Equivalence checking

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Intro: Equivalence and refinement checking in model based design

Refining statechart models

Properties expected from refinement relations

Introduction: Relations between models

- Equivalence between models:

Reference model ↔ Modified model

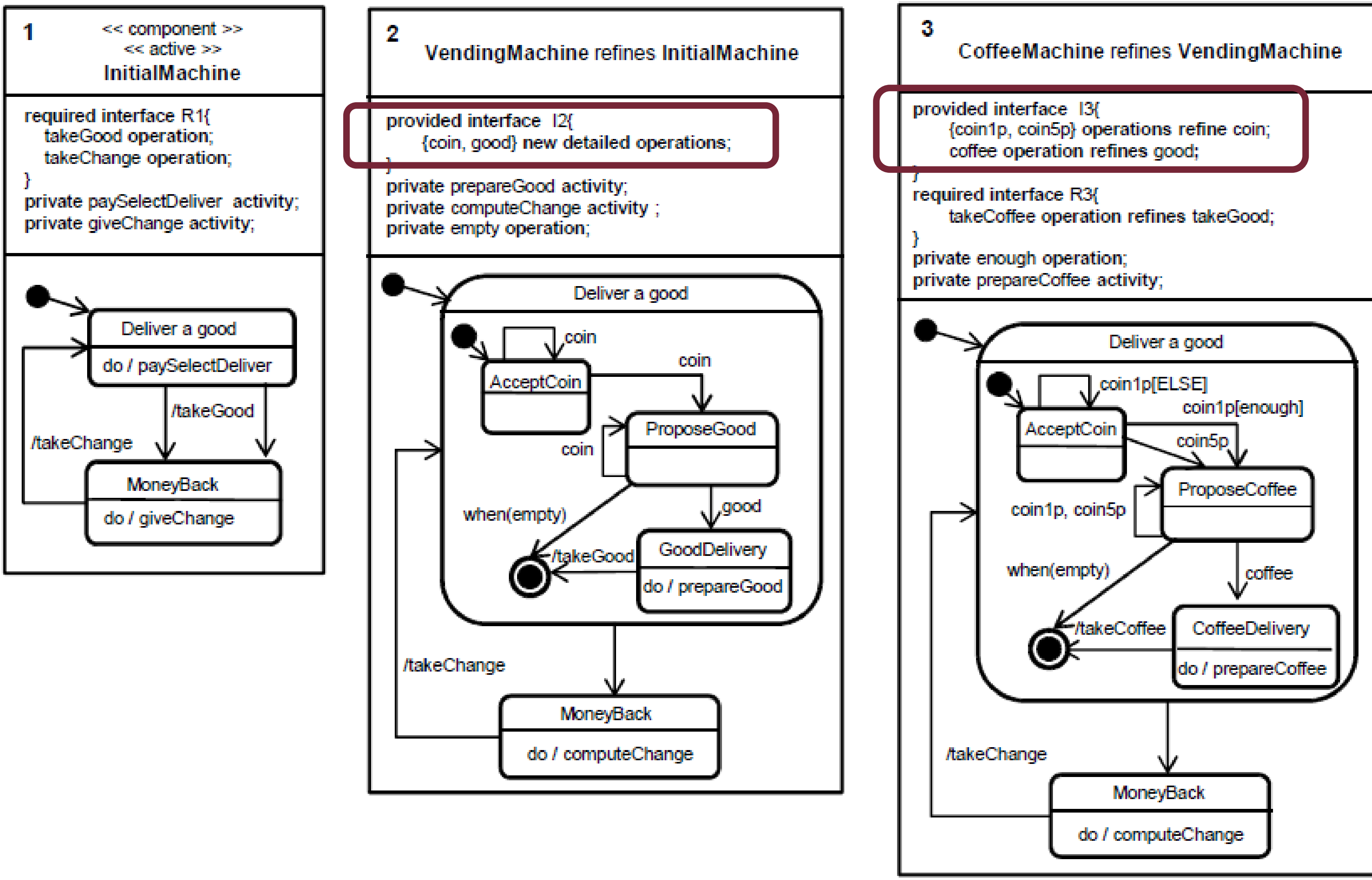
Specification (abstract) ↔ Implementation (concrete, more detailed)

