Formalizing Requirements: Temporal Logics

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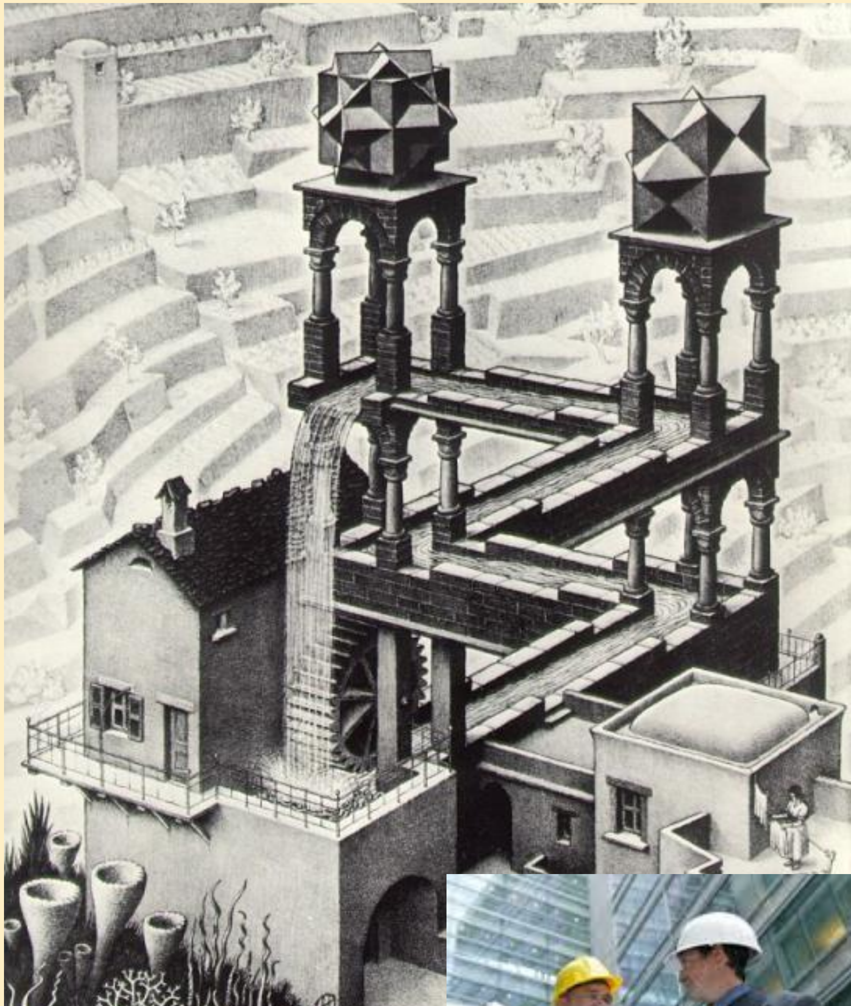
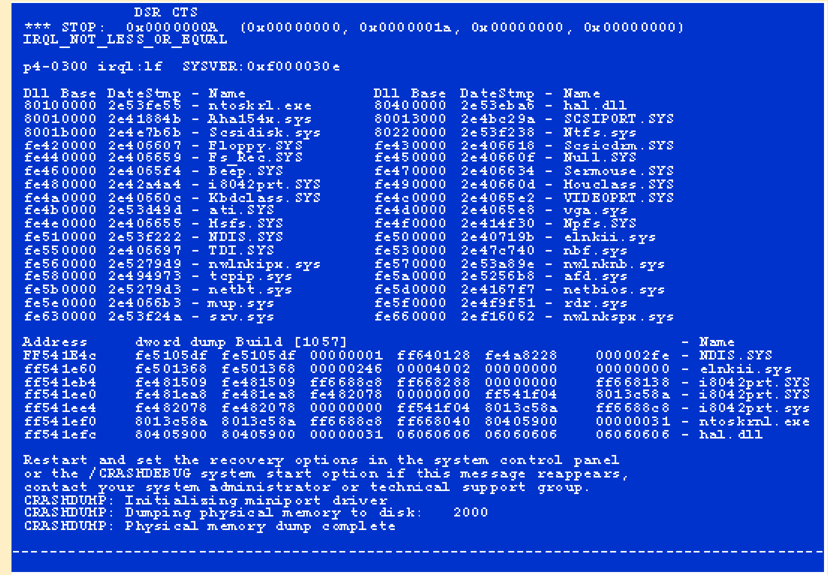
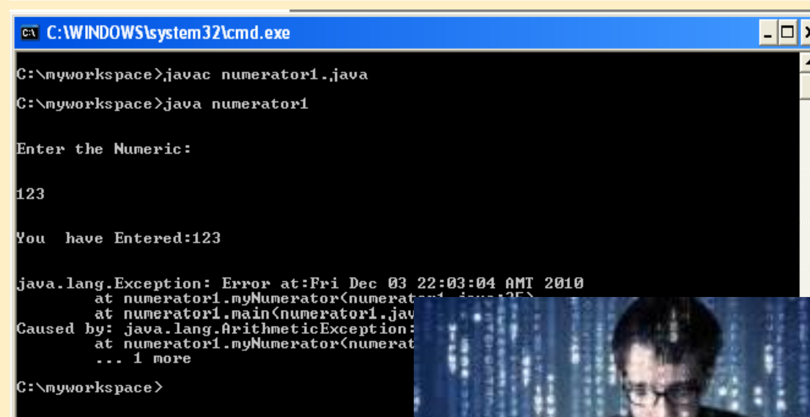



DSR CTS
*** STOP: 0x0000000A (0x00000000, 0x0000001a, 0x00000000, 0x00000000)
IRQL_NOT_LESS_OR_EQUAL
p4-0300 irql:1f SYSVER:0xf000030e
Dll Base DateStmp - Name Dll Base DateStmp - Name
80100000 2e53fe55 - ntoskrl.exe 80400000 2e53eba6 - hal.dll
80010000 2e41884b - Aha154x.sys 80013000 2e4bc29a - SCSIPORT.SYS
8001b000 2e4e7b6b - Scsidisk.sys 80220000 2e53f238 - Ntfs.sys
fe420000 2e406607 - Floppy.SYS fe430000 2e406618 - Scsicdrm.SYS
fe440000 2e406659 - Fs_Rec.SYS fe450000 2e40660f - Null.SYS
fe460000 2e4065f4 - Beep.SYS fe470000 2e406634 - Sermouse.SYS
fe480000 2e42a4a4 - i8042prt.SYS fe490000 2e40660d - Mouclass.SYS
fe4a0000 2e40660c - Kbdclass.SYS fe4c0000 2e4065e2 - VIDEOPRT.SYS
fe4b0000 2e53d49d - ati.SYS fe4d0000 2e4065e8 - vga.sys
fe4e0000 2e406655 - Msfs.SYS fe4f0000 2e414f30 - Npfs.SYS
fe510000 2e53f222 - NDIS.SYS fe500000 2e40719b - elnkii.sys
fe550000 2e406697 - TDI.SYS fe530000 2e47c740 - nbf.sys
fe560000 2e5279d9 - nwlnkipx.sys fe570000 2e53a89e - nwlnknb.sys
fe580000 2e494973 - tcpip.sys fe5a0000 2e5256b8 - afd.sys
fe5b0000 2e5279d3 - netbt.sys fe5d0000 2e4167f7 - netbios.sys
fe5e0000 2e4066b3 - mup.sys fe5f0000 2e4f9f51 - rdr.sys
fe630000 2e53f24a - srv.sys fe660000 2ef16062 - nwlnkspx.sys
Address dword dump Build [1057] - Name
FF541E4c fe5105df fe5105df 00000001 ff640128 fe4a8228 000002fe - NDIS.SYS
ff541e60 fe501368 fe501368 00000246 00004002 00000000 00000000 - elnkii.sys
ff541eb4 fe481509 fe481509 ff6688c8 ff668288 00000000 ff668138 - i8042prt.SYS
ff541ee0 fe481ea8 fe481ea8 fe482078 00000000 ff541f04 8013c58a - i8042prt.SYS
ff541ee4 fe482078 fe482078 00000000 ff541f04 8013c58a ff6688c8 - i8042prt.sys
ff541ef0 8013c58a 8013c58a ff6688c8 ff668040 80405900 00000031 - ntoskrnl.exe
ff541efc 80405900 80405900 00000031 06060606 06060606 06060606 - hal.dll
Restart and set the recovery options in the system control panel
or the /CRASHDEBUG system start option if this message reappears,
contact your system administrator or technical support group.
CRASHDUMP: Initializing miniport driver
CRASHDUMP: Dumping physical memory to disk: 2000
CRASHDUMP: Physical memory dump complete


C:\WINDOWS\system32\cmd.exe
C:\myworkspace>javac numerator1.java
C:\myworkspace>java numerator1
Enter the Numeric:
123
You have Entered:123
java.lang.Exception: Error at:Fri Dec 03 22:03:04 AMT 2010
at numerator1.myNumerator(numerat
at numerator1.main(numerator1.jav
Caused by: java.lang.ArithmeticException:
at numerator1.myNumerator(numerat
... 1 more
C:\myworkspace>

What is the point of formalizing requirements?

Motivating example: mutual exclusion

- 2 processes, 3 shared variables (H. Hyman, 1966)
  - **blocked0**: process 1 (P0) wants to enter
  - **blocked1**: process 2 (P1) wants to enter
  - **turn**: which process is allowed to enter (0 for P0, 1 for P1)

P0

```
while (true) {
    blocked0 = true;
    while (turn!=0) {
        while (blocked1==true) {
            skip;
        }
        turn=0;
    }
    // Critical section
    blocked0 = false;
    // Do other things
}
```

P1

```
while (true) {
    blocked1 = true;
    while (turn!=1) {
        while (blocked0==true) {
            skip;
        }
        turn=1;
    }
    // Critical section
    blocked1 = false;
    // Do other things
}
```

Is the algorithm correct?

