\mathrm{Var}(expr)$
- A binding of variable v: $b(v) \in \mathrm{Type}(v)$
- Evaluation (value) of expression $expr$ in binding b: $expr\langle b\rangle$

  where $v \in \mathrm{Var}(expr)$ and $b(v) \in \mathrm{Type}(v)$

Arc expressions

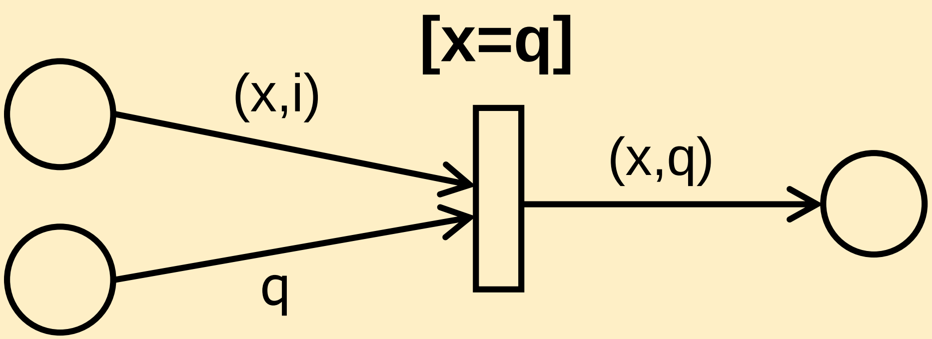
- May use variables
  - Variables have types (color sets): $\mathrm{Type}(v)$
  - Their value is an element of their types' multiset
- Closed arc expression: does not contain variables
- Open arc expression: contains variables that have to be bound to values
  - Binding: a specific value assignment to each variable
    - Arc expression can be evaluated with the given binding
  - Has type: $\mathrm{Type}(expr) = C(p)_{\mathrm{MS}}$
    - The color set (type) to which it is evaluated
  - Set of variables in the expression: $\mathrm{Var}(expr)$

Bound and unbound variables

- Bound variables
  - Value binding is determined by the incoming arcs
  - Consistency: a variable has only one value in each binding
    - For all in-arcs of the transition the same variable name denotes the same value
- Unbound variables
  - They can only be present in outgoing arc expressions
  - Enablement did not assign (bound) any value to them
  - Have to be bound at firing:
    - Can take any value from its color set
    - Number of possible bindings = cardinality of the color set
    - Non-deterministic choice

Guards

- Each guard is assigned to a transition
  - Expression over multisets
  - Evaluated to Boolean value
- The transition is enabled only if the guard is evaluated to “true”
  - “Filters” the enabled bindings



Enabling in colored Petri nets

- Binding of transitions
  - Valid binding: $\forall v \in \mathrm{Var}(t)\text{: } \mathrm{b}(v) \in \mathrm{Type}(v) \wedge \mathrm{G}(t)\langle b\rangle$

$$\mathrm{Var}(t) = \left\{ v \mid v \in \mathrm{Var}\left(\mathrm{G}(t)\right) \vee \exists a \in A(t) : v \in \mathrm{Var}\left(E(a)\right) \right\}$$

  - Set of all valid bindings: $\mathrm{B}(t)$
- A valid binding is enabled if
  - Guard is true
  - The input places contain enough colored tokens (cf. arc expressions $E^-(p,t)\langle b\rangle$) and the inhibitor arcs do not inhibit the firing (cf. arc expressions $E^h(p,t)\langle b\rangle$):

$$\forall p \in \bullet t : E^{-}(p,t)\langle b\rangle \leq M(p) \wedge E^{h}(p,t)\langle b\rangle > M(p)$$

Firing in colored Petri nets

- An enabled transition can fire if there is no enabled transition with higher priority, i.e.
  - The transitions with higher priority do not have enough tokens in their input places (see arc expressions $E^{-}(p,t')<b'>$) or their inhibitor arcs disable the firing (see arc expressions $E^{h}(p,t')<b'>$),

$$\forall t', \pi(t') > \pi(t) : \exists p \in \bullet t' :$$

$$E^{-}(p,t')\langle b' \rangle > M(p) \vee E^{h}(p,t')\langle b' \rangle \leq M(p)$$

  - Or their guards are not satisfied (not evaluated to true)

$$\neg G(t')\langle b' \rangle$$

Firing in colored Petri nets

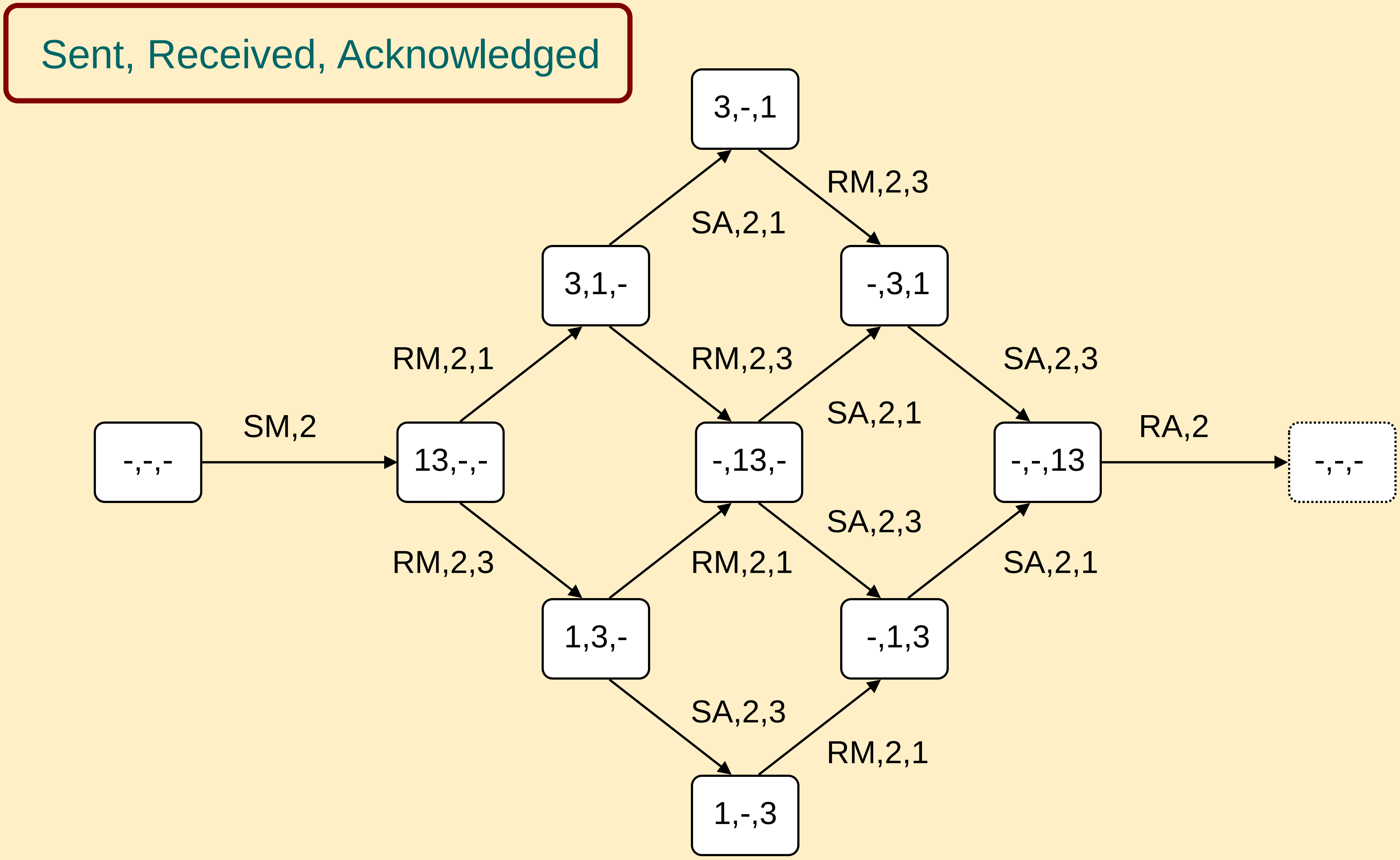
- Steps of firing:
  - Finding enabled bindings
    - Determined by incoming arc expressions and guards
  - Transition enabled with a given binding → it can fire
  - Firing: removal of colored tokens from incoming places, adding colored tokens to outgoing places

$$\forall p \in P : M'(p) = M(p) - \sum_{p \in \bullet t} E^{-}(p,t)\langle b \rangle + \sum_{p \in t \bullet} E^{+}(t,p)\langle b \rangle$$

  - Then M' directly reachable from M: $M \; [(t,b)\rangle \; M'$

Dynamic properties of colored Petri nets

Reachability graph (excerpt)



Dynamic properties of CPNs

- Extension of the uncolored Petri net properties to multisets
- Boundedness

  A place is bounded if the number of tokens in any state is bounded

  - n is an upper integer bound for p if $\forall M \in \left[M_0 \right\rangle : \ \left| M(p) \right| < n$
  - m is an upper multiset bound for p if $\forall M \in \left[M_0 \right\rangle : \ M(p) < m$

- Reversibility (home state)

  It is always possible to get back to a home state

  - M is a home state if $\forall M' \in \left[M_0 \right\rangle : \ M \in \left[M' \right\rangle$
  - X is a home group if $\forall M' \in \left[M_0 \right\rangle : \ X \cap \left[M' \right\rangle \neq \emptyset$

Dynamic properties of CPNs

- Liveness

  Liveness guarantees that some of the binding items remain active

  - Dead state (deadlock): no binding item is enabled

    $$\forall b \in BE: \quad \neg M\left[b\right\rangle$$

  - Dead transition: none of its bindings may become enabled

    $$\forall M' \in \left[M\right\rangle, b \in B(t): \quad \neg M'\left[b\right\rangle$$

  - Live transition: from each reachable state there is at least one trajectory starting where the transition is not dead (at least one binding will become active)

    $$\forall M' \in \left[M_0\right\rangle, \quad \exists M'' \in \left[M'\right\rangle, \exists b \in B(t): \quad M''\left[b\right\rangle$$

Dynamic properties of CPNs

- Fairness

  Fairness represents how often can a binding item fire

  – Impartial transition: fires infinitely often

  $$\forall b \in B(t),\ |\sigma| = \infty : \quad \mathrm{OC}_b(\sigma) = \infty$$