Expected behavior ↔ Provided behavior (e.g., protocol layers)

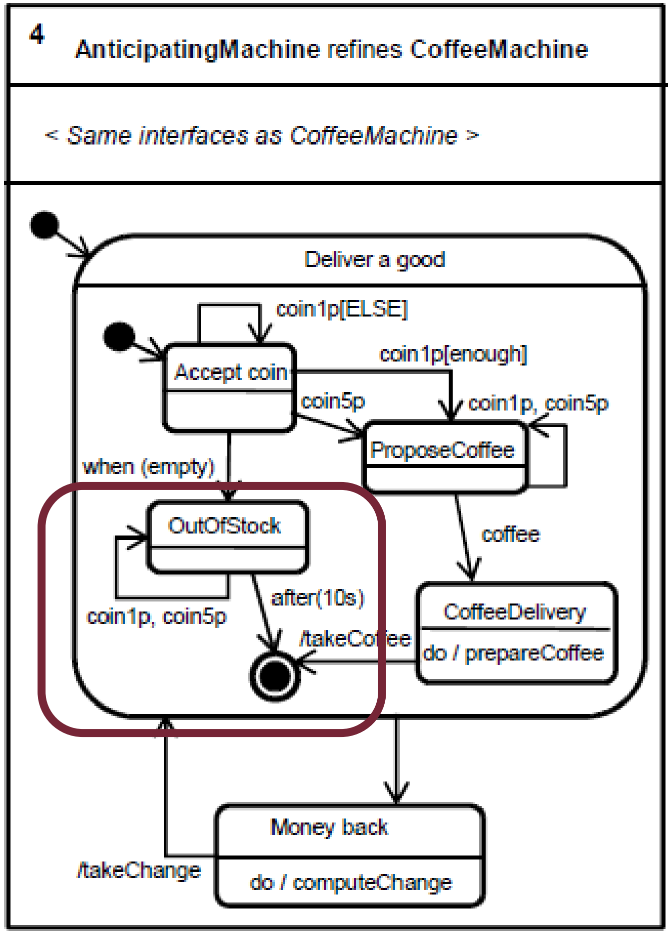
Fault-free “perfect” system ↔ Fault tolerant system in case of fault to be tolerated

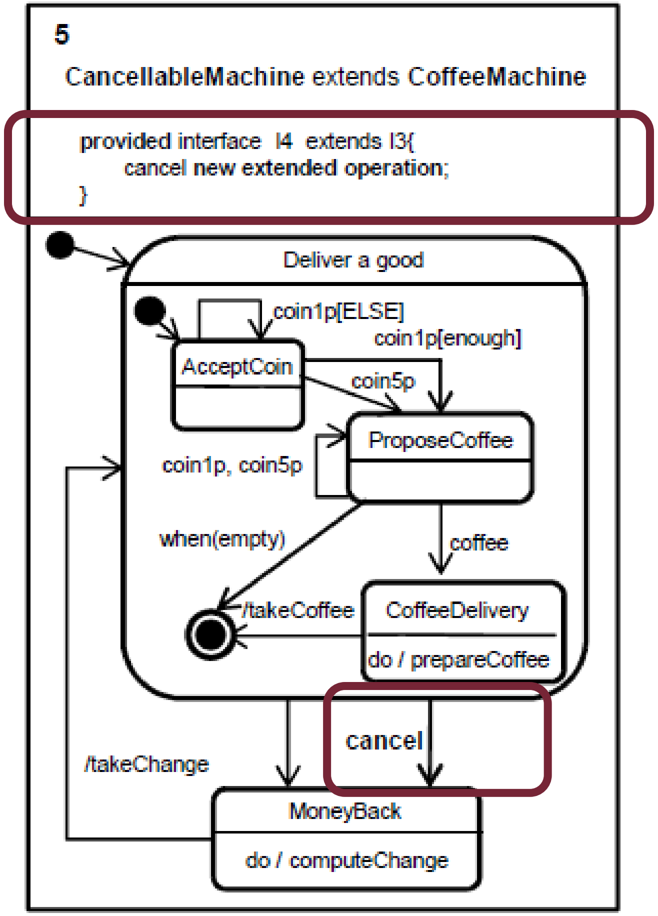
- Refinement between models:
  - Preserving original behavior and extending it in an allowed way
  - Reducing non-determinism in the model (with concrete conditions)