The model in UPPAAL (version 1)

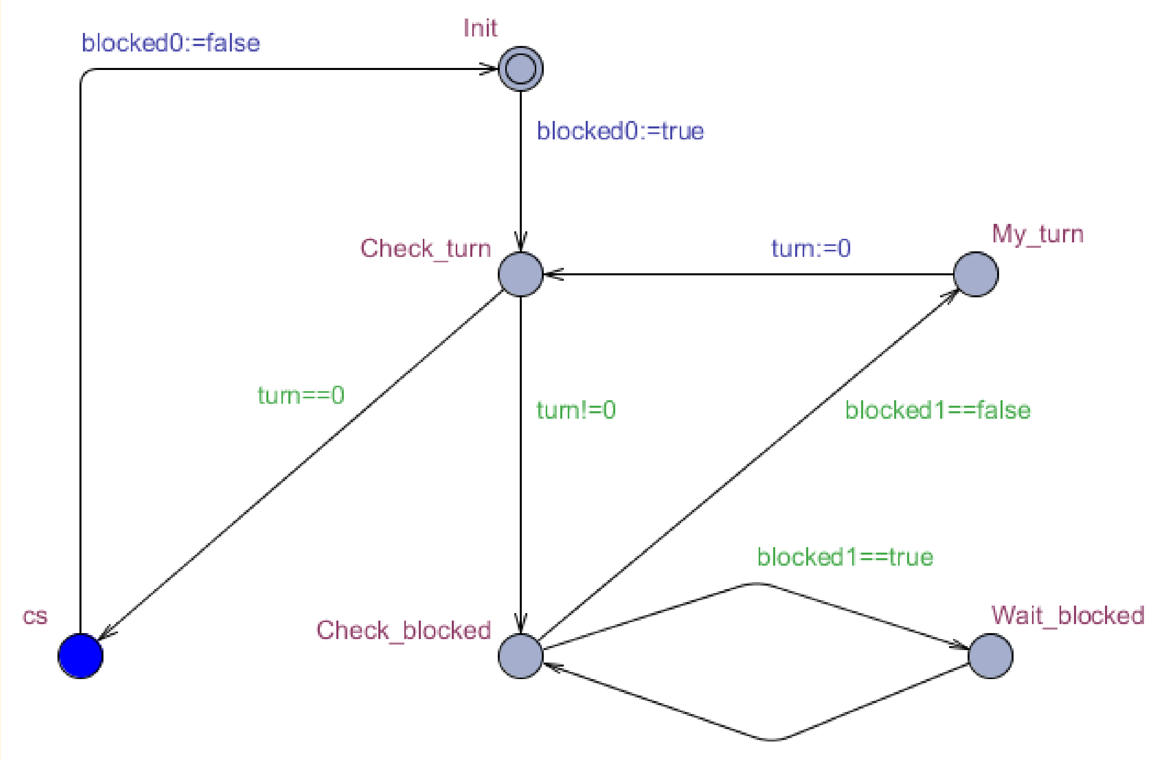
Declarations:

```
bool blocked0;
bool blocked1;
int[0,1] turn=0;
system P0, P1;
```

Modeling idioms used:

- Global variables
- Variables with restricted domain

Automaton P0:



P0

```
while (true) {
    blocked0 = true;
    while (turn!=0) {
        while (blocked1==true) {
            skip;
        }
        turn=0;
    }
    // Critical section
    blocked0 = false;
    // Do other things
}
```

The model in UPPAAL (version 2)

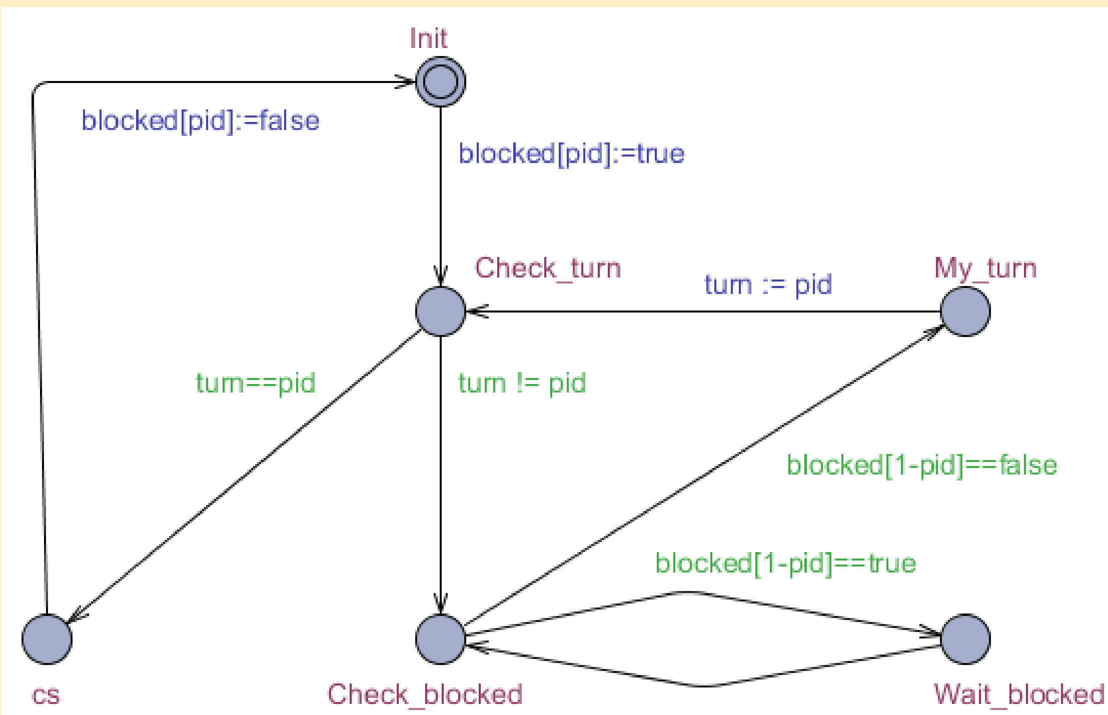
Declarations:

```
bool blocked[2];
int[0,1] turn;
P0 = P(0);
P1 = P(1);
system P0,P1;
```

Template P
with parameter pid:

Modeling idioms used:

- Global variables
- Variables with restricted domain
- Modeling common behavior with templates
- Template instantiation with parameters
- Variables of array type



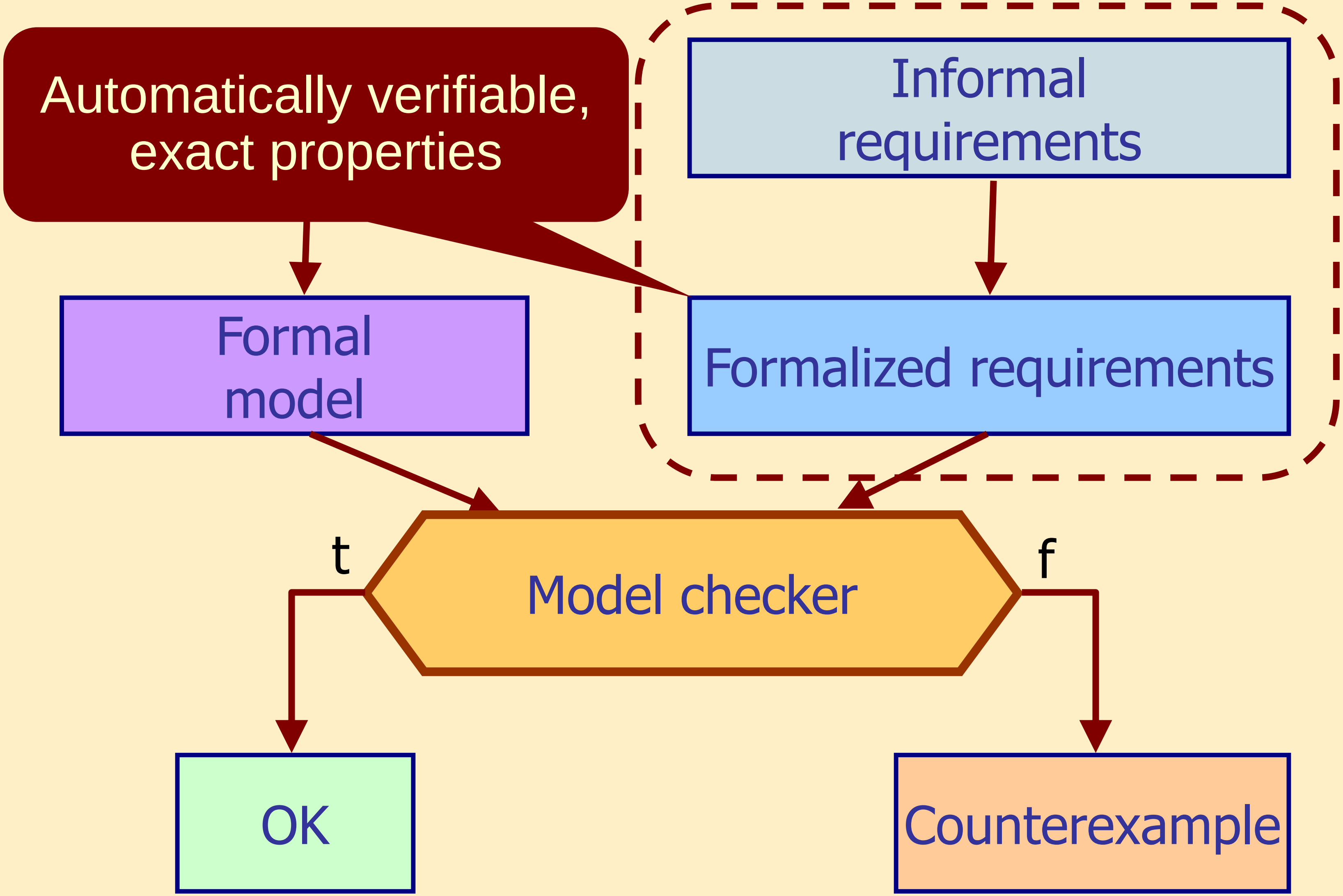
P0

```
while (true) {
    blocked0 = true;
    while (turn!=0) {
        while (blocked1==true) {
            skip;
        }
        turn=0;
    }
    // Critical section
    blocked0 = false;
    // Do other things
}
```

Properties to verifiy

- Mutual exclusion:
  - At most one process is allowed to be in the critical section
- The expected behavior is possible:
  - For P0 it is possible to enter the critical section
  - For P1 it is possible to enter the critical section
- Starvation freedom:
  - P0 will eventually enter the critical section
  - P1 will eventually enter the critical section
- Deadlock freedom:
  - It is not possible that processes are mutually waiting for each other

Our goal


Automatically verifiable, exact properties
Informal requirements
Formal model
Formalized requirements
Model checker
t
f
OK
Counterexample

Establishing and formalizing requirements

What are the typical requirements (in critical systems)?

What to formalize?