  – Fair transition: infinitely many enabling $\Rightarrow$ infinitely many firing

  $$\forall b \in B(t),\ |\sigma| = \infty : \quad \mathrm{EN}_b(\sigma) = \infty \Rightarrow \mathrm{OC}_b(\sigma) = \infty$$

  – Just transition: persistent enabling $\Rightarrow$ firing

  (there is no persistent enabling without firing)

  $$\forall b \in B(t), \forall i \geq 1 :$$

  $$\left[\mathrm{EN}_{b,i}(\sigma) \neq 0 \Rightarrow \exists k \geq i : \left[\mathrm{EN}_{b,k}(\sigma) = 0 \vee \mathrm{OC}_{b,k}(\sigma) \neq 0 \right] \right]$$

Structural properties of colored Petri nets

T invariant in CPNs

- Transition invariant

  A firing sequence σ that does not affect the state:

$$M'(p) = M(p) - \sum_{p \in \bullet t, b \in \sigma} E^{-}(p,t)\langle b\rangle + \sum_{p \in t\bullet, b \in \sigma} E^{+}(t,p)\langle b\rangle$$

where $M'(p) - M(p) = 0$ for all p

then $\displaystyle\sum_{p \in \bullet t, b \in \sigma} E^{-}(p,t)\langle b\rangle = \sum_{p \in t\bullet, b \in \sigma} E^{+}(t,p)\langle b\rangle$

P invariant in CPNs

- Place invariant

  Idea: Equation that is satisfied in every reachable state

  – Weighted token sum is constant:

  $$\mathrm{W}_{p_1}\left(M(p_1)\right)+\mathrm{W}_{p_2}\left(M(p_2)\right)+\ldots\mathrm{W}_{p_n}\left(M(p_n)\right)=\mathrm{m}_{\mathrm{inv}}$$

  – Weight function: maps the color sets of the places to a common multiset

  – W_P is a P invariant:

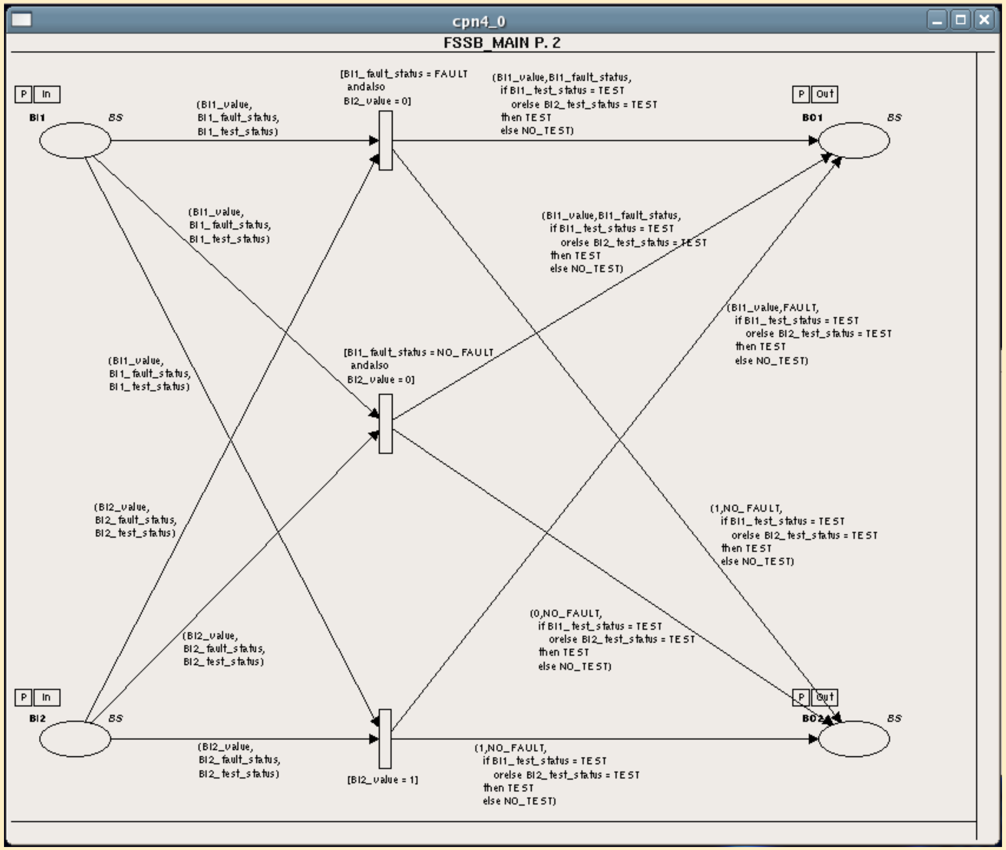
  $$\forall M\in\left[M_0\right\rangle:\quad\sum_{p\in P}W_p\left(M(p)\right)=\sum_{p\in P}W_p\left(M_0(p)\right)$$

Unfolding colored Petri nets

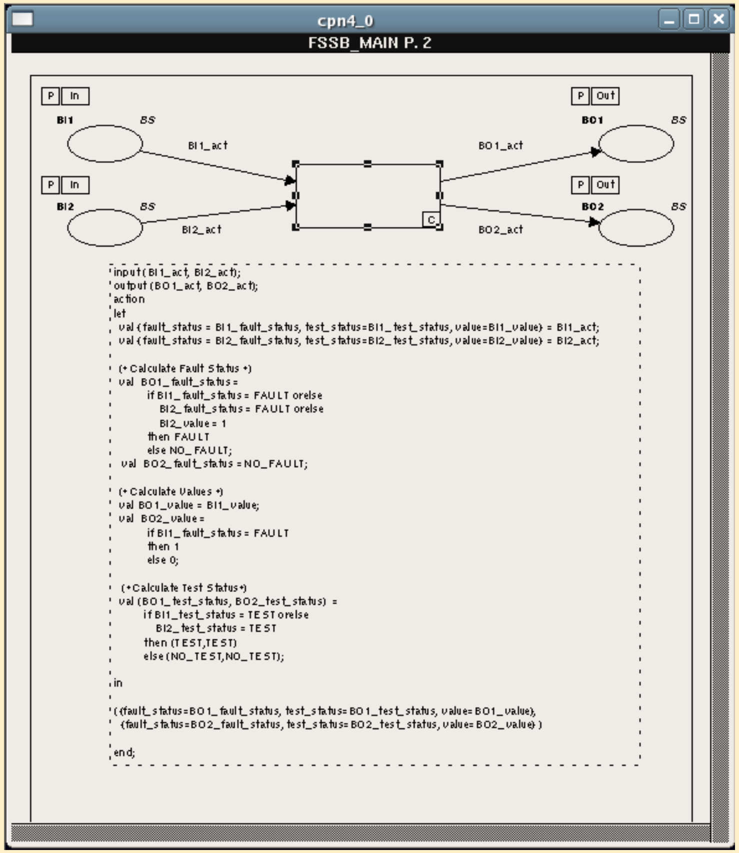
Possibilities to construct a CPN

- CPNs: information in both structure and data
- Extremities
  - Pure structural information, no data:
    - Uncolored (P/T) net (can be build as a CPN)
  - No structure, only data (data and control information):
    - 1 place + 1 transition, complex color sets and arc expressions
- We need the golden mean
  - To have a clean, readable CPN

Example: Modeling possibilities



Control flow expressed by the structure

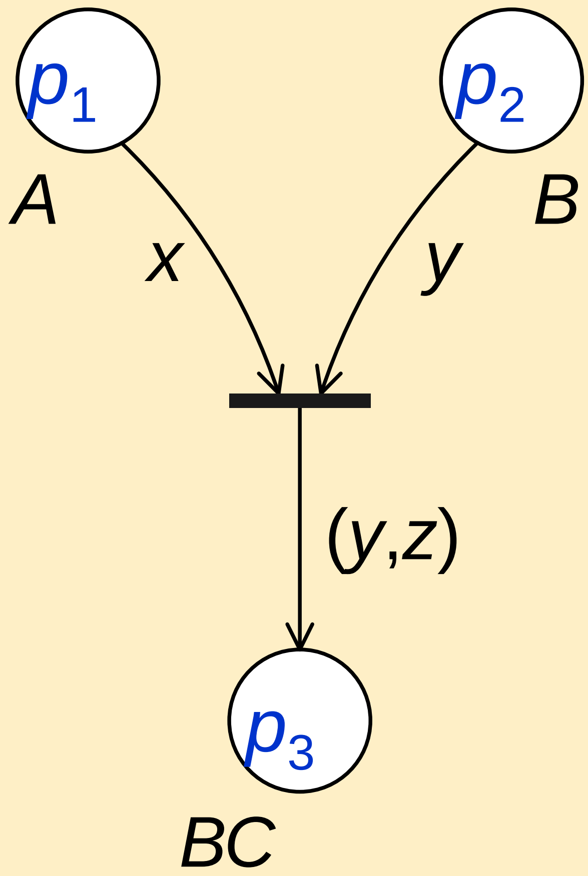


The same in code
("folded")

