Formal Methods (VIMIM100)	**Year 2017/2018, semester II**						17. May 2018.
Second Mid-term Exam, Group A	1.	2.	3.	4.	5.	6.	Σ
Name: _____							
NEPTUN code: _____	14 points	8 points	8 points	6 points	6 points	8 points	50 points

1. Theoretical questions   **If you work on a separate sheet, please always indicate it!**

1.1. For each of the following statements indicate (with an X) whether it is true, false or not decidable. | 3 points |

Statement	True	False	Not decidable
A Petri net with a source transition is always live.		X	
An inhibitor arc disables a transition if the place connected to the arc does not have tokens.		X	
Introducing priorities between transitions can always be avoided by using supplementary places.		X	

1.2. What are T-invariants in Petri nets? If you are using a formula, explain the notations! | 2 points |

A vector σ is a T-invariant if $W^T\sigma = 0$ holds, where W is the incidence matrix. Firing the transitions of a T-invariant does not change the marking.

1.3. Give the conditions for a transition to be enabled in *stochastic Petri nets* (SPN)! | 3 points |

- There are enough tokens on the input places (considering weights).
- If it became enabled at time t and generated a random delay d, then it was continuously enabled in the interval [t, t+d).

1.4. We are modeling a workflow with Petri nets, where tokens represent work items. Which properties of Petri nets can be used to analyze the following properties of the model? Give the *definition* of the properties as well! (1) The number of work items cannot grow arbitrarily large. (2) The number of work items is constant. | 2 points |

1) Boundedness: the number of tokens on each place is bounded (finite).
2) The relevant places have a P-invariant: the weighted sum of tokens is constant.

**1.5.** A short piece of code can be seen in the right.

a) Draw the *Control Flow Automaton* (CFA) model that represents the code! Use the number of the lines (0, 1, 2) for the locations! Represent assertion violations with a location labeled *err* and represent the normal ending of the program with a location labeled *end*.

b) We are using location and predicate abstraction with a single predicate *(x==0)* for model checking the CFA. What are the possible initial states in the abstract state space if the value of *x* can be arbitrary in the beginning? Give them in the following form: *(location, predicates)*!

c) Is the abstract path *(0, false)* → *(2, false)* → *(err, false)* a real or a spurious counterexample? Explain your answer!

```
    x : int
0: if (x < 0) {
1:     x := -x;
   }
2: assert(x>=0);
```
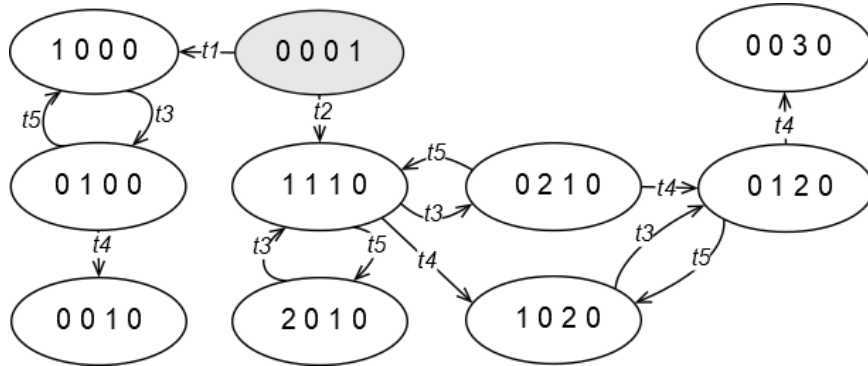
4 points

a)
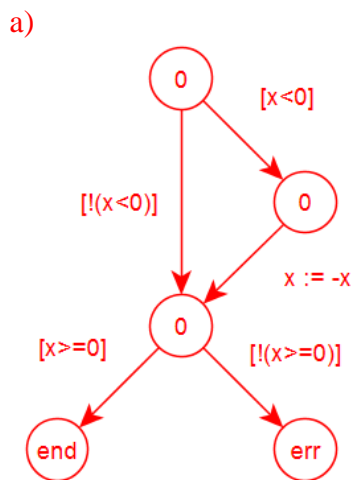
b)
(0, true) and (0, false)

c)
Counterexample is spurious because (0, false) → (2, false) requires !(x<0)
and (2, false) → (err, false) requires !(x>=0), yielding a contradiction.