Handling textual requirements

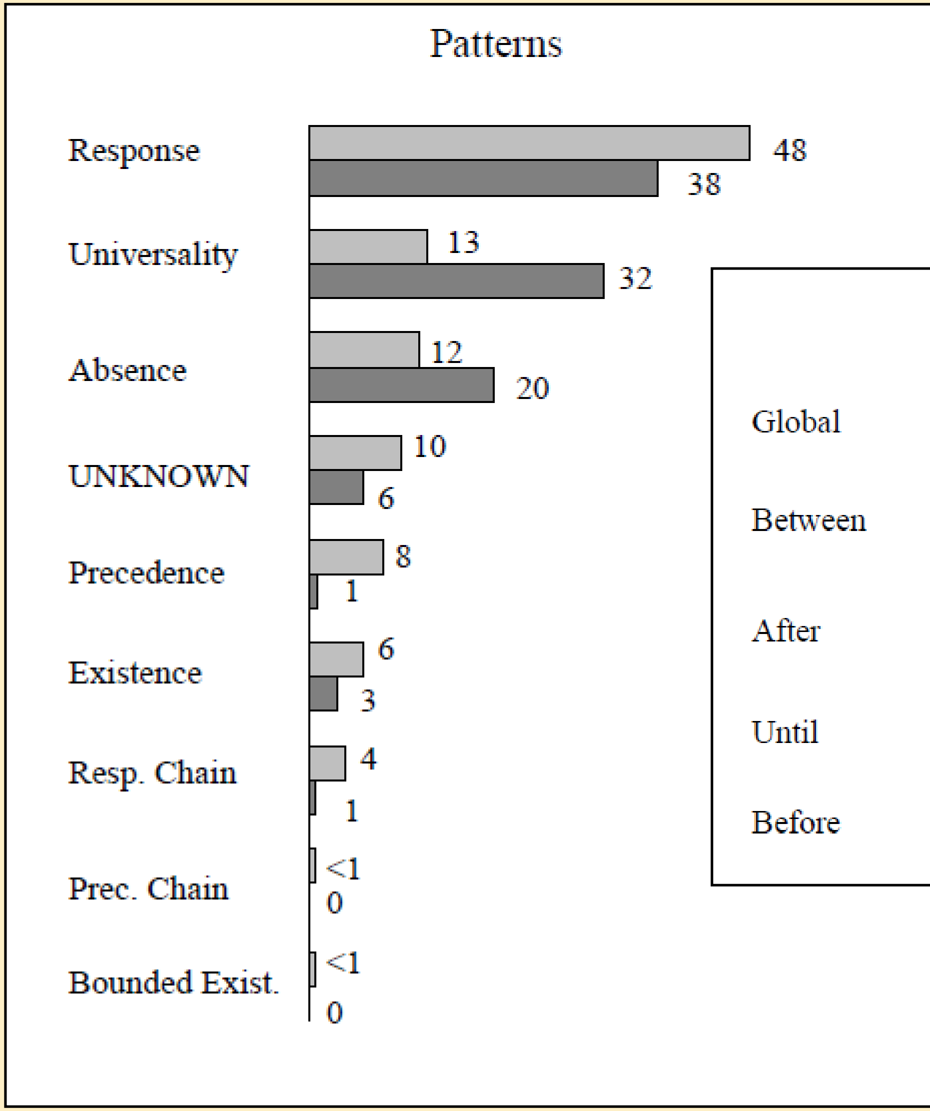
- Specifying a typical requirement: text

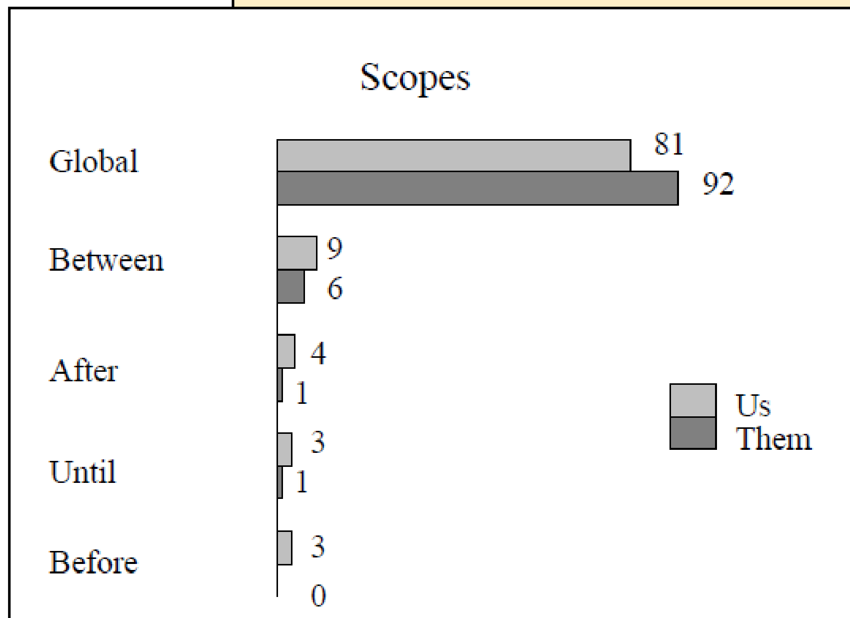
> If `alarm` is on and `alert` occurs, the output of `safety` should be true as long as `alarm` is on.

> If the `switch` is turned to `AUTO`, and the `light intensity` is `LOW` then the `headlights` should stay or turn immediately `ON`, afterwards the `headlights` should continue to stay `ON` in `AUTO` as long as the `light intensity` is not `HIGH`.

  - Is the textual description unambiguous?
  - Structure is not clear
    (condition, requirement, output, timing, ...)

The result of a survey





A significant proportion of requirements match to certain patterns

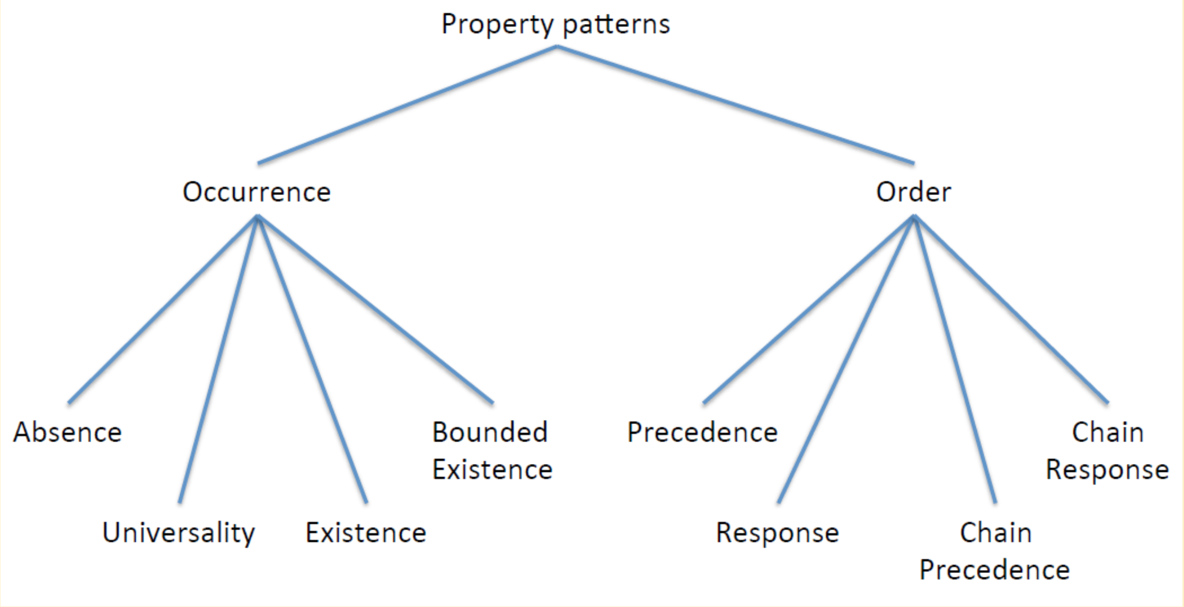
Figures: The distribution of matched patterns for requirements from two development teams

http://patterns.projects.cis.ksu.edu/documentation/patterns.shtml

Groups of patterns

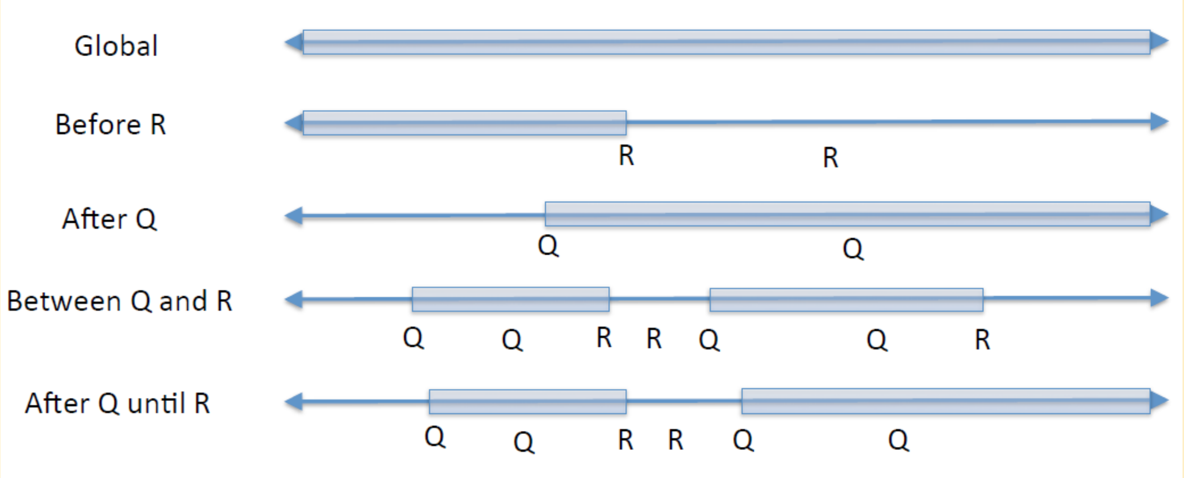
Pattern:

order or occurence



Scope:

relative to further events



Patterns

Occurrence:

- Absence: the referenced state/event never occurs
- Universality: the referenced state/event is always present
- Existence: the referenced state/event eventually occurs
- Bounded existence: the referenced state/event occurs at least k times

Order:

- Precedence: the referenced state/event preceeds an other state/event
- Response: the referenced state/event is proceeded by an other state/event
- Chain precedence: generalization of Precedence to sequences
- Chain response: generalization of Response to sequences

Examples of patterns

- Pattern Response in scope Global:

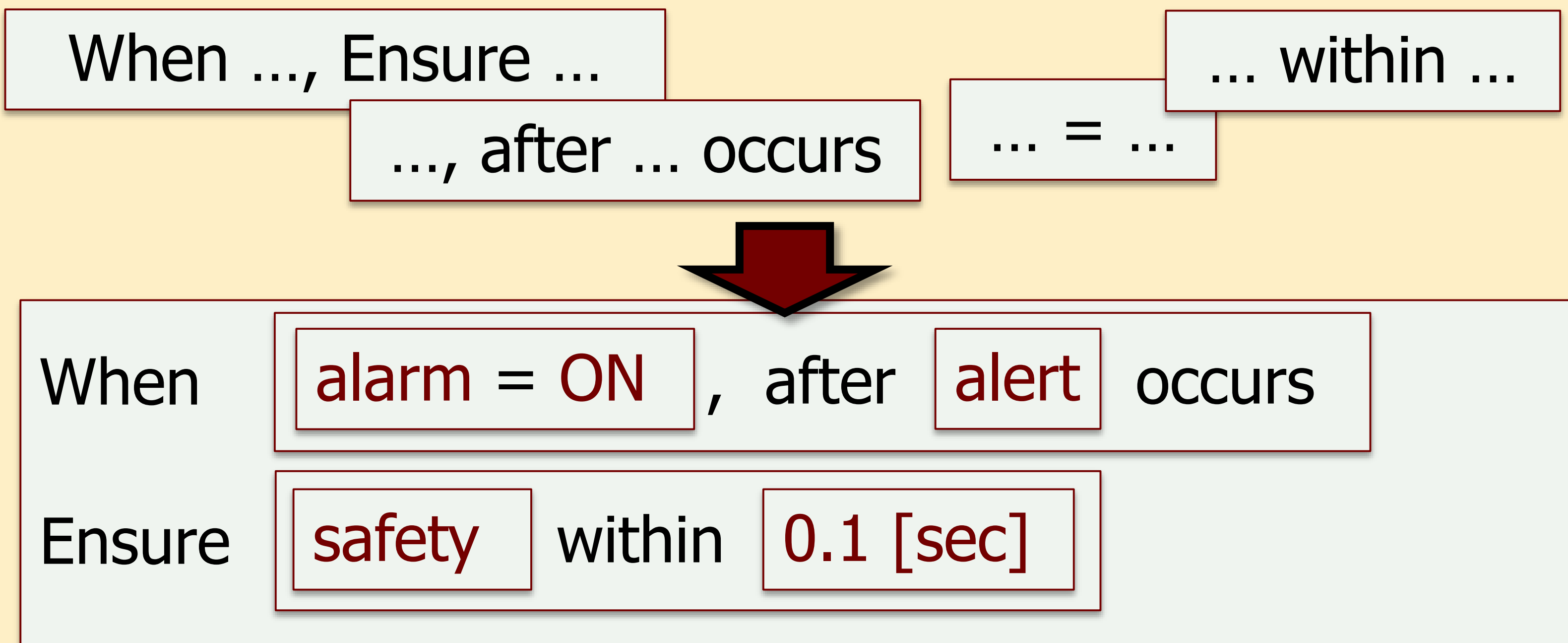
At any time during execution,
if event Request occurs,
then it should be proceeded by either Reply or Reject.

- Pattern Precedence in scope After:

After the occurrence of state NormalMode,
state ResourceGranted may only occur
if it is preceeded by state ResourceRequest.

A typical solution

- The use of textual templates*
  - Composing parameterized patterns
  - More transparent structure
  - Formal semantics can be assigned to the patterns



*examples by Dániel Darvas based on the tool STIMULUS

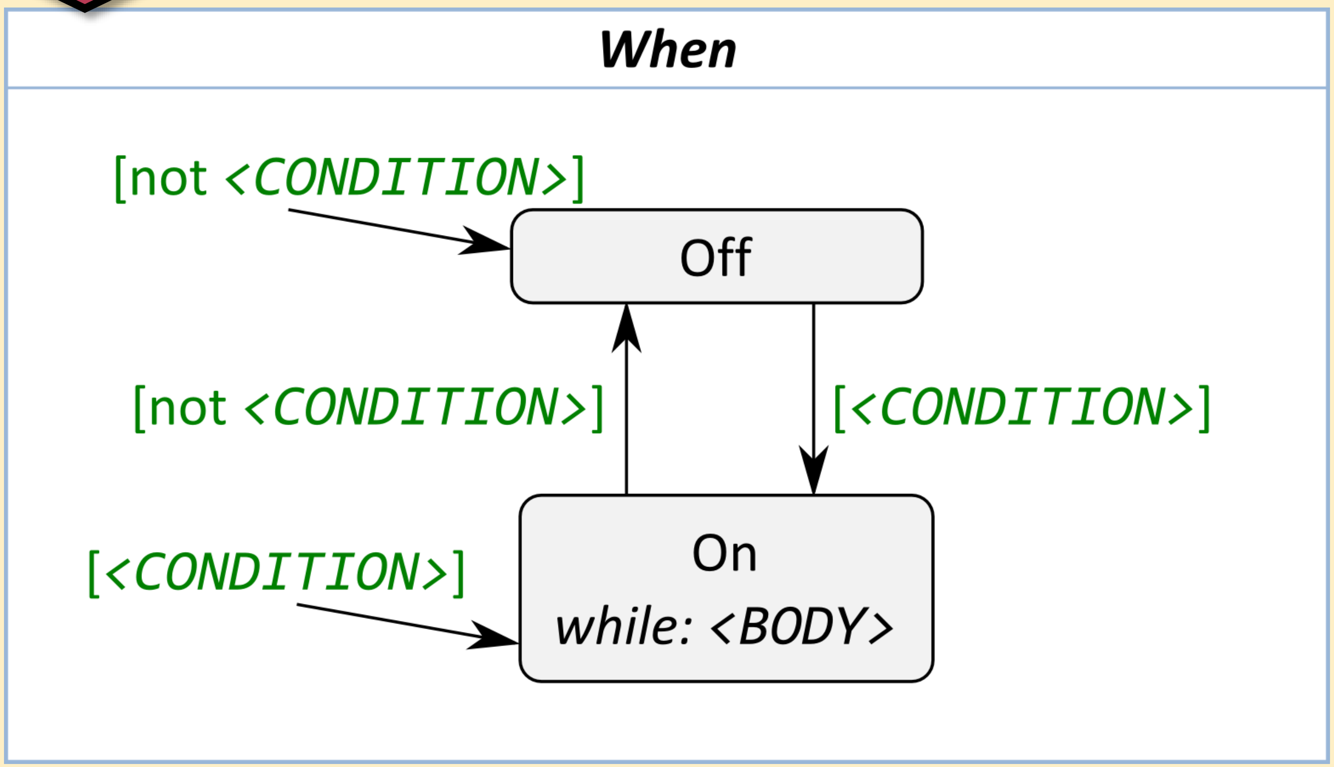
Example: Semantics of a pattern

When *<CONDITION>*, Ensure *<BODY>*

=

When *<CONDITION>* becomes true until it becomes false, do *<BODY>*

Here, the semantics of a block is given by a state machine

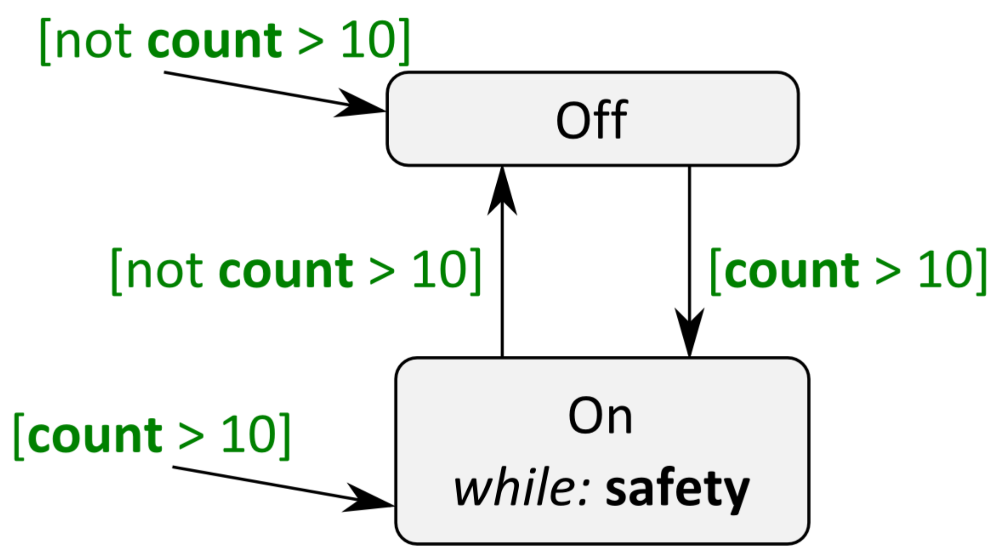


Example: Semantics of composite patterns

When number of occurrences of evt 10

Ensure safety

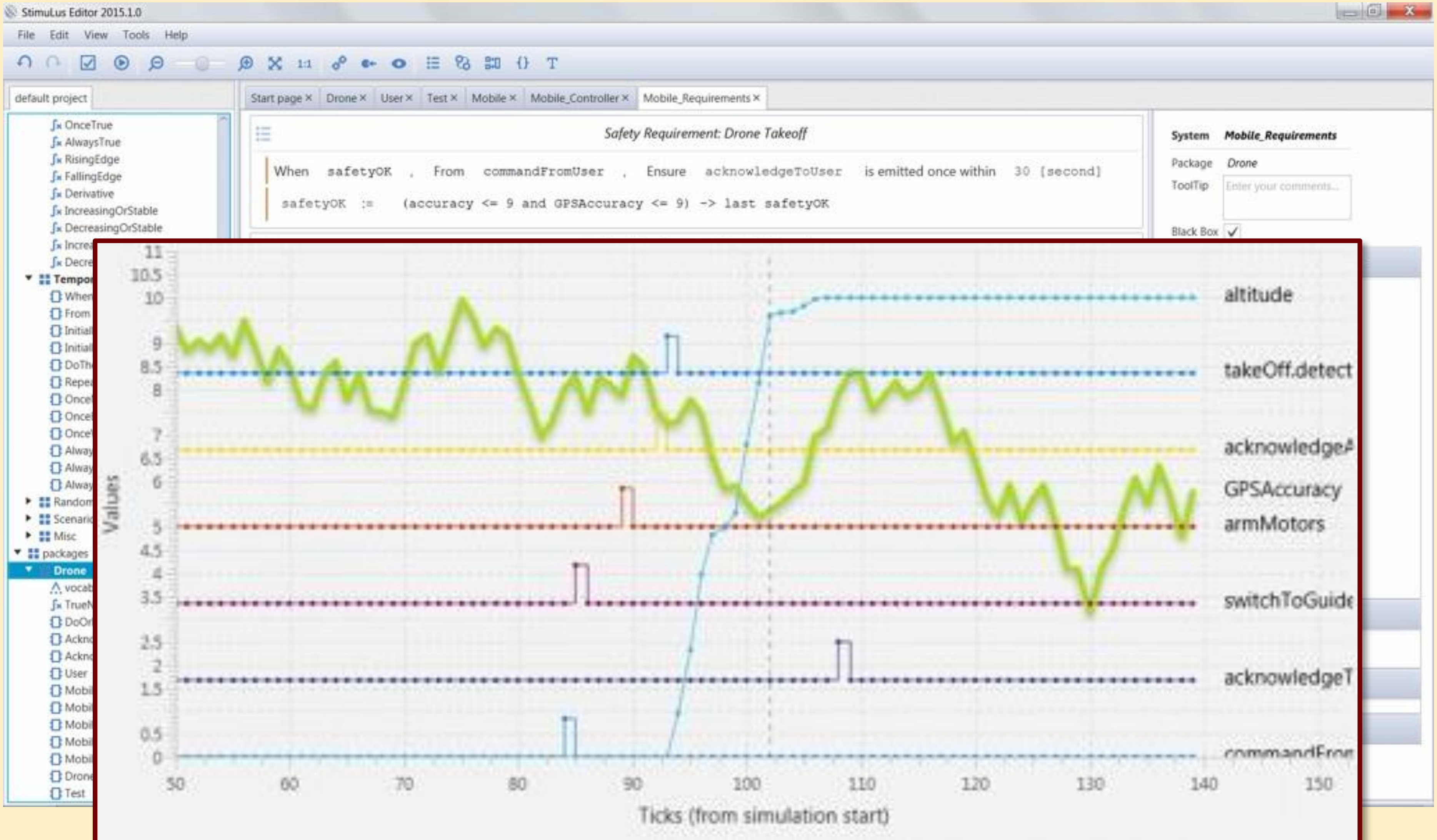
```
count := (0 -> last count) +
     (if evt then 1 else 0);
```