Unfolding

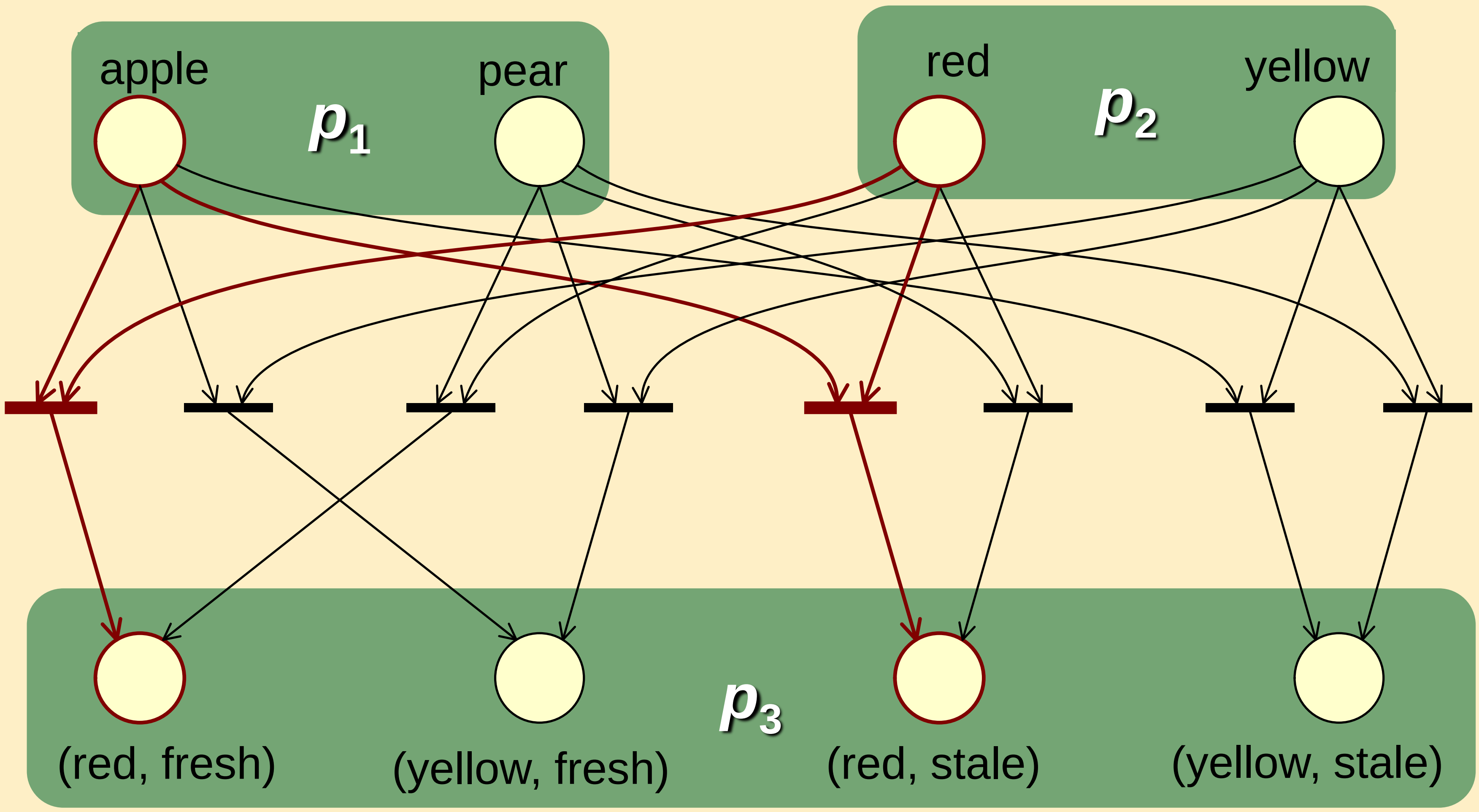
- Expressivity of CPNs (with priorities) equals to the expressivity of uncoloured PNs with inhibitor edges (and with priorities)
  - Each CPN has a corresponding uncolored PN with equivalent behavior (in the automaton theoretical sense $\rightarrow$ bisimulation for the steps)
  - Equivalent uncolored net: unfolded net
  - Unfolding:
    - Information of colored tokens is represented by the structure
    - Each event of the CPN has <u>exactly one</u> corresponding event in the unfolded net

Simple colored net




```
color A = with apple | pear;
color B = with red | yellow;
color C = with fresh | stale;
color BC = product B*C declare mult;
var x: A;
var y: B;
var z: C;
```

Unfolded, uncolored net



Example: A simple commit protocol

Problem description:

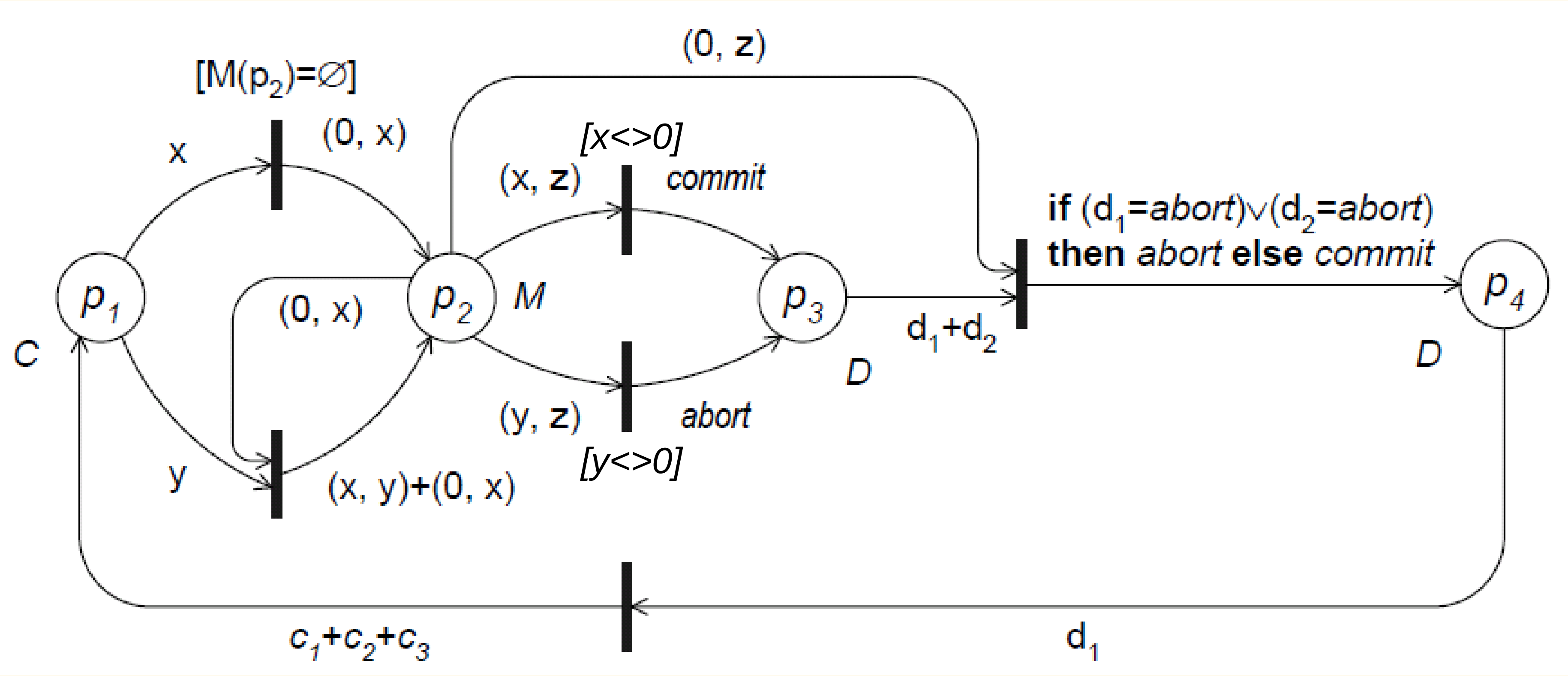
- The system consists of three components: c_1, c_2 és c_3
- One of them randomly becomes the coordinator which sends a request to the other two
- The response of another component is either an abort or commit vote
- Based on the vote of the two components the coordinator decides: the decision is commit if the two other components voted for commit, abort otherwise.

Example: Model of the simple commit protocol

- Three color sets are defined in the CPN model.
  Two of them are simple color sets:
  $C = \{0, c_1, c_2, c_3\}$ representing components,
  $D = \{\text{commit, abort}\}$ representing votes/decisions.
  One compound color set:
  $M = C \times C$ for requests (originator and target);
  the $(0, x)$-like token represents that
  the coordinator does not receive a request
- Five variables are used, their types: $x, y, z \in C$;
  and $d1, d2 \in D$
- The if in the arc expression has the common intuitive meaning (as in programming languages)
- In the initial state the place p_1 has 3 tokens:
  $M(p_1)=c_1+c_2+c_3$, the other places are empty
- Empty set is denoted by $\varnothing$

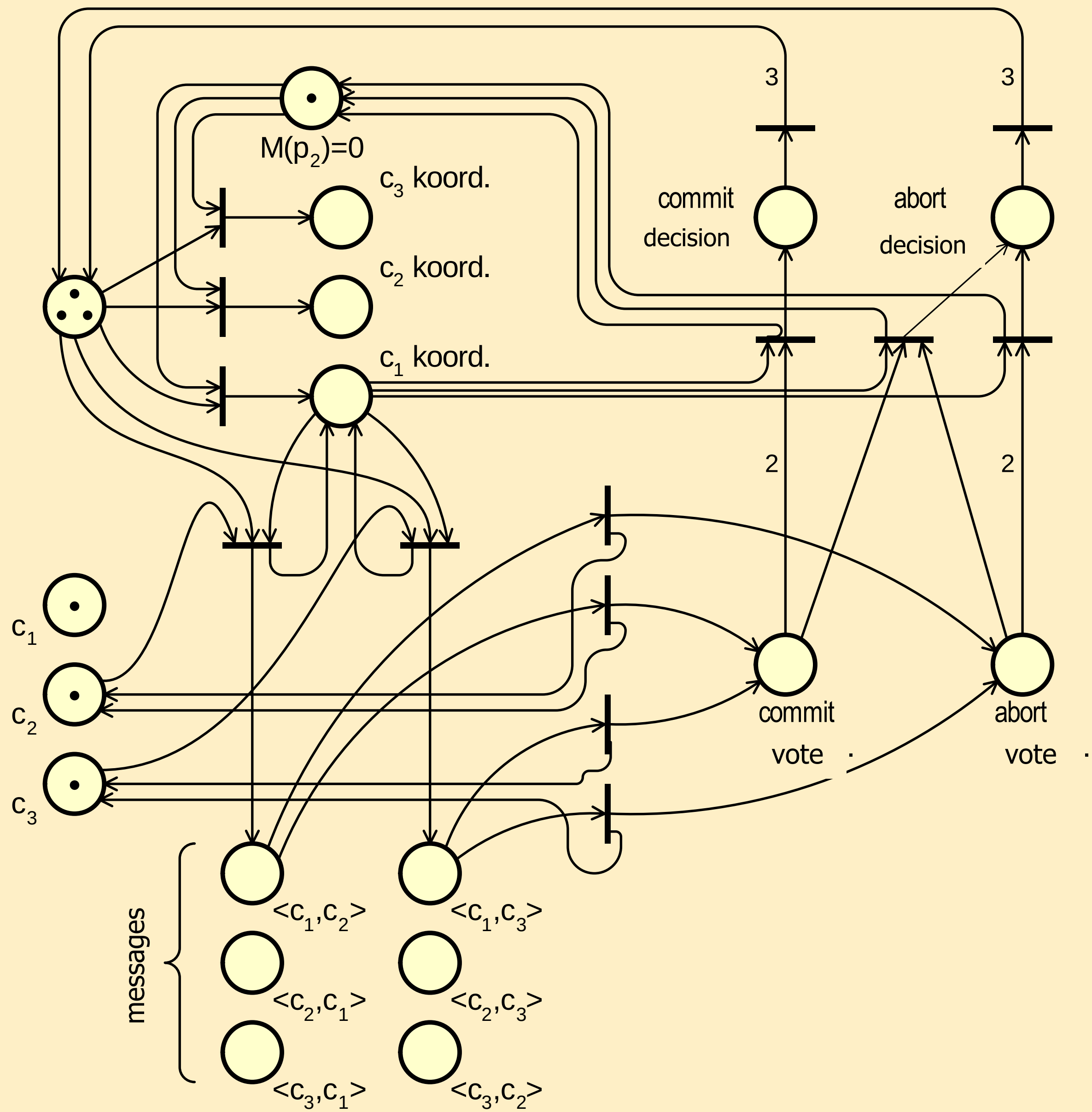
Example: Model of the simple commit protocol