2. Dynamic properties

The figure below represents the state space of a Petri net as a *reachability graph*. The net contains 5 transitions denoted by t_1, ..., t_5. The states are denoted by token distribution vectors, for example the vector (0 1 0 0) represents $m(p_1) = 0$, $m(p_2) = 1$, $m(p_3) = 0$ and $m(p_4) = 0$. The initial state is marked with a darker background.

2.1. Answer the following questions based on the graph above. *No explanation* is needed here. | 8 points |

		True	False	Not decidable			True	False	Not decidable
(a)	The net is bounded	X	☐	☐	(e)	Transition t1 is L0-live	☐	X	☐
(b)	The net has no deadlocks	☐	X	☐	(f)	Transition t2 is L1-live	X	☐	☐
(c)	The net has a home state	☐	X	☐	(g)	Transition t3 is L2-live	X	☐	☐
(d)	The net is reversible	☐	X	☐	(h)	Transition t4 is persistent	☐	X	☐

Explanations (not needed in the exam):

a) Reachability graph is finite.
b) For example (0 0 3 0) has no outgoing edges.
c) No state can be reached from any state reachable from it.
d) (0 0 3 0) can be reached from the initial state, but not vice versa.
e) t1 can fire, for example from the initial state.
f) t2 can fire, for example from the initial state.
g) There is a reachable cycle containing t3.
h) At (0 1 0 0) t4 is enabled, but firing t5 disables it under (1 0 0 0).

3. Invariants   Please provide the solution on a new sheet!

A Petri net is given in the right.
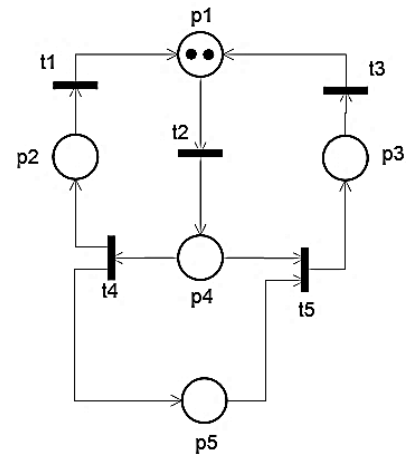
3.1. Give the weighted *incidence matrix* of the net!

$W =$

	p1	p2	p3	p4	p5
t1	1	-1	0	0	0
t2	-1	0	0	1	0
t3	1	0	-1	0	0
t4	0	1	0	-1	1
t5	0	0	1	-1	-1

2 points

3.2. Check if the following vector is a P-invariant of the net (explain your answer)!

$(1, 1, 1, 1, 0)^T$

2 points

Yes, because W multiplied by the vector is 0.

3.3. Check if the following vector is a T-invariant of the net (explain your answer)!

$(1, 0, 1, 1, 0)^T$

2 points

No, because W^T multiplied by the vector is not 0.

3.4. Does the following CTL expression hold for the Petri net with the given initial marking? The notation *m(pi)* describes the number of tokens in place *pi*. Explain your answer!

2 points

$\mathbf{AG}(m(p1) + m(p2) + m(p3) + m(p4) + m(p5) = 2)$

The weighted sum of tokens is 2 for the initial state, but (1 1 1 1 1) is not a P-invariant, therefore the expression does not hold.

4. Modeling with Petri nets

4.1. Draw an extended, but non-colored Petri net model based on the following description by completing the partial model below with only *tokens, transitions and arcs*! (If you are not sure, first draw a draft version on a separate sheet!)

6 points

- The money making machine is *idle*, it has 5 units of *ink,* 5 units of *paper* and 5 units of *metal.*
- When the machine is idle, we can press the buttons for making a *coin* or printing a *banknote,* which selects the type of money. These buttons can be pressed regardless of the available supplies (ink, paper, metal).
- Printing a banknote requires 1 unit of paper and 2 units of ink, while making a coin only requires 1 unit of metal.