The use of formalized requirements

- Validation
  - Executions can be generated that satisfy the requirement
  - Executions can be evaluated w.r.t. to the requirements
    - Are we specifying what we think we do?
    - Is the set of requirements complete and unambiguous?
- Formal verification
  - Verification of design (models)
- Generating a test oracle
  - Verification of implementation (in a testing environment)
- Documentation
  - Readable, but formalized and validated

Example: Argosim Stimulus tool



http://argosim.com/

The takeaways

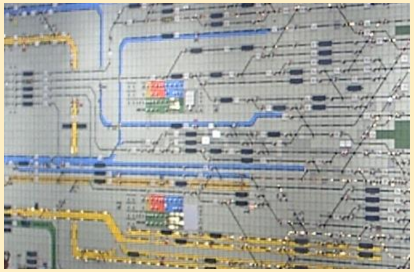
- The majority of properties match certain patterns
  - If … then …, While … ensure …, After …
  - Occurrence/order of states/events
- More complex requirements can be composed from simpler ones
  - Parametrization: properties of a state/event
  - Nesting
- Formalization of requirements helps
  - Analysis of requirements: validity, completeness, consistency
  - Verifiaction of design: exhaustive analysis of executions
  - Test evaluation, runtime monitoring: components can be automatically generated

Temporal requirements

What kind of requirements do we formalize?

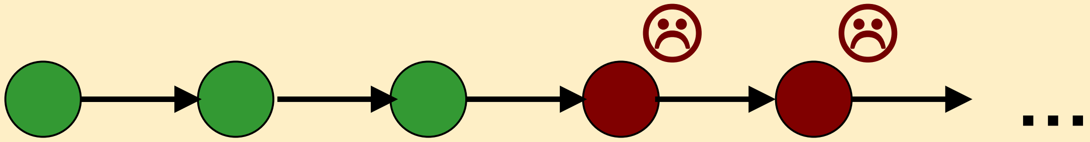
- Verification: Model <-> Many requirements
  - Functional: logically correct behavior <- our current goal
  - Extra-functional: performance, reliability, … <- later
- Goal: verifying reachability of states
  - System (model): we know local properties of states
    - Name, valuation of variables, mode of operation, …
  - Requirements: order of occurrence of states
    - Is a desirable state reachable? -> Liveness properties
    - Are we avoiding dangerous states? -> Safety properties

    Can be verified by exhaustive expolation of the state space!
- Important in state based, event driven systems

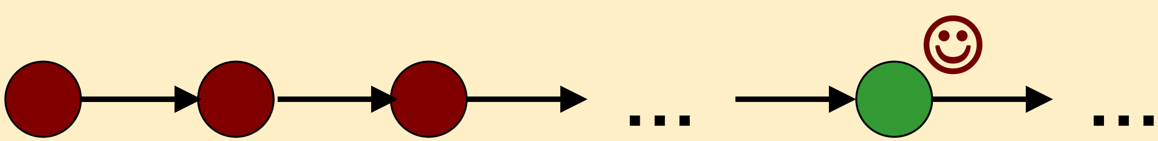

Safety properties

- Expresses freeness from dangerous situations
  - "In all states, the pressure is below the critical level"
  - "The press machine only operates with closed barriers."
- Examples from Computer Science:
  - Deadlock freedom: no deadlock can occur
  - Mutual exclusion: at most one process in the critical section
  - Data confidentiality: no unauthorized accesses
- Universal property on reachable states:
  - "In all reachable states it holds that …"
  - Formulates an invariant
- If a sequence of states violates it:
  then already a finite prefix of the sequence violates it



Liveness properties

- Expresses reachability of a desirable state
  - "After start the press machine emits the finished product."
  - "After the disturbance the system stabilizes."
- Example from Computer Science:
  - "The process gets served"
  - "The sent message arrives"
  - "The process provides the expected result on its output"
- Existential property on reachable states
  - "There exists a reachable state such that …"
  - Formulates occurrence
- If a sequence of states violates it:
  then it can be extended so that it satisfies the property

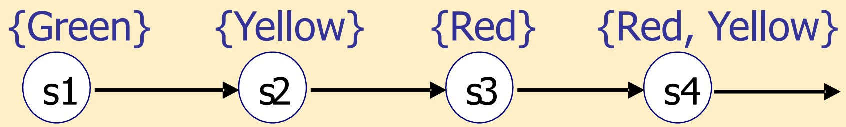


What kind of description language is needed?

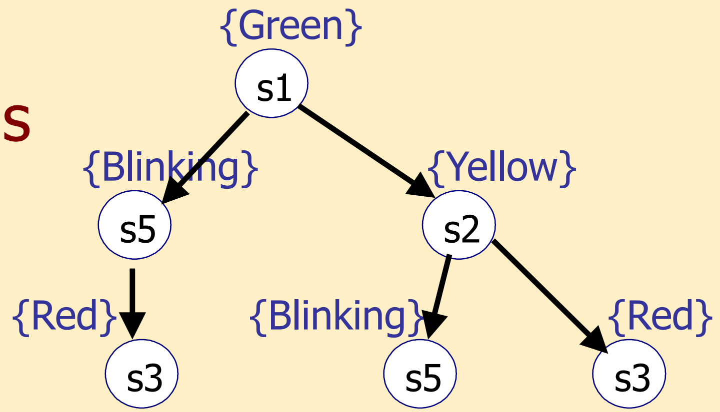
- Reachability: occurrence and order of states
  - Order: logical time
    - Current point in time: current state
    - Subsequent points in time: next state(s)
  - Temporal connectives can be used to express requirements
- Temporal logics:
  - Formal system for evaluating changes in logical time
  - Temporal connectives:
    "always", "at some point", "before", "while" …
    (correspond to typical requirement patterns)

Classification of temporal logics

- Linear:
  - We consider individual executions of the system
  - Each state has exactly one subsequent state
  - Logical time along a linear timeline (trace)



- Branching:
  - We consider trees of executions of the system
  - Each state possibly has many subsequent state
  - Logical time along a branching timeline (computation tree)



Temporal logics

Where can we use temporal logics?

- Goal: examining the state space

The simplest mathematical model: Kripke structure

- We express local properties of states by labeling

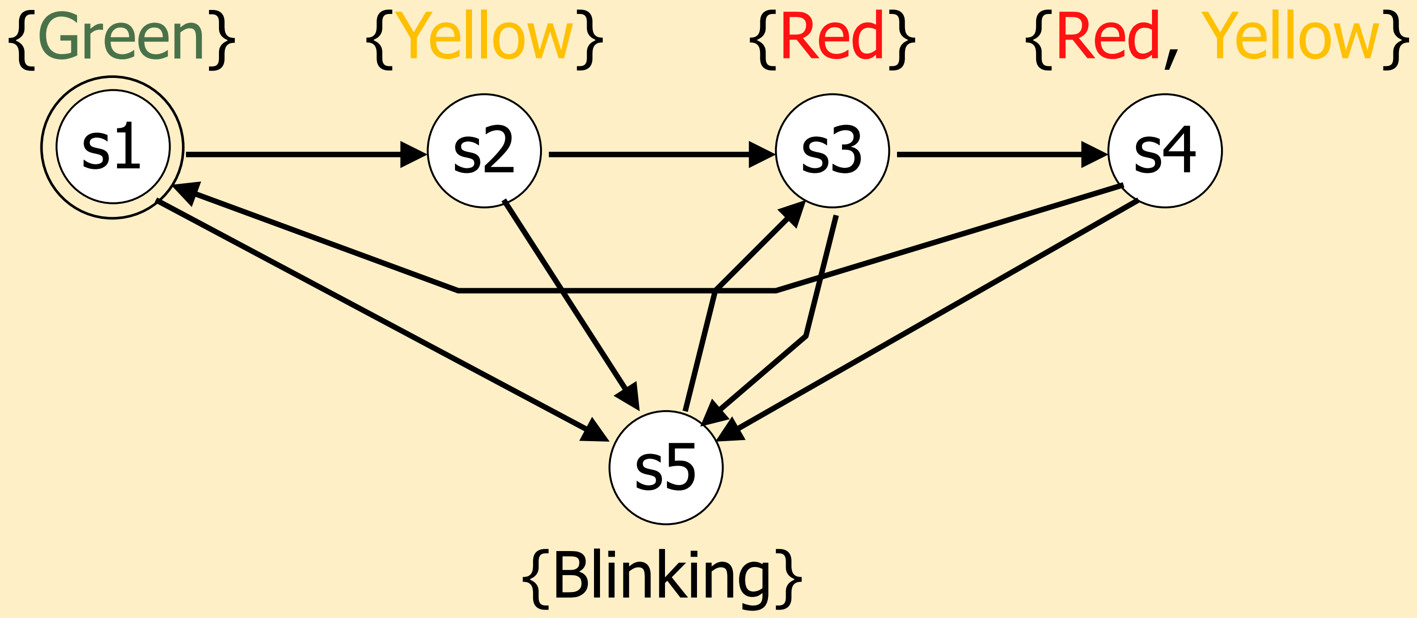
A Kripke structure KS over a set of atomic propositions $AP = \{P, Q, R, \dots\}$ is a tuple (S, I, R, L) where

- $S = \{s_1, s_2, \dots, s_n\}$ is a finite set of states,
- $I \subseteq S$ is the set of initial states,
- $R \subseteq S \times S$ is the set of transitions and
- $L : S \rightarrow 2^{AP}$ is the labeling of states by atomic propositions

Example for KS

Traffic light

- $AP = \{\text{Green}, \text{Yellow}, \text{Red}, \text{Blinking}\}$
- $S = \{s_1, s_2, s_3, s_4, s_5\}$