- Colored Petri net model:
  - p_1: Participants (tokens c_1, c_2, c_3 in initial state)
  - p_2: Requests
  - p_3: Votes
  - p_4: Decision



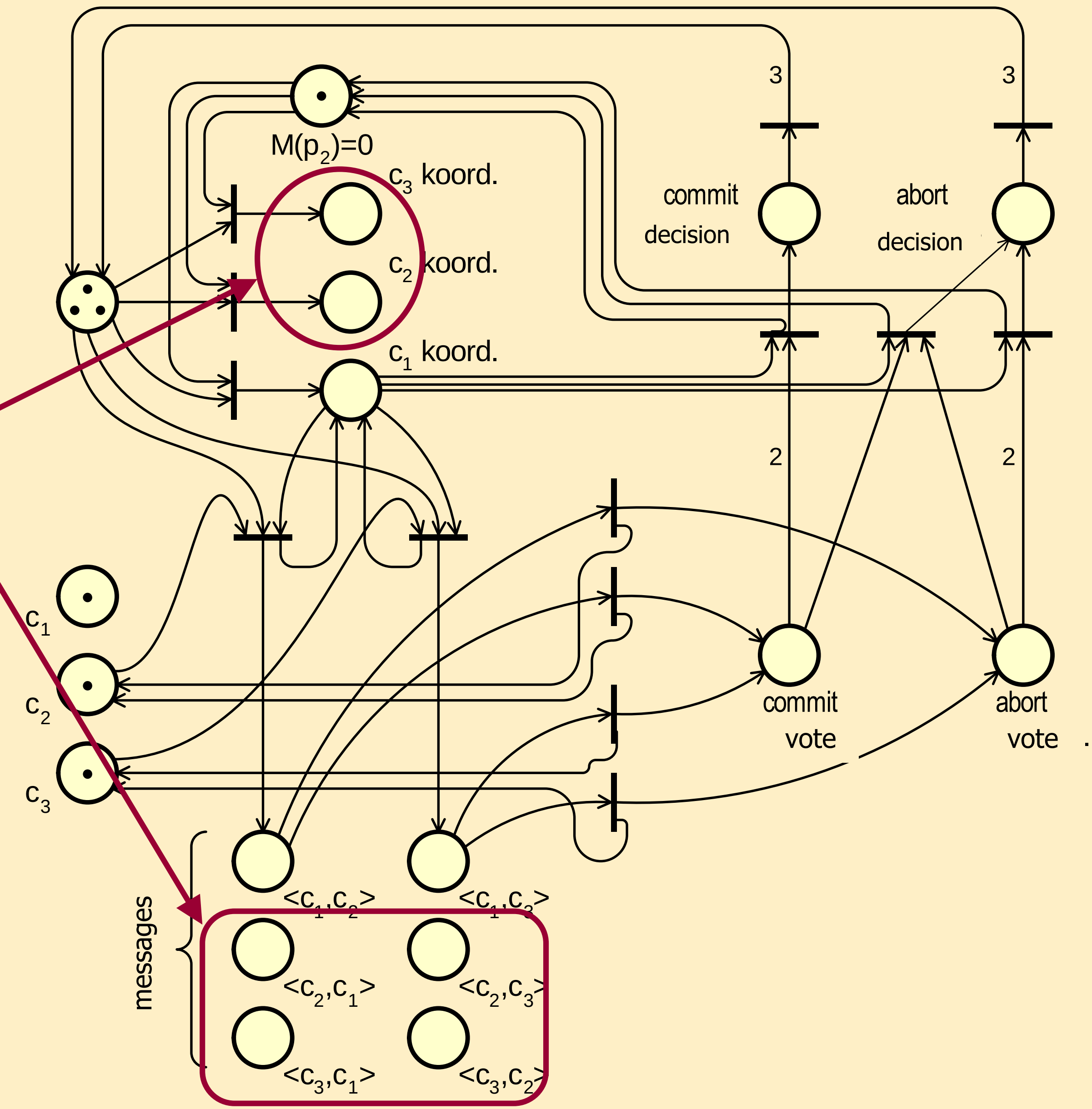
Example: Model of the simple commit protocol

- Partially unfolded (uncolored PN) model: c_1 is the coordinator
- Simple optimizations were done in the structure and events (firings)



Example: Model of the simple commit protocol

Similar nets needed for these parts too



Hierarchical colored Petri nets

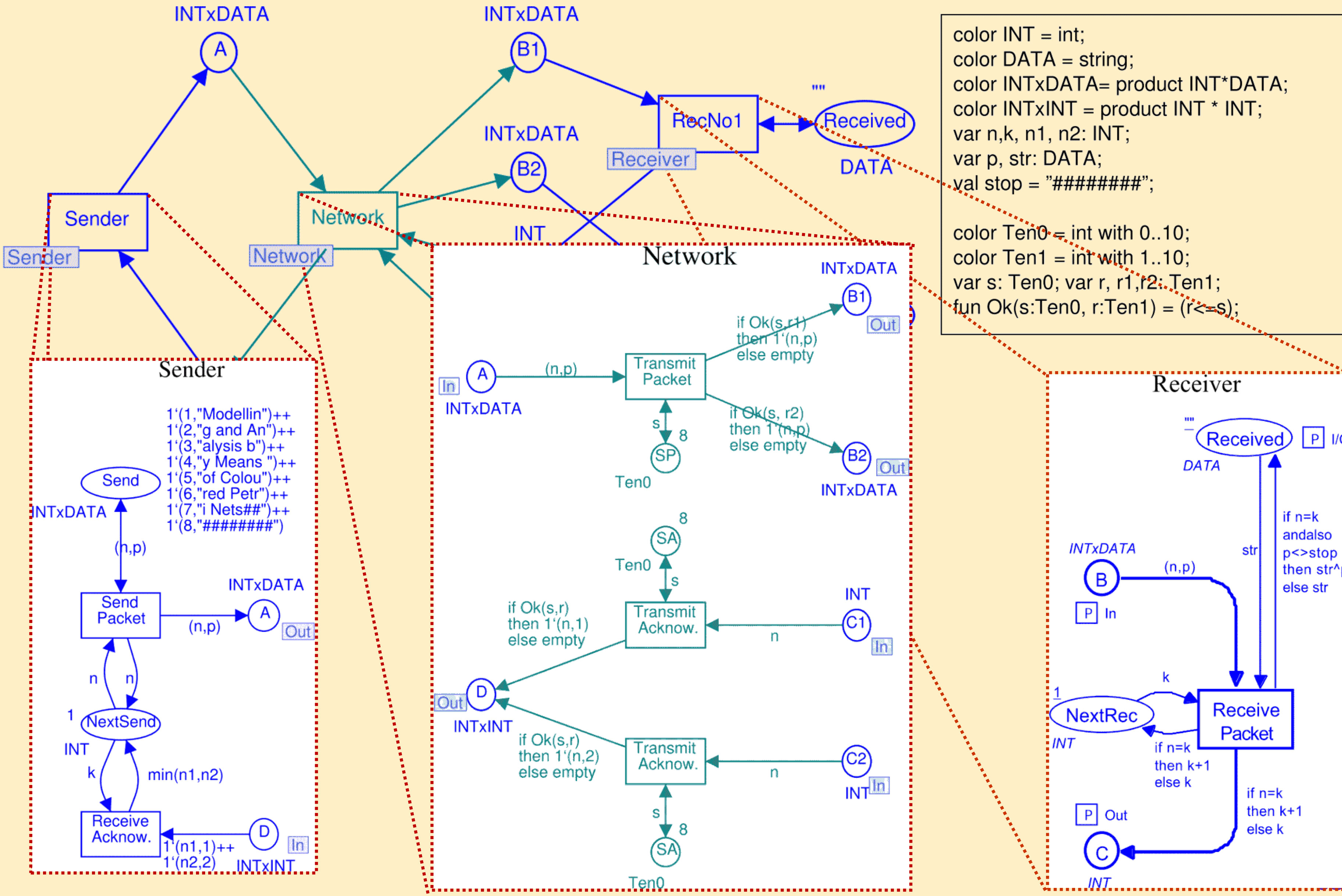
Hierarchical colored Petri nets

- Integration of subnets into a complex CPN hierarchically
  - Pages: Colored Petri net models (subnets)
    - Page number, page name: alternatives to refer to the subnet
    - The pages can be instantiated (on any level of the hierarchy)
    - The marking (token distribution) is unique for each instance
  - Hierarchy: Structure of the pages
    - Main (prime) page: topmost level
    - Secondary page instances (subpages)
      - Identification: page-instance ID number
      - Page-hierarchy graph

Tools of hierarchical composition

1. Coarse (substitute) transition
   – Representation of a subpage
   – Interfaces between pages: places
     1. On main page: “Socket” places $\rightarrow$ insertion point of subnets
     2. On subpage: “Port” places $\rightarrow$ connection points of the subnet, port type: input, output, input-output (bidirectional), general
2. Fusion places
   – Places with same name, multiple instances, denoting the same place at different locations
   – Tokens are added / removed simultaneously to / from each instance

