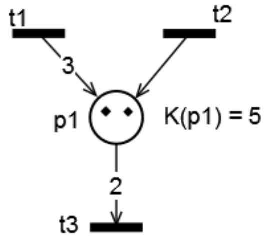
Formal methods sample exercises

Theoretical questions

What does it mean when a place in a Petri net has *finite capacity*? Draw the equivalent, infinite capacity net corresponding to the net below (using the transformation from the lectures).



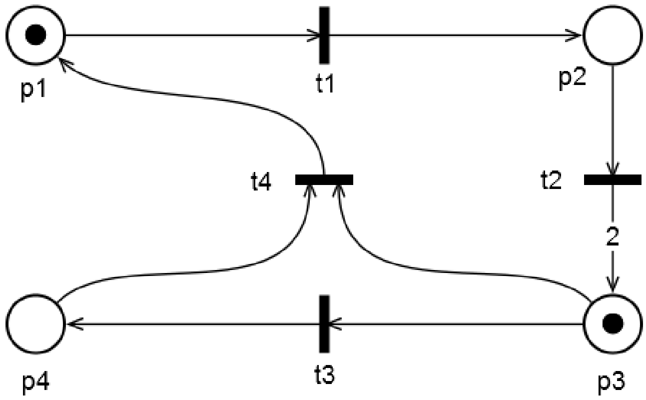
Give the *formal definition* of P-invariants (including the meaning of the variables in the definition). Give an example for the practical usage of P-invariants.

A Petri net is given below described by its topology. Give the *reduction steps* based on the *property preserving transformations* (known from the lectures) and the final result (draw the reduced Petri net).

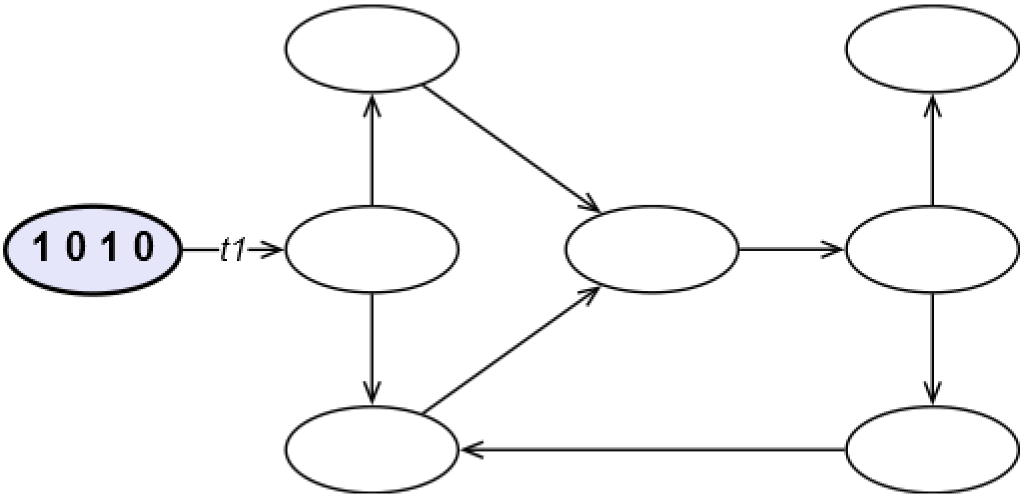
- $\bullet p_1 = \emptyset$
- $\bullet p_2 = \{t_1, t_2\}$
- $\bullet t_1 = \{p_1\}$
- $\bullet t_2 = \{p_1\}$

State space, dynamic properties

Investigate the dynamic properties of the following Petri net based on its reachability graph.



First, complete the reachability graph of the Petri net below. The graph is **missing** *edges*, *edge labels* and *state labels*. Label edges with *transitions* and states with *token distribution vectors*. (e.g. the state with token distribution m(p_1) = 0, m(p_2) = 1, m(p_3) = 2, m(p_4) = 3 should be labeled with 0 1 2 3). The initial state is the one on the left side with thick border.