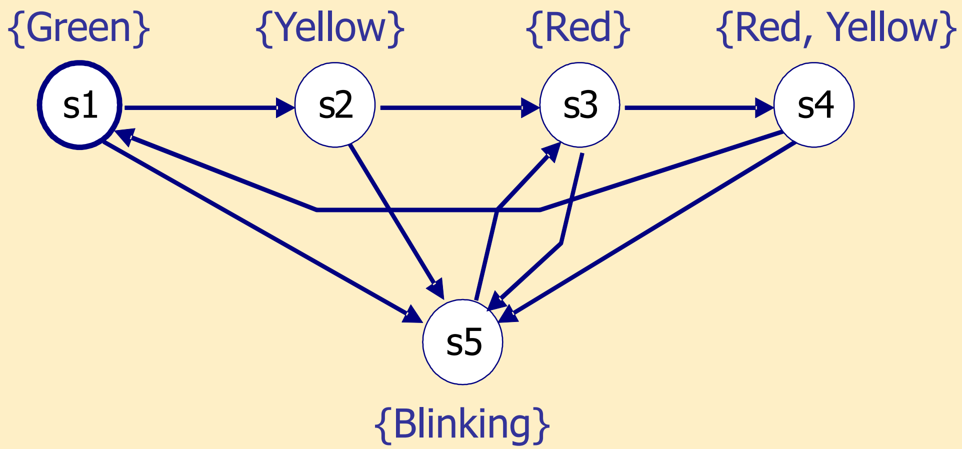


Linear Temporal Logic: LTL

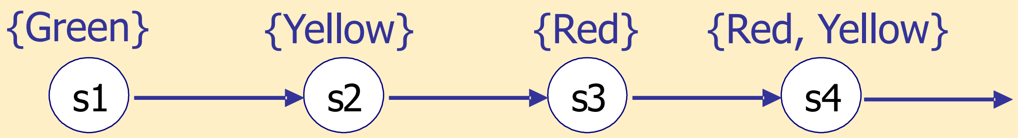
Linear Temporal Logic

- Interpreted over paths of a Kripke structure
  - e.g. the effects of a concrete input

The model (KS):



A path (sequence of states):



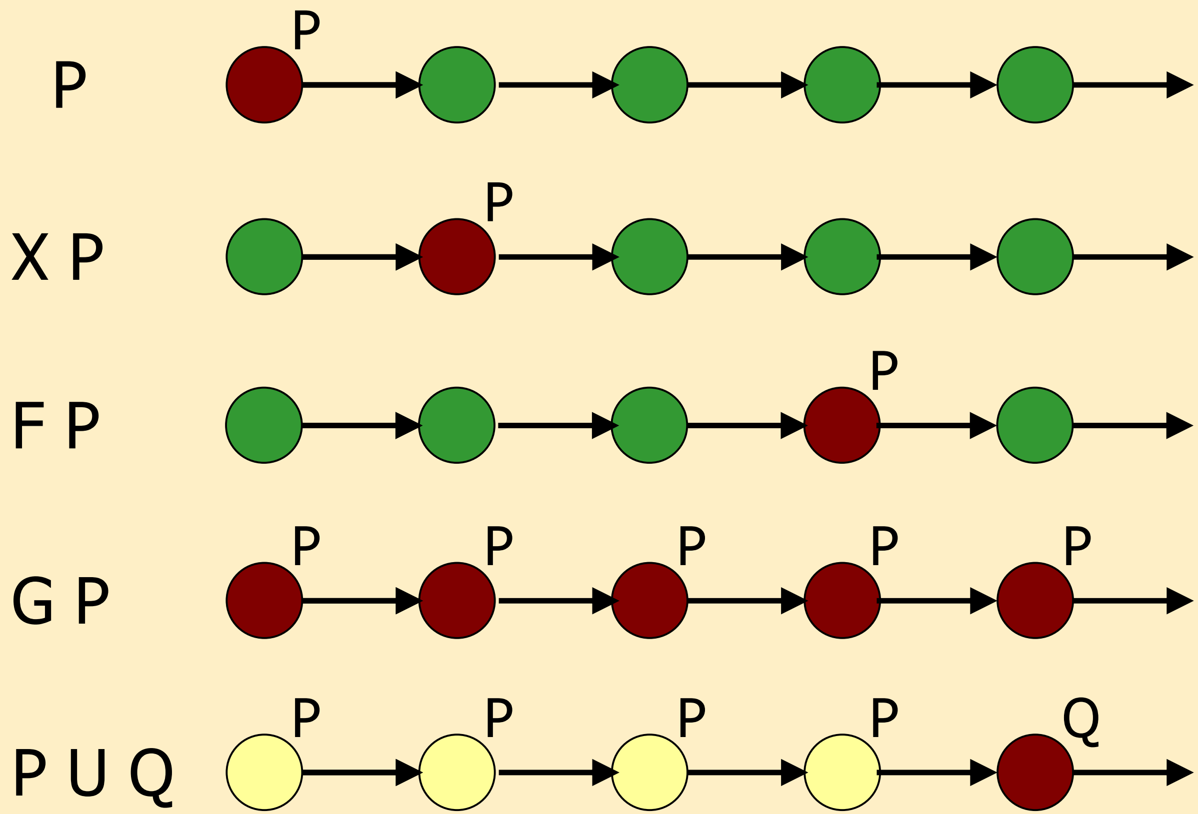
Linear temporal logic – Formulas

Construction of formulas: p, q, r, ...

- Atomic propositions (elements of AP): P, Q, ...
- Boolean connectives: $\wedge$, $\vee$, $\neg$, $\Rightarrow$
  $\wedge$: conjunction, $\vee$: disjunction, $\neg$: negation , $\Rightarrow$: implication
- Temporal connectives: X, F, G, U informally:
  - X p: "neXt p"
    p holds in the next state
  - F p: "Future p"
    p holds somewhere on the subsequent path
  - G p: "Globally p"
    p holds in all states of the subsequent path
  - p U q: "p Until q"
    p holds at least until q, which holds at the subsequent path

LTL temporal connectives

For a path of a Kripke structure



LTL examples I.

- $p \Rightarrow Fq$

  If p holds in the initial state, then eventually q holds.
  - Example: Start $\Rightarrow$ F End
- $G(p \Rightarrow Fq)$

  For all states, if p holds, then eventually q holds.
  - Example: G (Request $\Rightarrow$ F Reply)
    For a request, a reply always arrives
- $p \; U \; (q \vee r)$

  Starting from the initial state, p holds until q or r eventually holds.
  - Example: Requested U (Accept $\vee$ Refuse)
    A continuous request either gets accepted or refused
- $(p \wedge G(p \Rightarrow Xp)) \Rightarrow Gp$

  Formalization of the mathematical induction principle - always holds

LTL examples II.

- GF p

  After any states along the path, p will eventually hold

  - There is no state after which p does not hold eventually
  - Example: GF Running
    The start state is reached from all states

- FG p

  After some state, p will continuously hold.

  - Example: FG Normal
    After an initial transient the system operates normally

Formalizing requirements: Example

Consider an air conditioner with the following modes:

AP={Off, On, Error, MildCooling, StrongCooling, Heating, Ventilating}

- Potentially more than one labels!
  - E.g. {On, Ventilating}
- When formalizing requirements, we might not yet know all potential behaviors
  - We assume only the labels on states

Example (cont.)

AP={Off, On, Error, MildCooling, StrongCooling, Heating, Ventilating}

- The air conditioner can (and will) be turned on:

  $\mathbf{F}$ On
- At some point, the air conditioner always breaks down

  $\mathbf{GF}$ Error
- If the air conditioner breaks down, it eventually gets repaired

  $\mathbf{G}\ (\text{Error} \Rightarrow \mathbf{F}\ \neg\text{Error})$
- If the air conditioner breaks down, it cannot heat:

  $\mathbf{G}\ \neg(\text{Error} \wedge \text{Heating})$

Example (cont.)

AP={Off, On, Error, MildCooling, StrongCooling, Heating, Ventilating}

- The air conditioner can only break down when turned on:

  $\mathbf{G}\ (\mathbf{X}\ \text{Error} \Rightarrow \text{On})$

- After heating, the air conditioner must ventilate:

  $\mathbf{G}\ ((\text{Heating} \wedge \mathbf{X}\ \neg\text{Heating}) \Rightarrow \mathbf{X}\ \text{Ventilate})$

  but it may also break down:

  $\mathbf{G}\ ((\text{Heating} \wedge \mathbf{X}\ \neg\text{Heating}) \Rightarrow \mathbf{X}\ (\text{Ventilate} \vee \text{Error}))$

- After ventilation the air conditioner must not cool strongly until it performs some mild cooling:

  $\mathbf{G}\ ((\text{Ventilating} \wedge \mathbf{X}\ \neg\text{Ventilating}) \Rightarrow \mathbf{X}(\neg\text{StrongCooling}\ \mathbf{U}\ \text{MildCooling}))$

LTL formal intepretation

- So far we discussed the logic only informally
  Questions arise, e.g.:
  - Does F p hold if p holds in the first state?
  - Does p U q hold if q holds in the first state?
- To enable formal verifiaction, we need the following:
  - Syntax:
    What are the well-formed formulas?
  - Semantics:
    When does a given formula hold for a given model?

LTL syntax

The set of well-formed formulas (wff) in LTL are given as follows.

Let $P \in AP$ and p and q be wffs. Then

- **L1**: P is a wff.
- **L2**: $p \wedge q$ and $\neg p$ are wffs.
- **L3**: $p \, \mathsf{U} \, q$ and $\mathsf{X} \, q$ are wffs.

Precedence rules:

$$\mathsf{X}, \mathsf{U} > \neg > \wedge > \vee > \Rightarrow > \equiv$$

Derived connectives

- true holds for all states
  false holds in no state
- $p \vee q$ means $\neg(\neg p \wedge \neg q)$
  $p \Rightarrow q$ means $\neg p \vee q$
  $p \equiv q$ means $p \Rightarrow q \wedge q \Rightarrow p$
- F p means true U p
  G p means $\neg F(\neg p)$
  p WU q means $G(p) \vee (p \text{ U } q)$
- "Before" connective:
  p WB q = $\neg((\neg p) \text{ U } q)$ (weak before)
  p B q = $\neg((\neg p) \text{ U } q) \wedge F\ q$ (strong before)

Informally:
p must occur before q
(B: and q must occur)

LTL semantics – Notation

- $M = (S, I, R, L)$ Kripke structure
- $\pi = (s_0, s_1, s_2, \ldots)$ a path of M where
  $s_0 \in I$ and $\forall i \geq 0$: $(s_i, s_{i+1}) \in R$
  - $\pi^i = (s_i, s_{i+1}, s_{i+2}, \ldots)$ the suffix of π from i

- $M,\pi \models p$ denotes (logical entailment):
  In Kripke structure M, along path π, p holds

The semantics of LTL defines when a wff holds over a path (i.e. it defines the entailment relation).