Example: hierarchical version of the simple protocol




```
color INT = int;
color DATA = string;
color INTxDATA= product INT*DATA;
color INTxINT = product INT * INT;
var n,k, n1, n2: INT;
var p, str: DATA;
val stop = "########";

color Ten0 = int with 0..10;
color Ten1 = int with 1..10;
var s: Ten0; var r, r1,r2: Ten1;
fun Ok(s:Ten0, r:Ten1) = (r<=s);
```

Example CPN:
Distributed database manager

Specification of the distributed database manager

- n different servers; local copy on each server, managed by a local database manager

  $DBM = \{d_1, d_2, \ldots, d_n\}$, $n \geq 3$
- Database operations:
  - Modification of local data
  - Change notification of the other database managers which will update
- State of the system:
  - Active: handling the update is in progress
  - Passive: handling the update is finished
- States of database managers:

  Inactive, Performing (updating), Waiting (for acknowledgement)
- Notification about changes: with messages
  - Message header: sender and receiver database manager

    $MES = \{(s,r) \mid s,r \in DBM \wedge s \neq r\}$, $Mes(s) = \sum_{r \in DBM-\{s\}} 1`(s,r)$
  - Message states: Unused, Sent, Received, Acknowledged

Distributed database: Declarations

Declaration field

```
val n = 4;
color DBM = index d with 1..n;
color PR = product DBM * DBM;
fun diff(x,y) = (x<>y);
color MES = subset PR by diff;
color E = with e;
fun Mes(s) = mult'PR(1`s, DBM--1`s)
var s, r : DBM;
```

Meaning:

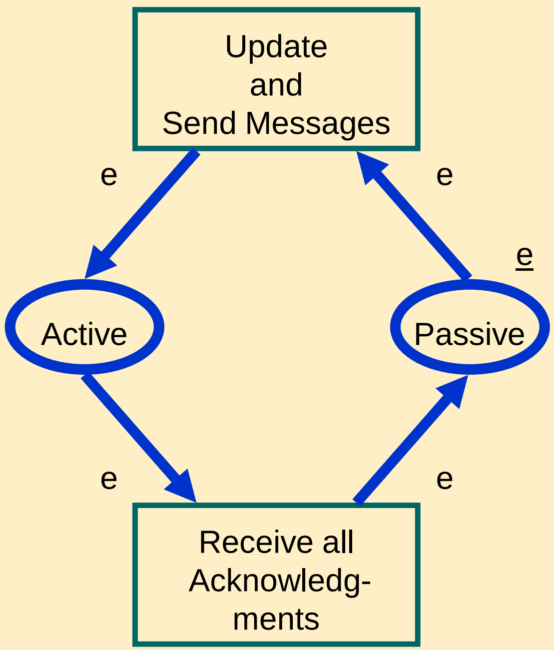
$$\text{DBM} = \{\text{d}_1, \text{d}_2, \ldots, \text{d}_n\}$$

$$\text{MES} = \{(s,r) \mid s,r \in \text{DBM} \wedge s \neq r\}$$

$$\text{Mes(s)} = \sum_{r \in \text{DBM-\{s\}}} 1'(s,r)$$

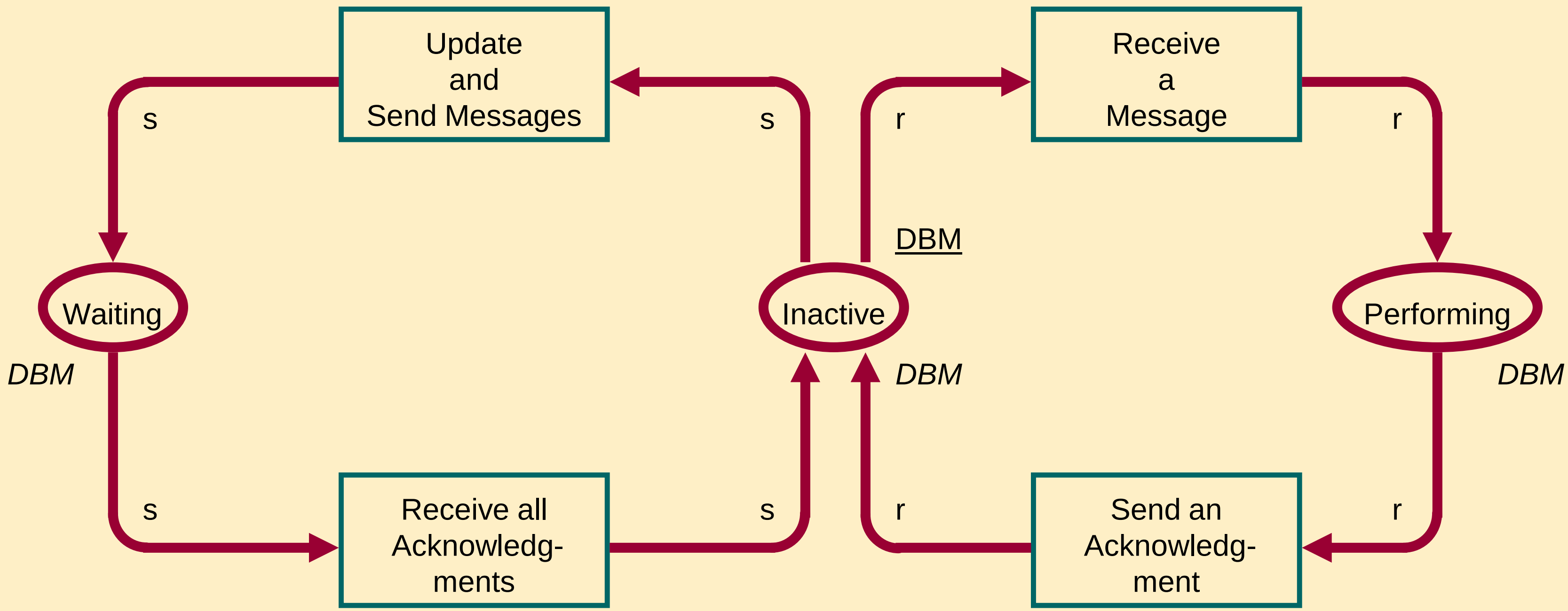
- DBM: database managers
- PR: DBM pairs
- MES: possible messages (headers)
- Mes(s): messages that can be sent by the DBM s
- E: simple token (uncolored)

Distributed database: System component



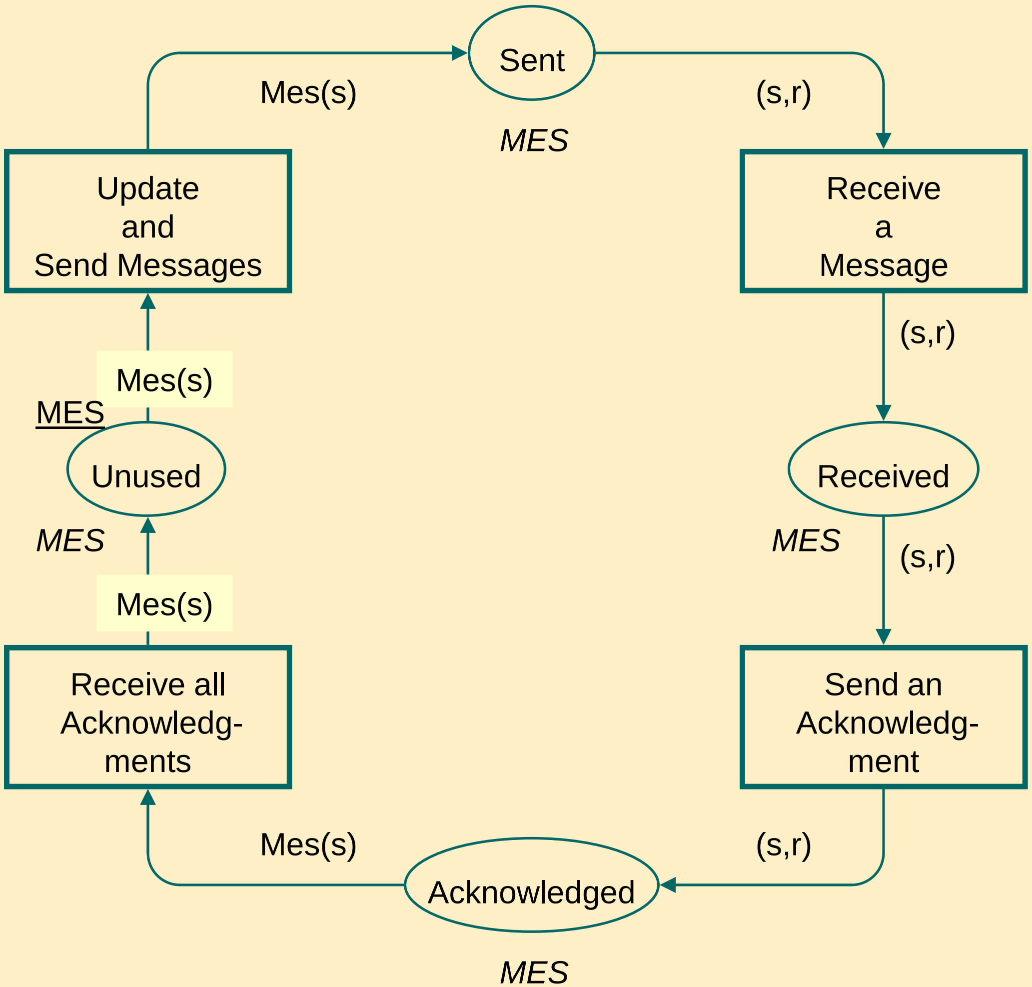
- System states denoted by a single token, initially ‘Passive’