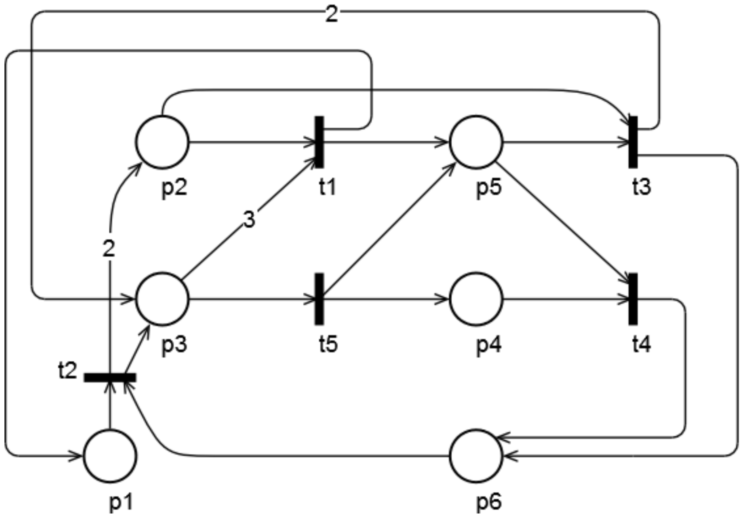
Then, observe the reachability graph and the Petri net and answer the following questions. Explanation is not needed in this subtask.

		true	false	not decidable			true	false	not decidable
(a)	The reachability and the coverability graphs are identical	☐	☐	☐	(e)	The sequence *t1*, *t2*, *t4* forms a T-invariant	☐	☐	☐
(b)	The net is not persistent	☐	☐	☐	(f)	Transitions *t3* and *t4* are bounded fair	☐	☐	☐
(c)	The net contains deadlock(s)	☐	☐	☐	(g)	The state (0 1 0 1) is a home state	☐	☐	☐
(d)	Transition *t3* is L2-live	☐	☐	☐	(h)	The net is globally fair	☐	☐	☐

Invariants

The Petri net below is given along with its incidence matrix $\mathbf{W^T}$.

$$\mathbf{W^T} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{array} \begin{bmatrix} t_1 & t_2 & t_3 & t_4 & t_5 \\ 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & \mathbf{B} & 0 & 0 \\ \mathbf{A} & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & \mathbf{C} & 1 \\ 1 & 0 & -1 & -1 & 1 \\ 0 & -1 & 1 & 1 & \mathbf{D} \end{bmatrix}$$



The incidence matrix is missing some values, denoted by letters. What are the numbers that should be written in the place of the letters?

A = B = C = D =

Pick the P-invariants for the net from the following vectors! Write a letter *M* next to the box if the invariant is *minimal*!

(a) $(4,2,1,0,1,1)^T$ ☐ ......

(b) $(10,5,2,1,1,2)^T$ ☐ ......

(c) $(3,1,0,2,0,1)^T$ ☐ ......

(d) None of the vectors is a P-invariant. ☐ ......

Check *using the state equation* (with explanation), if the following vectors are T-invariants for the net!

(a) $(2,2,2,0,0)^T$ ☐

(b) $(0,1,0,1,3)^T$ ☐

(c) $(1,2,1,1,3)^T$ ☐

(d) None of the vectors is a T-invariant. ☐

Modeling with Petri nets

Create and draw a Petri net (**not colored**) model of a programmer based on the description below. A part of the model is given below, complete this model.

- The programmer is either *working*, *having fun* or *sleeping*.
- The programmer has 5 units of energy each day.
- If the programmer is working or having fun, he/she can consume one unit of energy.
- If the programmer is working and has consumed at least 3 units of energy, he/she can start having fun.
- If the programmer is having fun and he/she has no more energy, he/she can go to sleep.
- If the programmer is sleeping and all of its energy is refilled, he/she can start working.
- While the programmer is sleeping, one unit of energy can be refilled.