LTL semantics

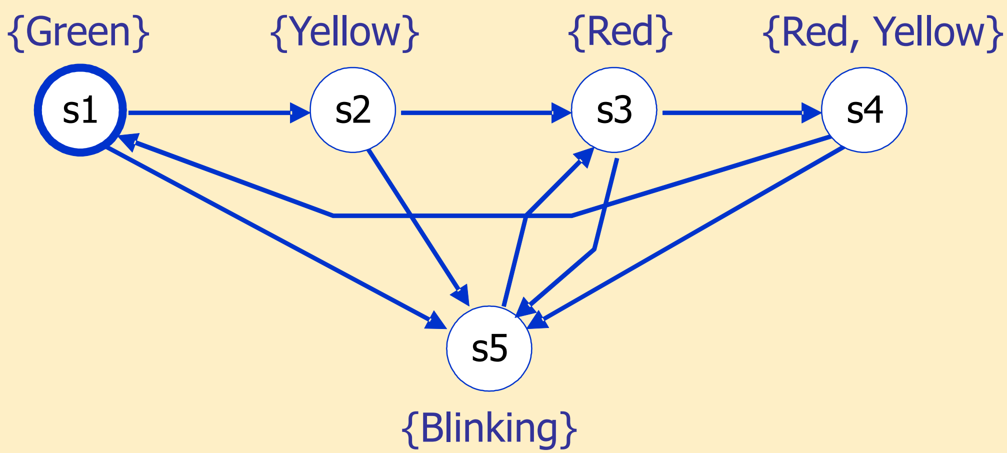
Defined recursively w.r.t. syntactic construction rules

- **L1**: $M,\pi \models P$ iff $P \in L(s_0)$
- **L2**: $M,\pi \models p \wedge q$ iff $M,\pi \models p$ and $M,\pi \models q$
  $M,\pi \models \neg q$ iff not $M,\pi \models q$.
- **L3**: $M,\pi \models (p \; U \; q)$ iff
  $\pi^j \models q$ for some $j \geq 0$ and
  $\pi^k \models p$ for all $0 \leq k < j$

$M,\pi \models X \; p$ iff $\pi^1 \models p$

Interpreting LTL formulas, example

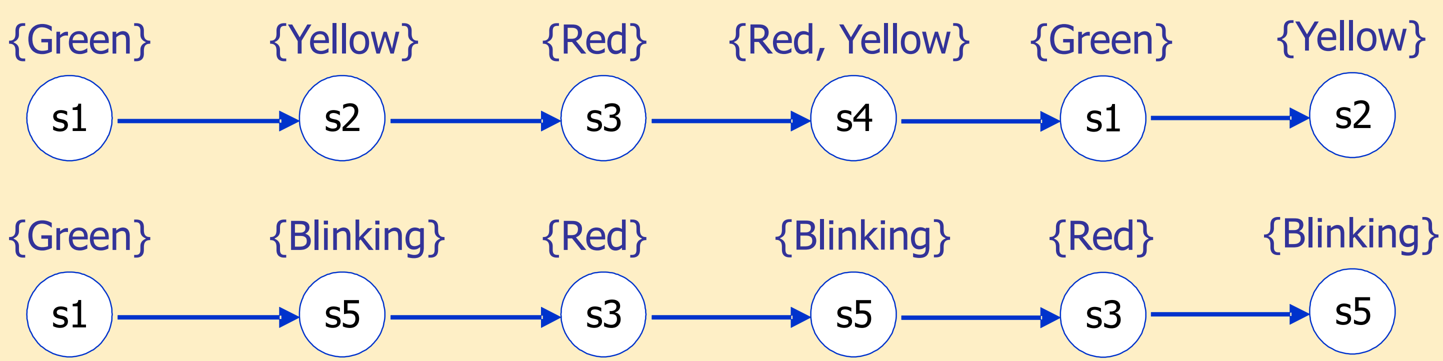
- Kripke structure M :



- Paths:

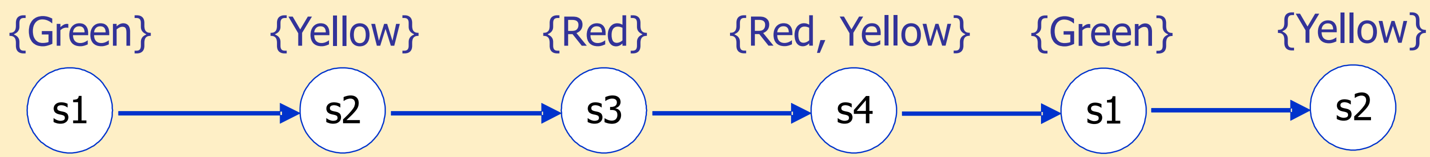
M,π_1

M,π_2



Examples (cont.)

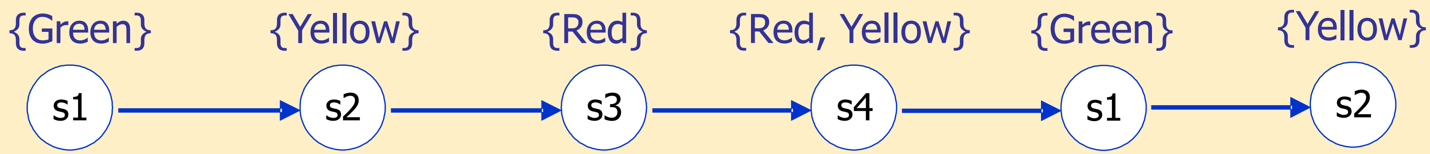
M,π_1



- $M,\pi_1 \models$ Green, as Green $\in L(s_1)$
- not $M,\pi_1 \models$ Red, as Red $\notin L(s_1)$
- not $M,\pi_1 \models$ Green U Red,
  as Red $\notin L(s_1)$, Red $\notin L(s_2)$ and Green $\notin L(s_2)$
- $M,\pi_1 \models$ F Red, as Red $\in L(s_3)$
  More precisely: $\pi_1^2 \vDash Red$

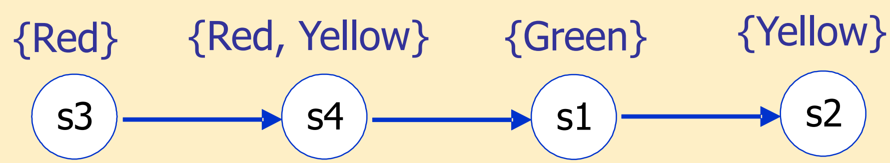
Examples (cont.)

M,π_1



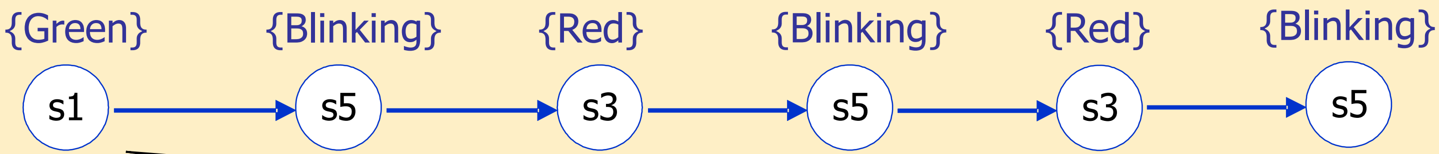
- $M,\pi_1 \models F\ (Red\ U\ Green)$,
  as there exists a suffix for which
  (Red U Green) holds:

π_1^2



Examples (cont.)

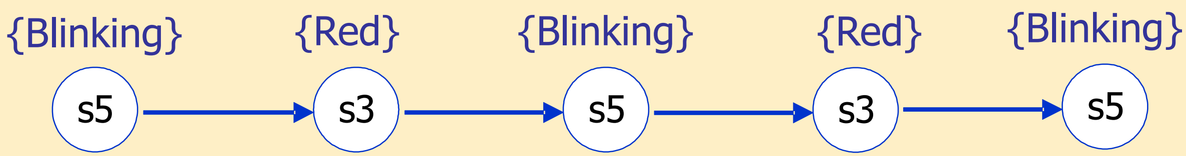
M, π_2



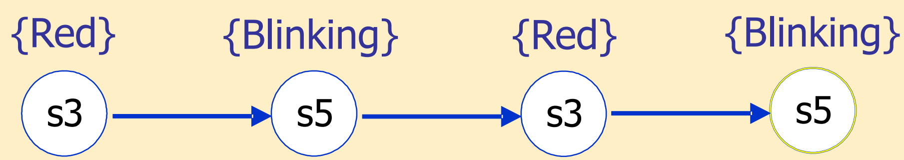
Also true for π_2^0:
$p \Rightarrow q$ is true if p is false

- $M, \pi_2 \models F\,(\text{Blinking} \Rightarrow X\ \text{Red})$,
  as there exists a suffix such that Blinking $\Rightarrow$ X Red

π_2^1

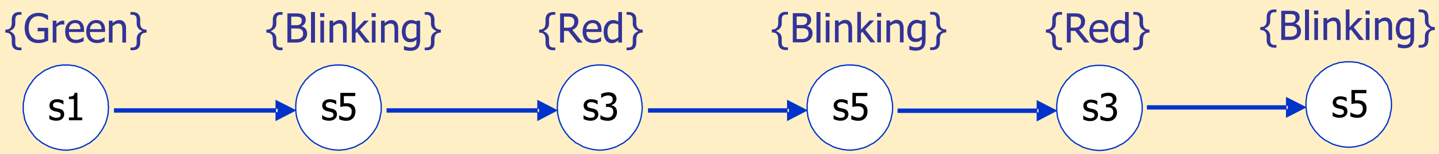


π_2^2



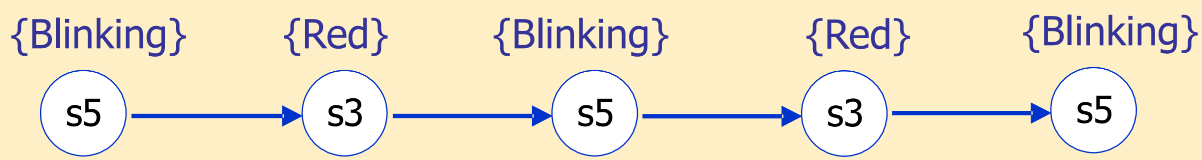
Examples (cont.)