- The machine starts *cropping paper* and *warming up* at the same time to print a banknote, and it starts *measuring metal* and *warming up* at the same time for making a coin. If there are not enough supplies (ink, paper, metal) the operation is stopped.
- If the machine is *ready* and the *required materials* are available (metal was measured or paper was cropped) the machine starts *making money.*
- After making money, it is *ready* and the machine is idle again (and a new choice can be made again).
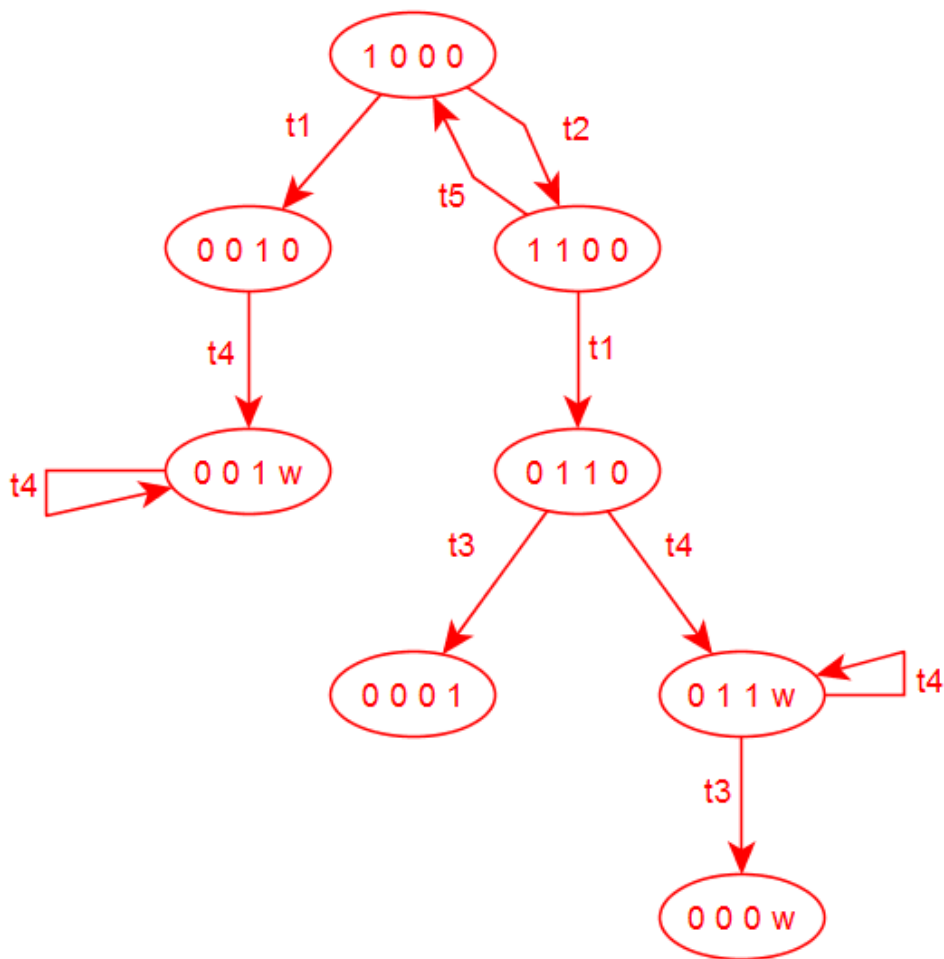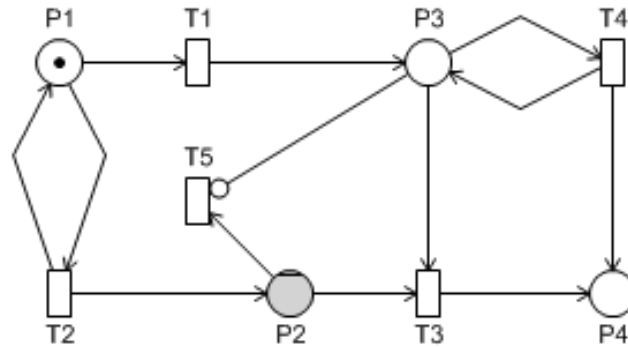- If there is not enough ink for a banknote, a LED indicates that the machine *ran out of ink.*
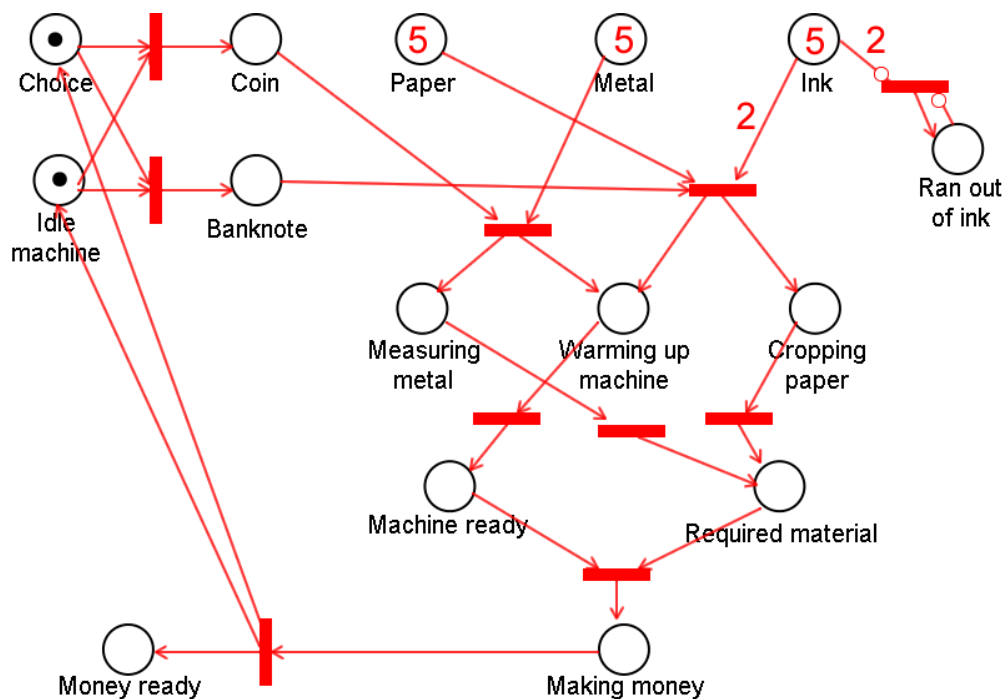
5. Coverability graph   Please provide the solution on a new sheet!