Example: Refinement in statechart models (1)

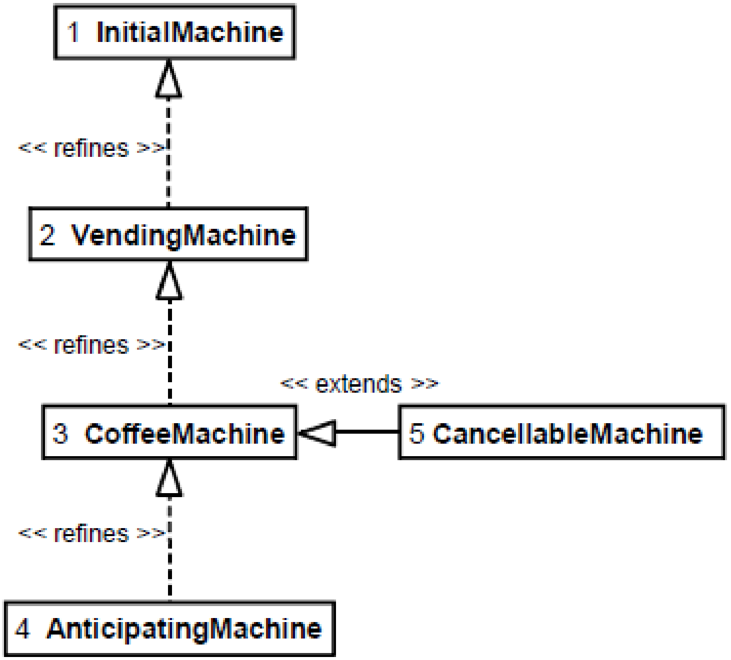


Example: Refinement in statechart models (2)





What is expected: Checking well-defined relations



- “Refines” relation: to keep existing behavior (with proper mapping of events and actions) with refinements
- “Extends” relation: to allow controlled changes in existing behavior

What do we expect from a refinement relation?

Informal expectations:

- Reflexive and transitive
- Not symmetric
- Keeping liveness property: The refined model shall be able to provide the behavior that the original model is able to provide
  - With proper mapping of events and actions of the refined model
  - Assuming fairness: Keeping the liveness property in case of fair behavior (i.e., in case of choices, all potential behaviors will eventually occur)
- Composability:
  - Subsequent refinements result in refinement
  - Refinement and extension result in extension
- …

Formal treatment:

- Precise definitions of the relations are required!

Definition of the relations

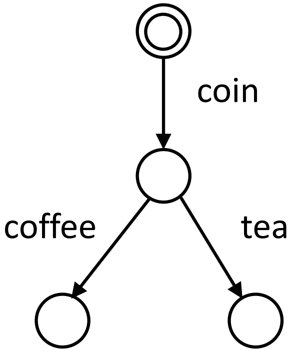
- Relations are defined on low-level models, typically on Labeled Transition System (LTS)
- Recap: The definition of LTS