Distributed database: Database managers



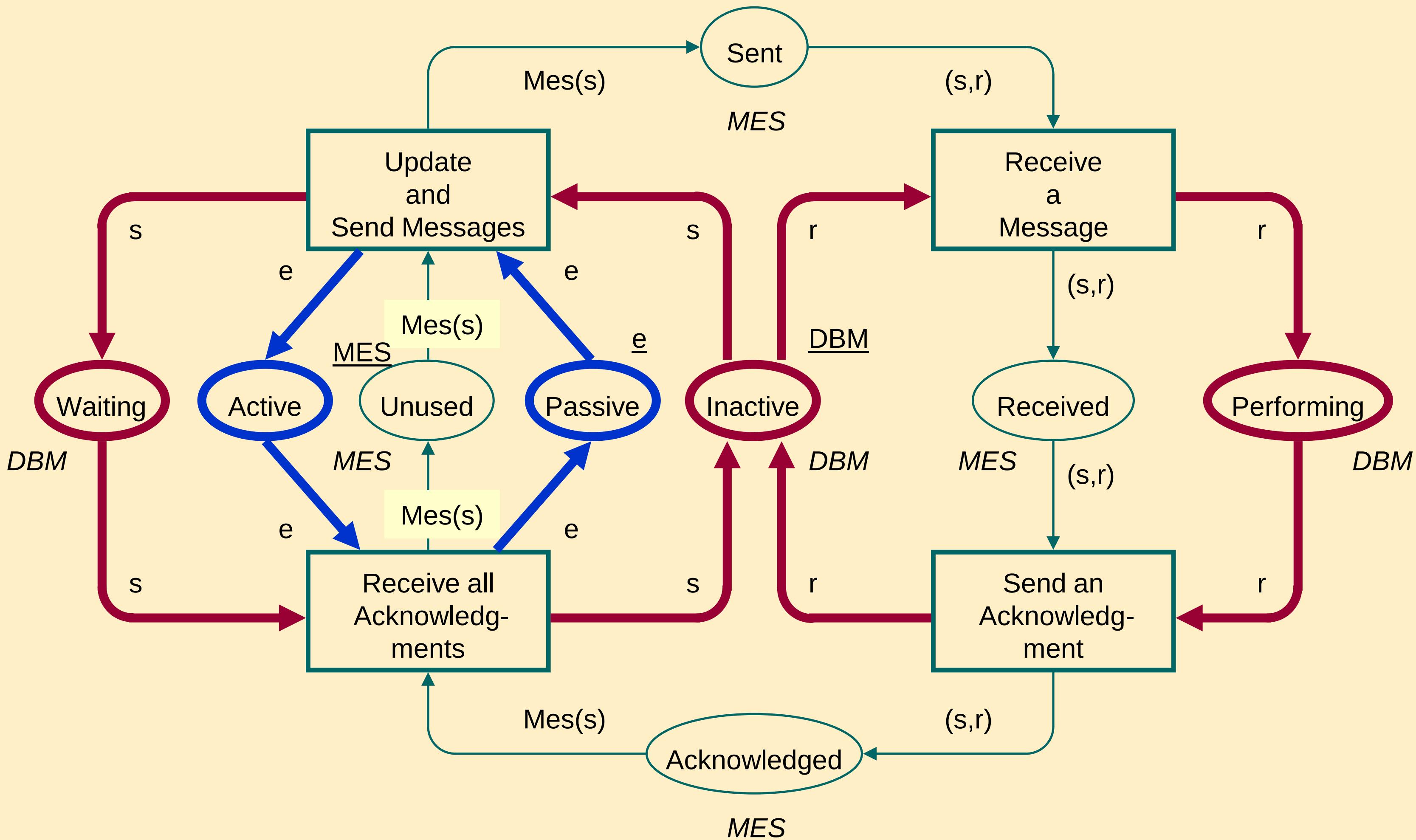
- DBMs are grouped by states, each group is represented by one place
- Initially each DBM is inactive; later it can change or update

Distributed database: Messages



- Places: message buffers
- A DBM sends notifications to the others; one from the set of possible messages

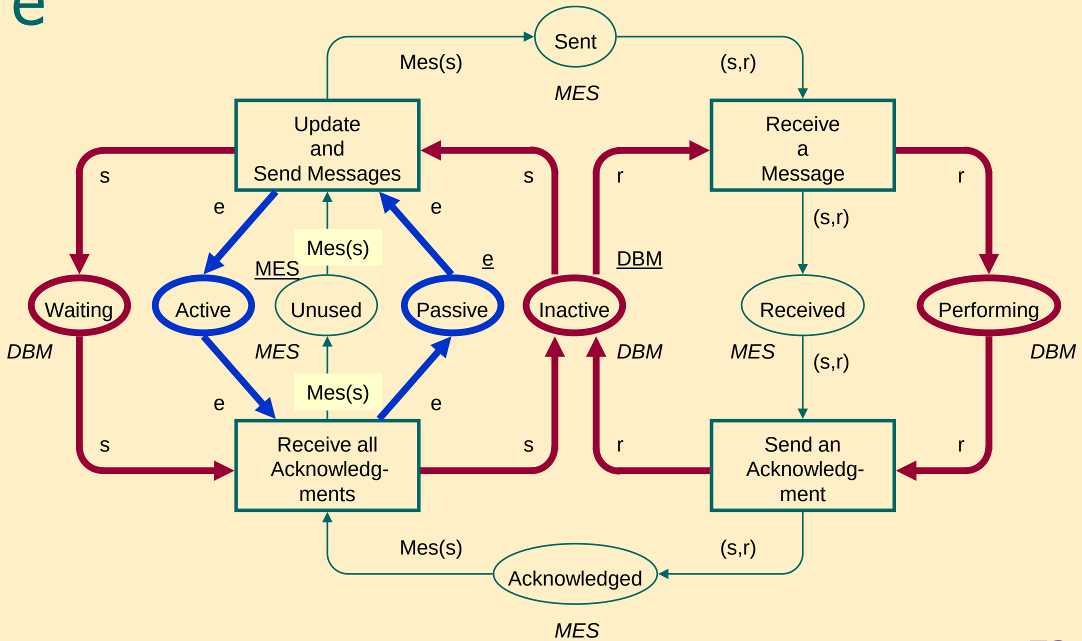
Distributed database: Complete CPN model



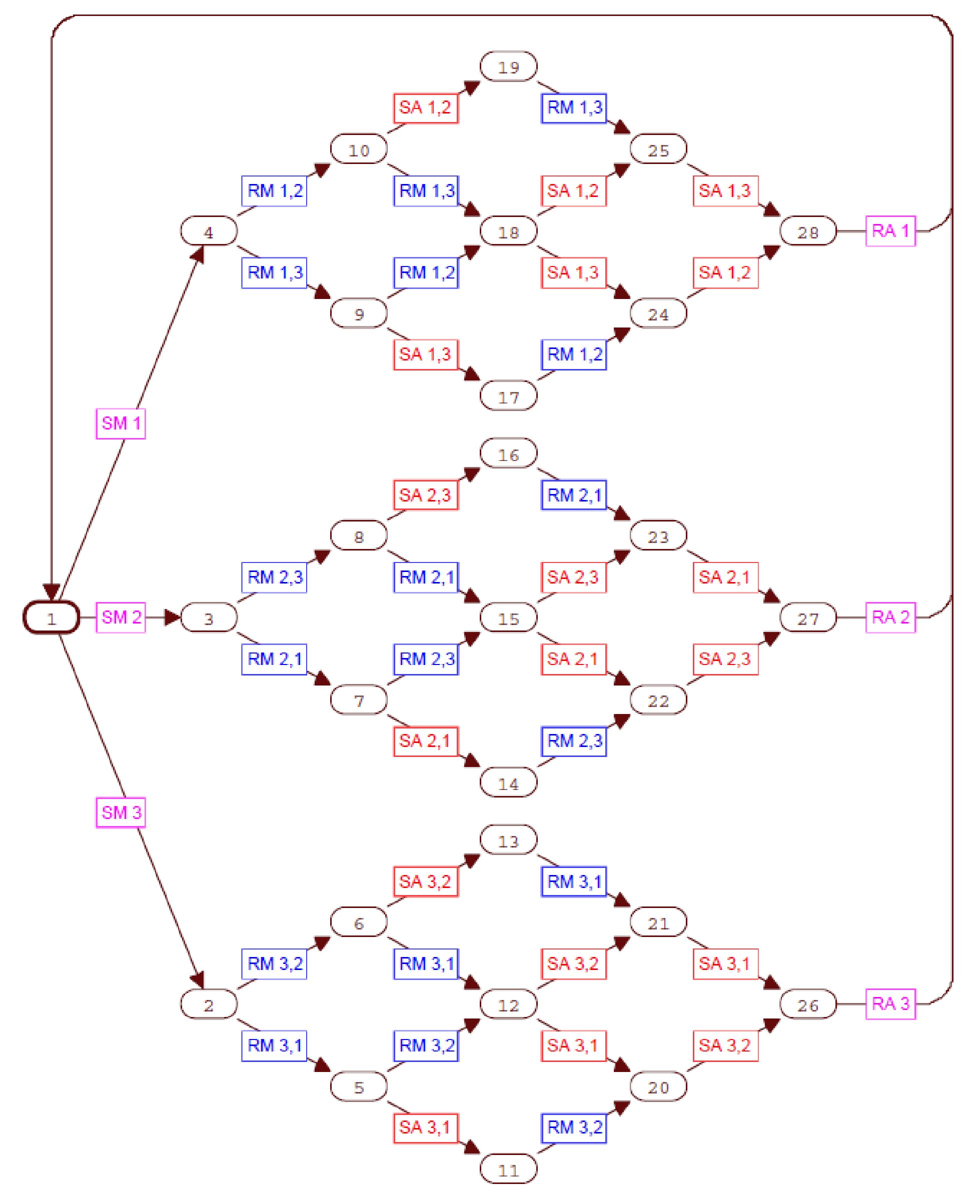
- Active and Passive places: only one DBM performs change at the same time, then waits

Particularities of the model

- Causality
  - Update and Send → Receive → Send Ack → Receive Ack
- Conflict
  - Update and Send
    enabled for each binding item s,
    but only one can fire
- Concurrency
  - Receive a Message
    for binding items
    (s,r) that are
    concurrent
    with themselves



Reachability graph for n=3



- Occurrence graph
- Abbreviated transition names:
  - SM: Update and Send Messages
  - RM: Receive a Message
  - SA: Send an Acknowledgment
  - RA: Receive all Acknowledgments

Dynamic properties: boundedness

	Multiset	Integer
– Inactive	**DBM**	**n**
– Waiting	**DBM**	**1**
– Performing	**DBM**	**n - 1**
– Unused	**MES**	**n*(n - 1)**
– Sent, Received, Acknowledged	**MES**	**n - 1**
– Passive, Active	**E**	**1**