The following Petri net is given where place P2 has a finite capacity of K(P2)=1. Other places have infinite capacity. Draw the *coverability graph* for the Petri net! Label arcs of the graph with transitions!
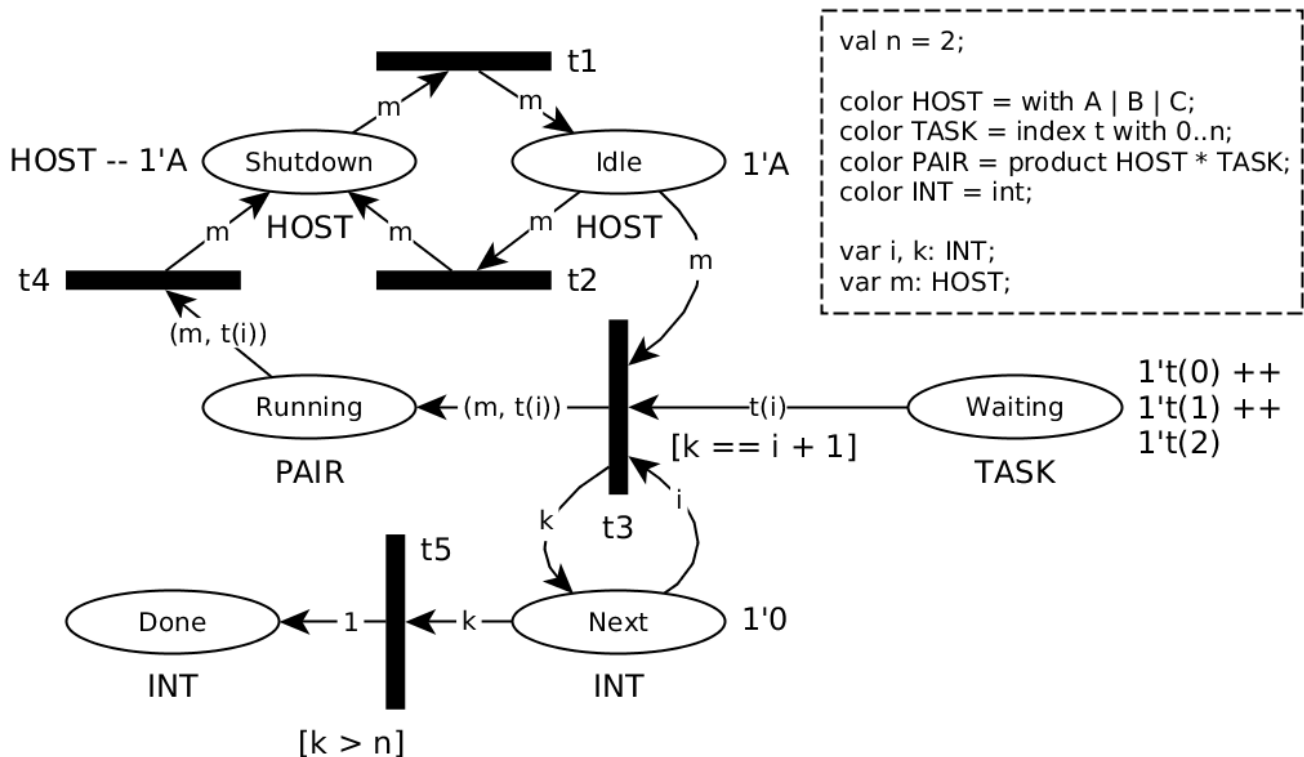
6 points