M,π_2



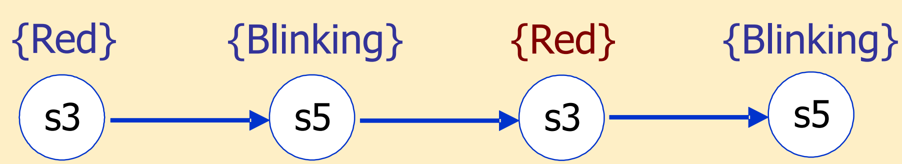
- M,π_2 |= XF (XX Red), as for suffix

$\pi_2{}^1$



F (XX Red) holds, as
it has a suffix such that XX Red holds:

$\pi_2{}^2$



Extending LTL for LTSs

- Expresses properties of transitions: labeling by actions
- Exactly one action per transition
- Application: modeling of communication and protocols

A labeled transition system LTS over a set of actions $Act = \{a, b, c, \dots\}$ is a triple $(S, I, \rightarrow)$ where

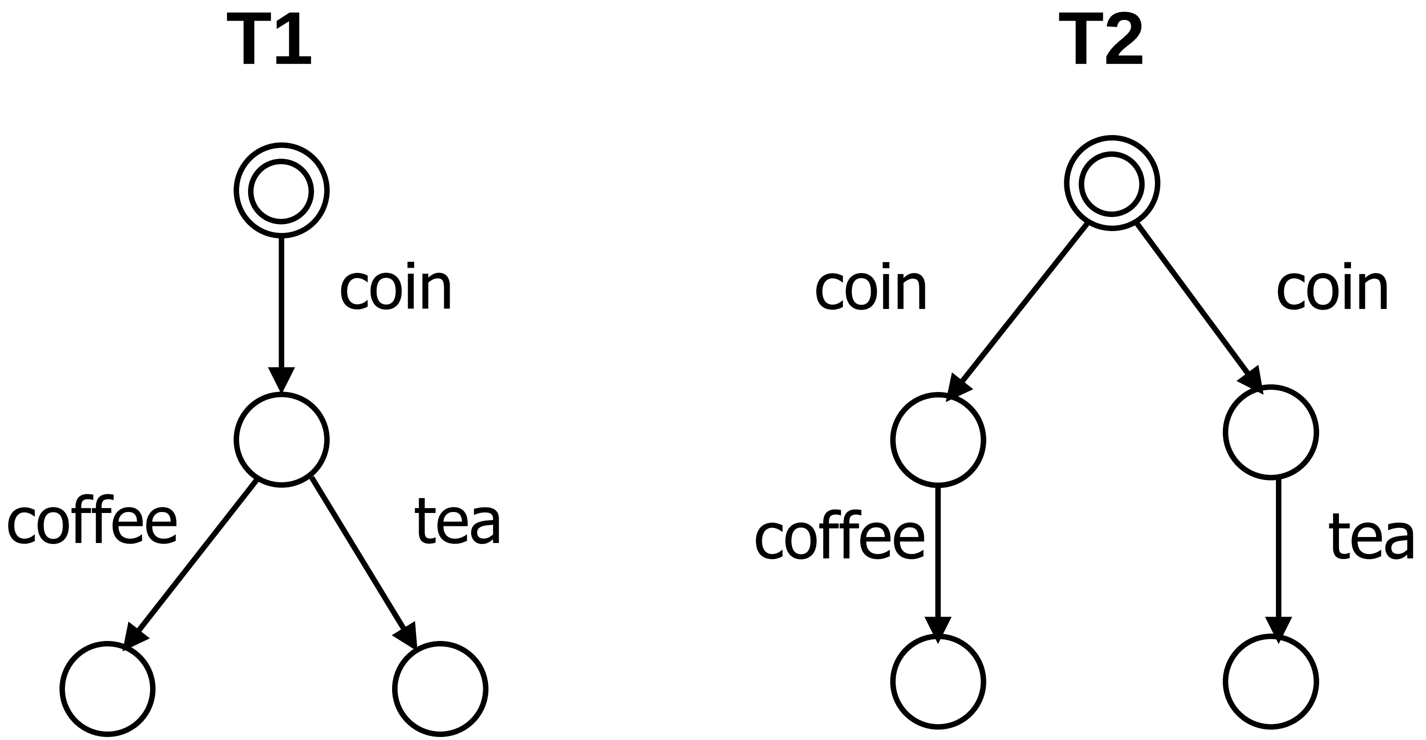
- $S = \{s_1, s_2, \dots, s_n\}$ is a finite set of states,
- $I \subseteq S$ is the set of initial states,
- $\rightarrow : S \times Act \times S$ is the set of transitions

We denote by $s \xrightarrow{a} s'$ iff $(s, a, s') \in \rightarrow$.

Example for LTS

Vending machine

- $Act = \{\text{coin}, \text{coffe}, \text{tea}\}$



LTL for LTSs

A path now is an alternating sequence of states and actions:

- $\pi = (s_0, a_1, s_1, a_2, s_2, a_3, \ldots)$

Extending syntax:

- **L1***: If $a \in Act$ then (a) is a wff.

The corresponding case in semantics:

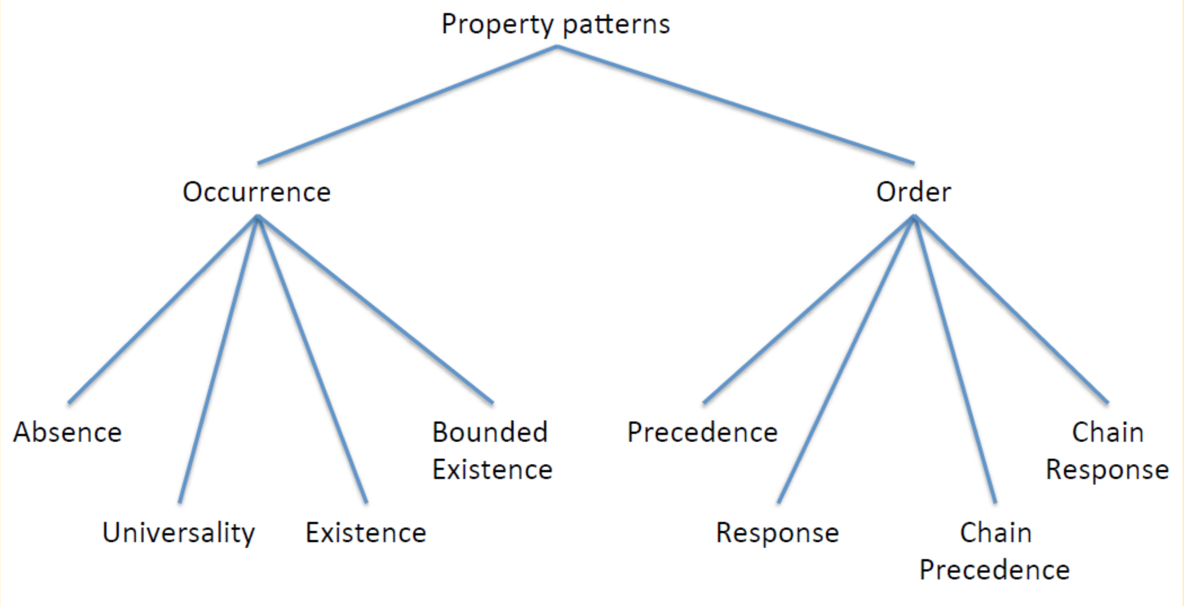
- **L1***: $M,\pi \models (a)$ iff. $a_1=a$
  where a_1 is the first action in π.

This way we can describe requirements of communicating systems.

Where we started:
typical patterns for requirements

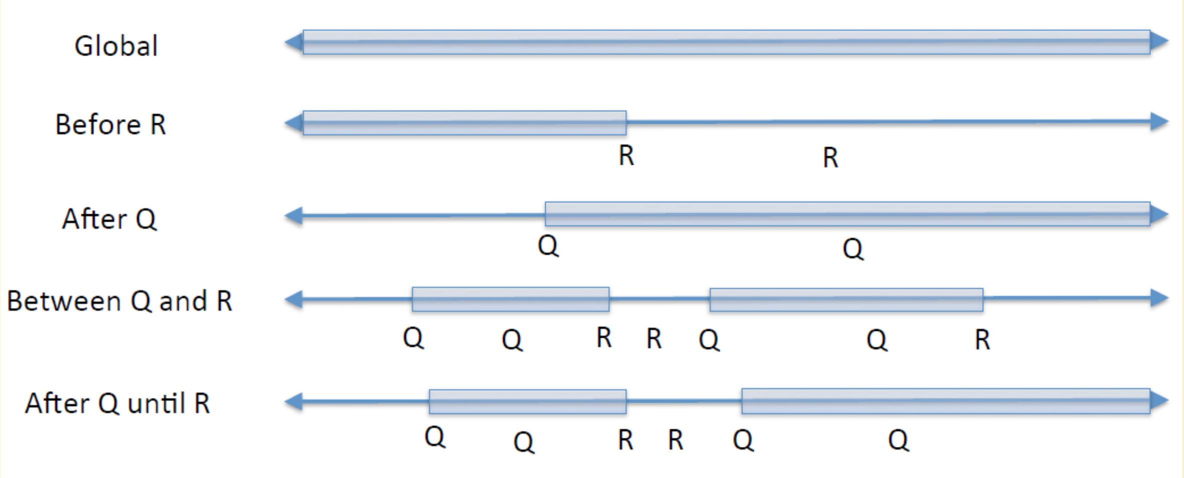
Pattern:

order or
occurence



Scope:

relative to
further events



Formalization of patterns (examples)

After is inclusive, *Before* is exclusive

Universality within scope	**Property in LTL**
P occurs in each step of the execution globally.	$G\ P$
P occurs in each step of the execution before Q.	$F\ Q \rightarrow (P\ U\ Q)$
P occurs in each step of the execution after Q.	$G(Q \rightarrow G\ P)$
P occurs in each step of the execution between Q and R.	$G((Q \wedge \neg R \wedge F\ R) \rightarrow (P\ U\ R))$

Existence within scope	**Property in LTL**
P occurs in the execution globally.	$F\ P$
P occurs in the execution before Q.	$\neg\ Q\ WU\ (P \wedge \neg\ Q)$
P occurs in the execution after Q.	$G\ (\neg Q) \vee F\ (Q \wedge F\ P)$
P occurs in the execution between Q and R.	$G((Q \wedge \neg R \wedge F\ R) \rightarrow (\neg R\ U\ (P \wedge \neg R)))$

Formalization of textual requirements (examples)

If α and β holds, then α has to remain true as long as β is true as well.

$$\mathbf{G}\big((\alpha \wedge \beta) \rightarrow (\alpha\ \mathbf{U}\ \neg\beta)\big)$$

If `alarm` is on and `alert` occurs, the output of `safety` should be true as long as `alarm` is on.

$$\mathbf{G}((\text{alarm} = \text{ON} \wedge \text{alert}) \rightarrow \text{X}(\text{safety}\ \mathbf{U}\ \neg\text{alarm}))$$

LTL summary

- Formalization of requirements
- Temporal logics
  - Linear temporal logic
  - Branching time temporal logic
- LTL
  - Connectives
  - Syntax
  - Semantics
- Interpretation of LTL formulas
- Formalization of requirements in LTL