Dynamic properties: Liveness, fairness

- Liveness Properties
  - Dead markings: None
  - Dead transition instances: None
  - Live transition instances: All
- Fairness Properties
  - Impartial transition instances:
    - Update and Send Messages
    - Receive a Message
    - Send an Acknowledgment
    - Receive all Acknowledgments
  - Fair transition instances:
    - None
  - Just transition instances:
    - None

- Impartial transition: Fires infinitely often
- Fair transition: Infinitely many enabling → infinitely many firing
- Just transition: Persistent enabling → firing

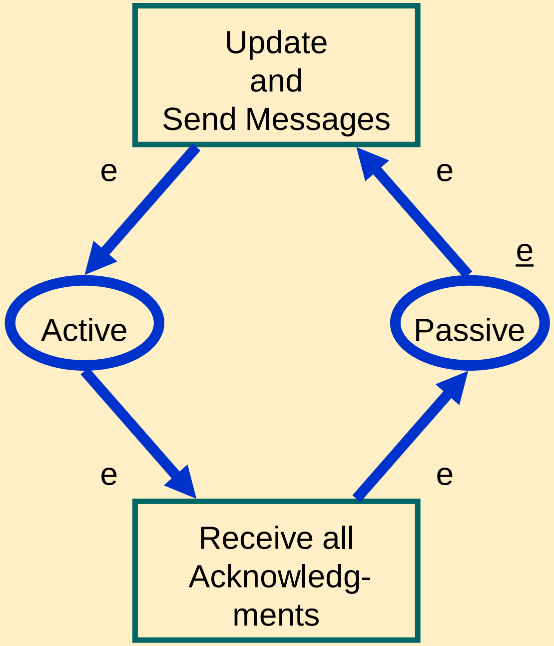
Structural properties: P invariants

- M(Active) + M(Passive) = 1`e
- M(Inactive) + M(Waiting) + M(Performing) = DBM
- M(Unused) + M(Sent) + M(Received) + M(Acknowledged) = MES

- M(Performing) – Rec(M(Received)) = $\emptyset$
  - Function Rec() for token mapping: Rec(s,r) = r
- M(Sent) + M(Received) + M(Acknowledged) – Mes(M(Waiting)) = $\emptyset$
  - Function Mes() for token mapping : Mes(s): the messages can be sent by DBM s
- M(Active) – Ign(M(Waiting)) = $\emptyset$
  - Function Ign() turns tokens with any color into token with color $e \in E$