6. Colored Petri nets   Please provide the solution on a new sheet!

The following colored Petri net is given with its definition block. Types of places are denoted by capital words and the actual marking is written next to each place (which is non-empty).


```
val n = 2;

color HOST = with A | B | C;
color TASK = index t with 0..n;
color PAIR = product HOST * TASK;
color INT = int;

var i, k: INT;
var m: HOST;
```

6.1. Answer the following questions:
- a) Enumerate the enabled transition(s) with binding(s) under the actual marking!
  t1 with m = B or m = C
  t2 with m = A
  t3 with m = A, i = 0, t(i) = t(0), k = 1, (m, t(i)) = (A, t(0))
- b) Select one of the enabled bindings for t3 and give the marking after firing!
  Waiting: 1't(1) ++ 1't(2)
  Idle:
  Next: 1'1
  Running: 1'(A, t(0))
- c) Is the net bounded with the given initial state? Explain your answer!
  Yes. The initial marking is bounded and every transition can either fire a bounded number of times (t3, t4, t5) or produces the same amount of tokens as it consumes (t1, t2).
- d) Give a firing sequence (transitions and bindings) which leads to a state where the place *Done* has a token!
  t1 with m = B
  t1 with m = C
  t3 with m = A, i = 0, k = 1, t(i) = t(0), (m, t(i)) = (A, t(0))
  t3 with m = B, i = 1, k = 2, t(i) = t(1), (m, t(i)) = (B, t(1))
  t3 with m = C, i = 2, k = 3, t(i) = t(2), (m, t(i)) = (C, t(2))
  t5 with k = 3

- e) Is there a T-invariant in the net? Explain your answer!
  Yes, for example t1, t2.

8 points