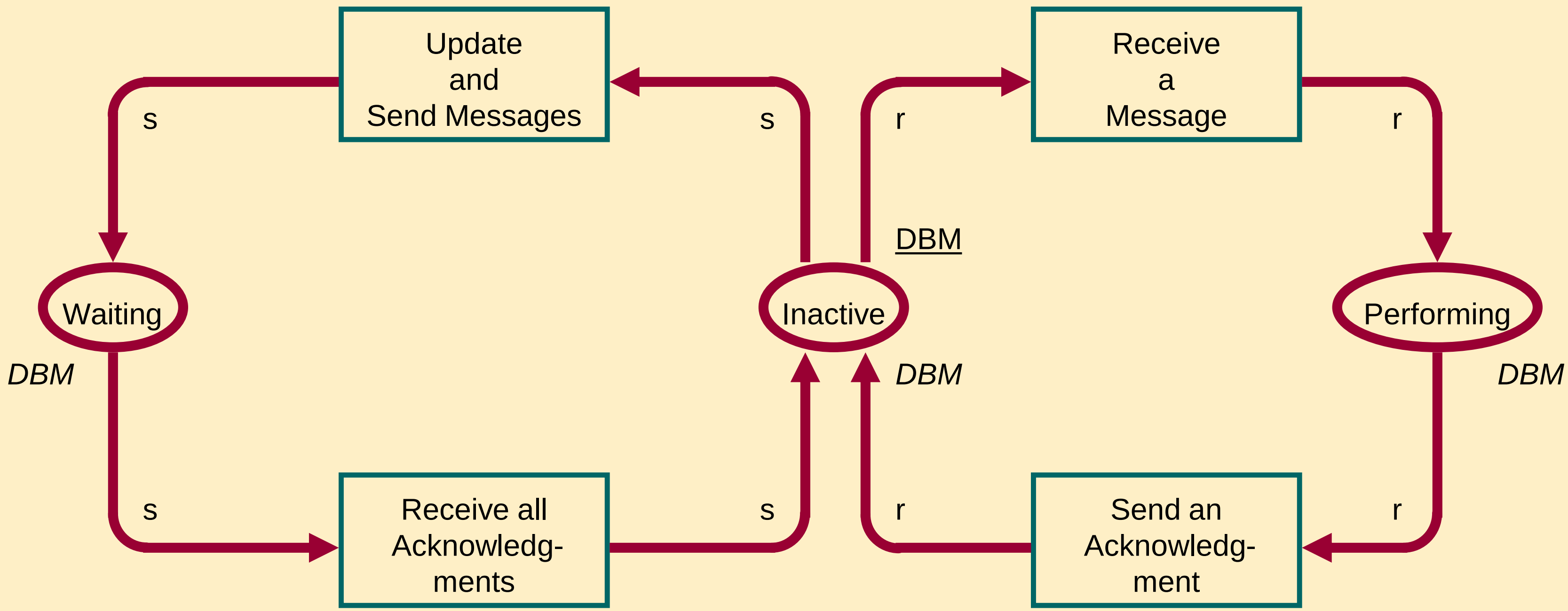
P invariant: the state of the system

M(Active) + M(Passive) = 1`e



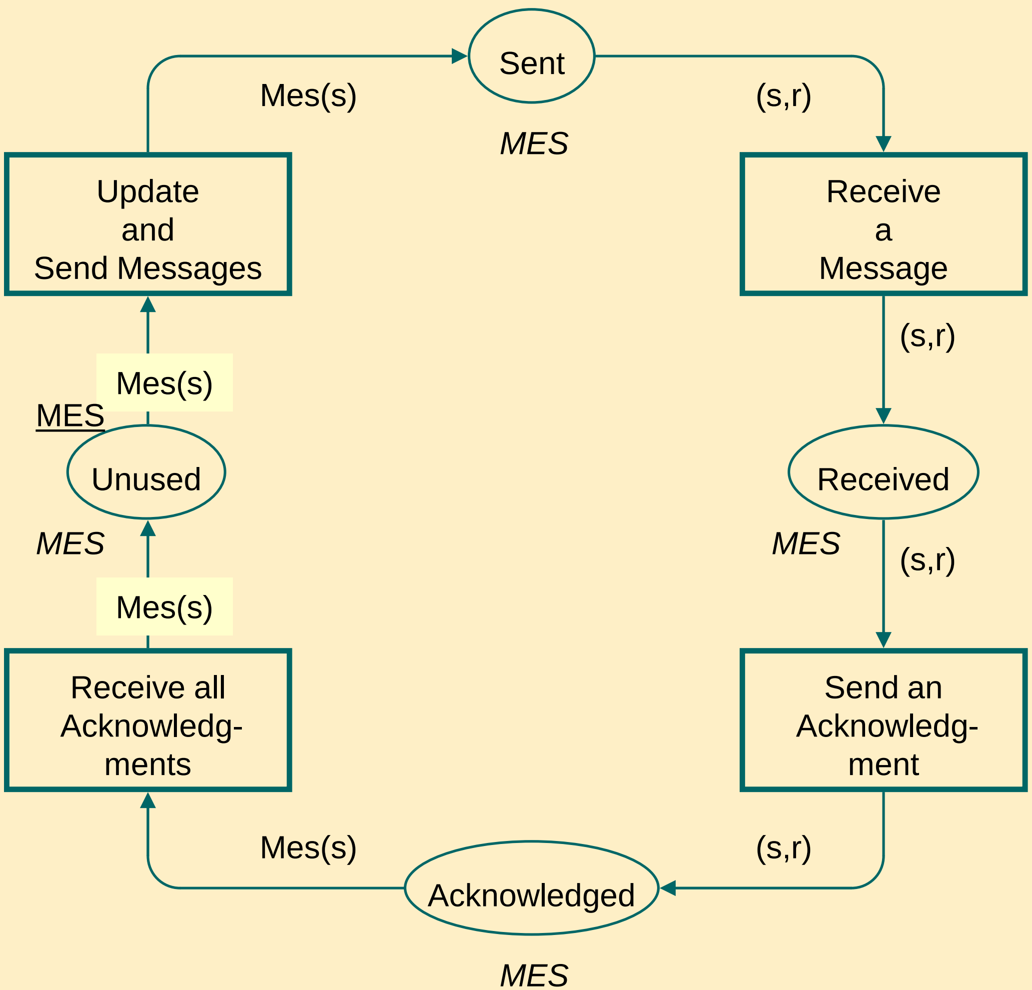
P invariant: database managers

$$M(\text{Inactive}) + M(\text{Waiting}) + M(\text{Performing}) = DBM$$

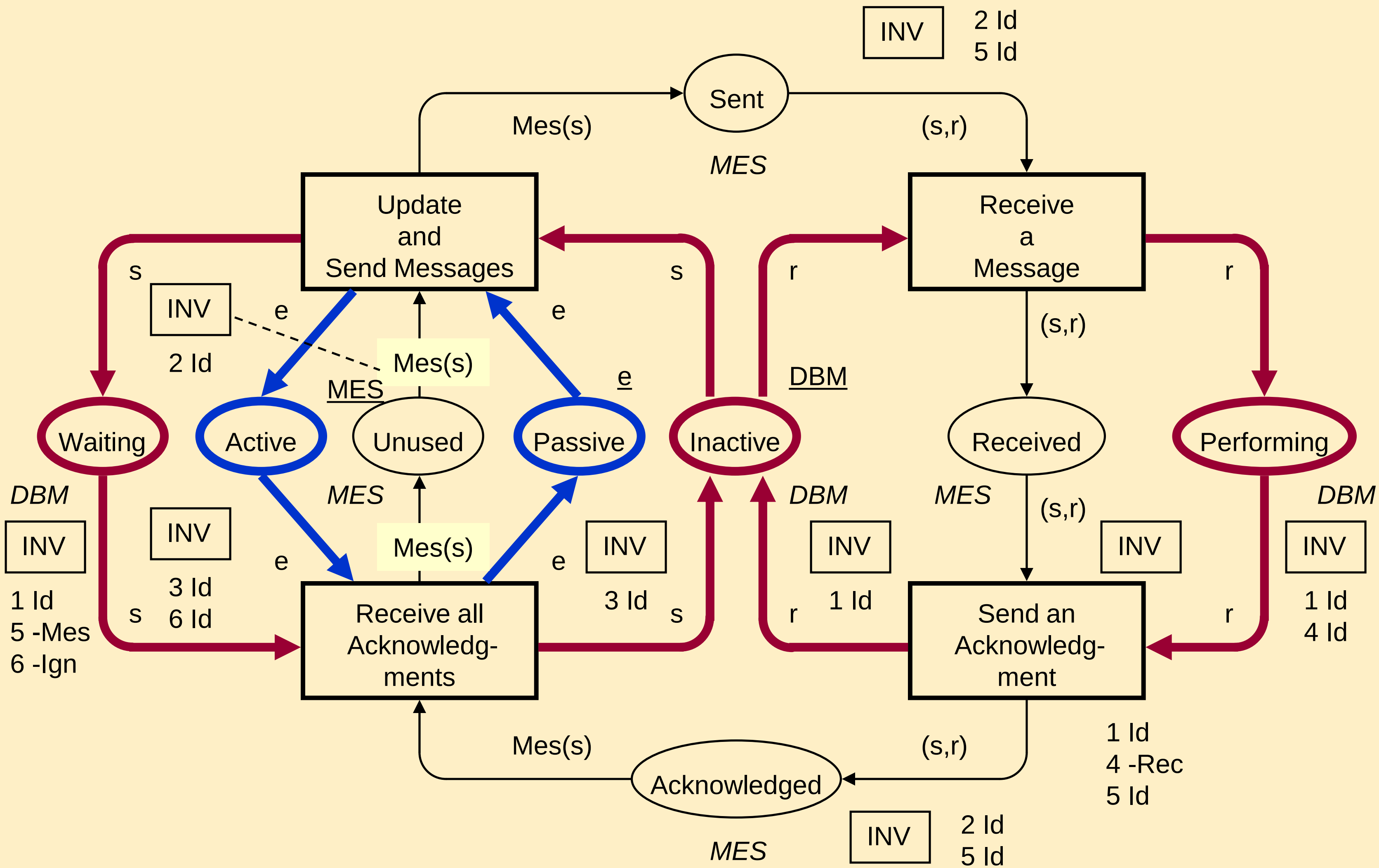


P invariants: messaging subsystem

M(Unused) + M(Sent) + M(Received) + M(Acknowledged) = MES

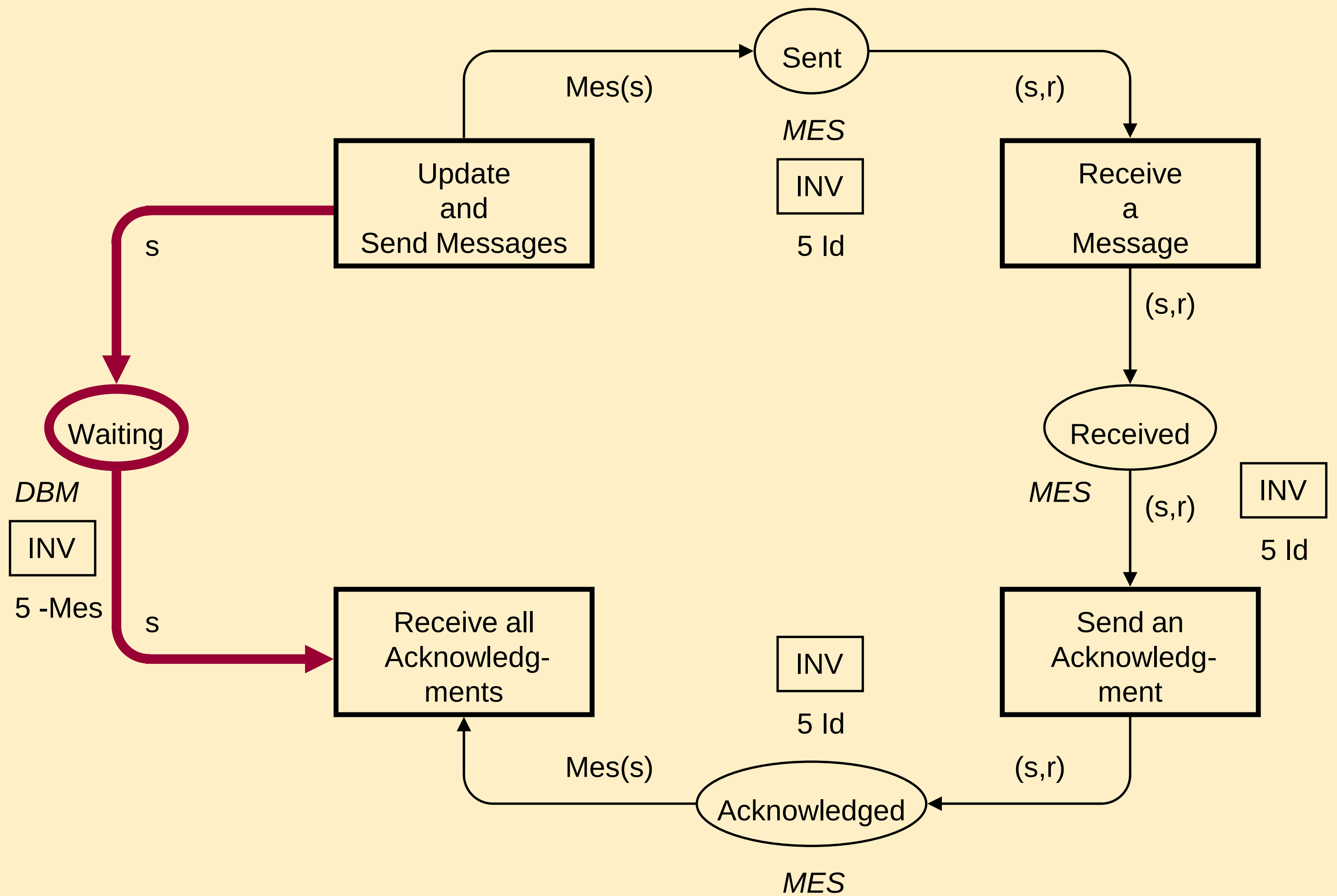


P invariants of the model

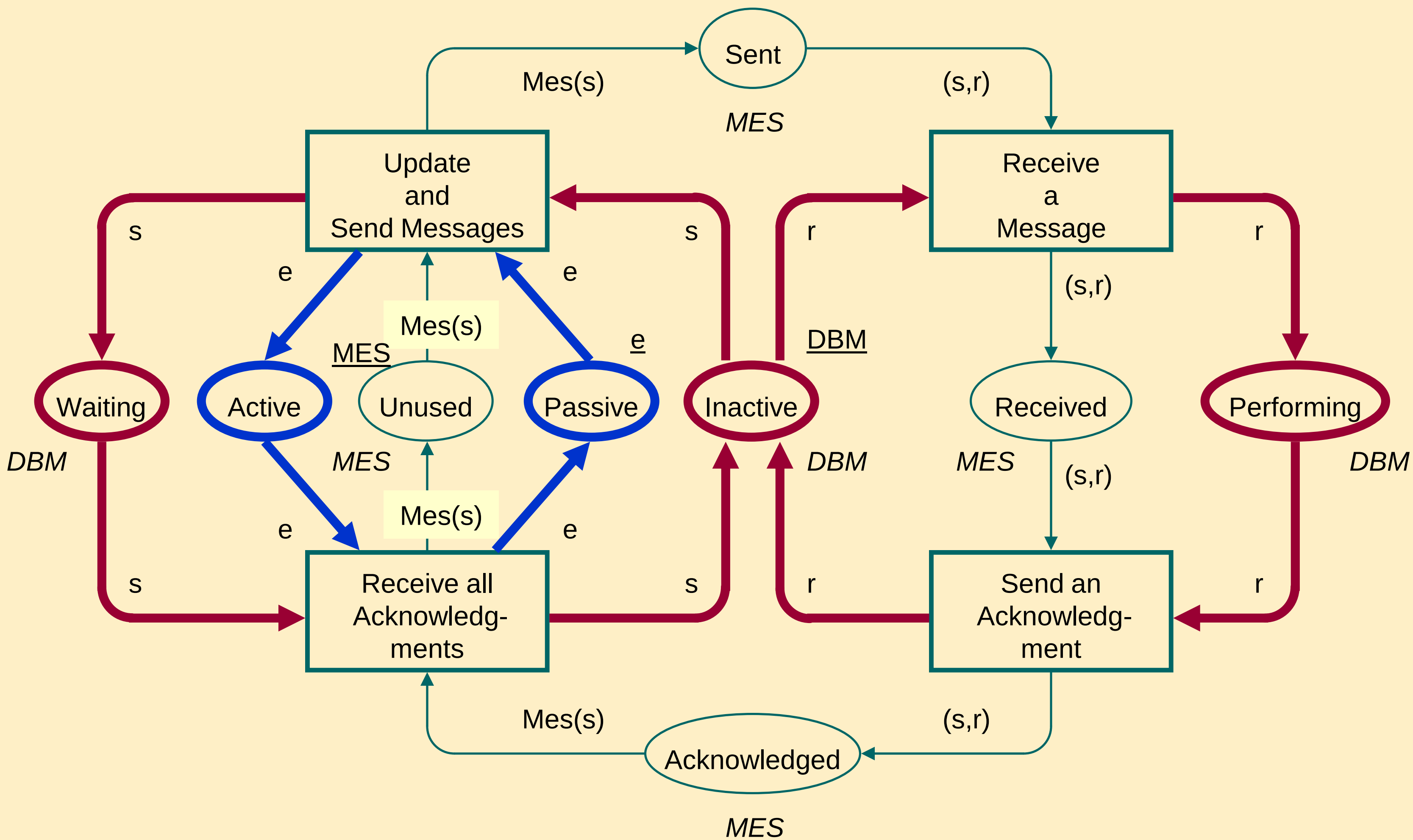


One of the P invariants

$$M(Sent) + M(Received) + M(Acknowledged) - Mes(M(Waiting)) = \varnothing$$



The complete CPN model (reminder)



Messaging unfolded for n=3