$$LTS = (S, Act, \rightarrow)$$

S set of states

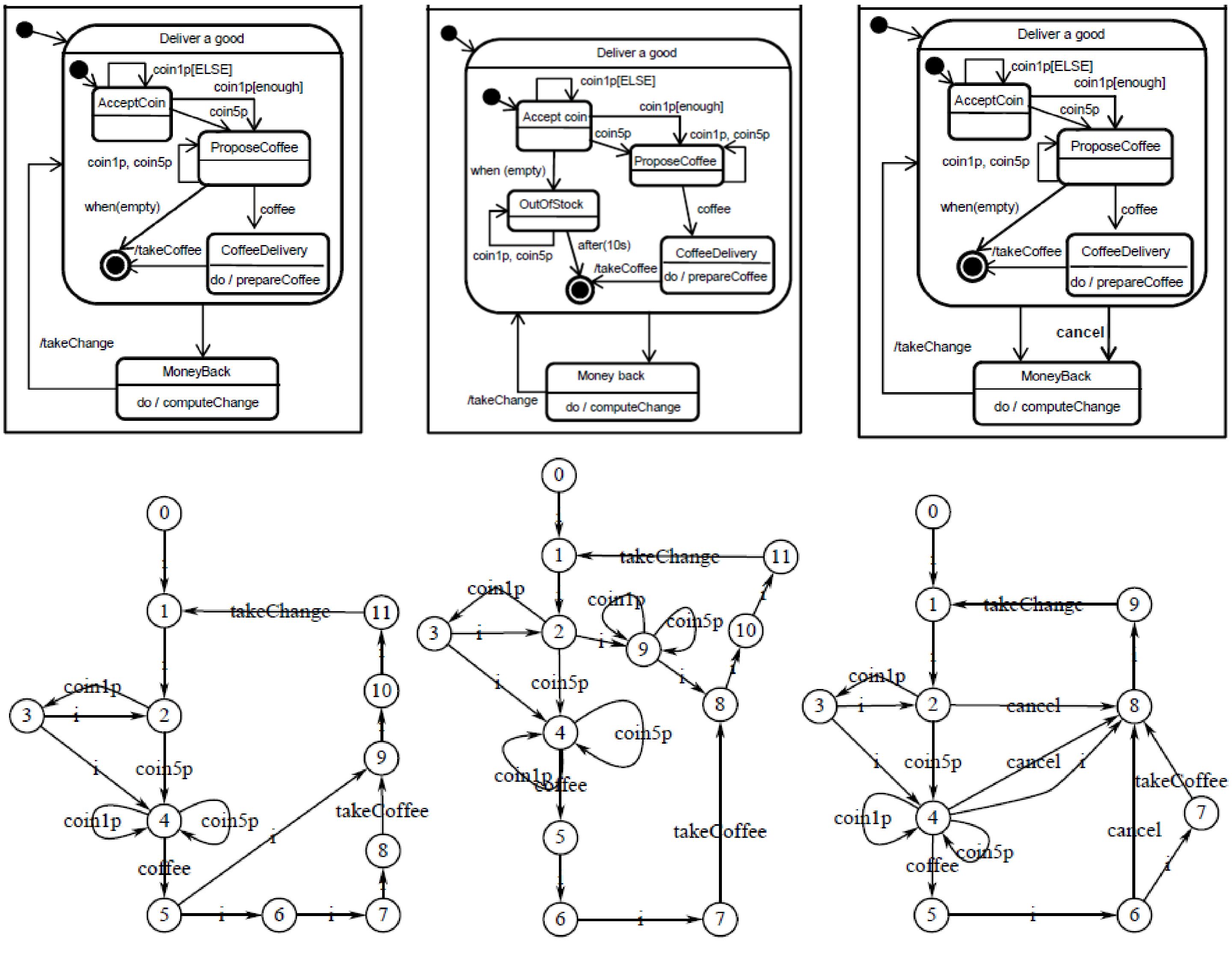
Act set of actions

$\rightarrow \subseteq S \times Act \times S$ state transition relation



- LTS may be derived from higher-level formalisms (using operational semantics)
  - E.g., statecharts, Petri-nets, process algebra, …

Example: Mapping LTS from statechart



a. LTS_3: CoffeeMachine b. LTS_4: AnticipatingMachine c. LTS_5: CancellableMachine

Equivalence relations

Trace equivalence

Strong bisimulation equivalence

Weak bisimulation (observational) equivalence

Classification of relations

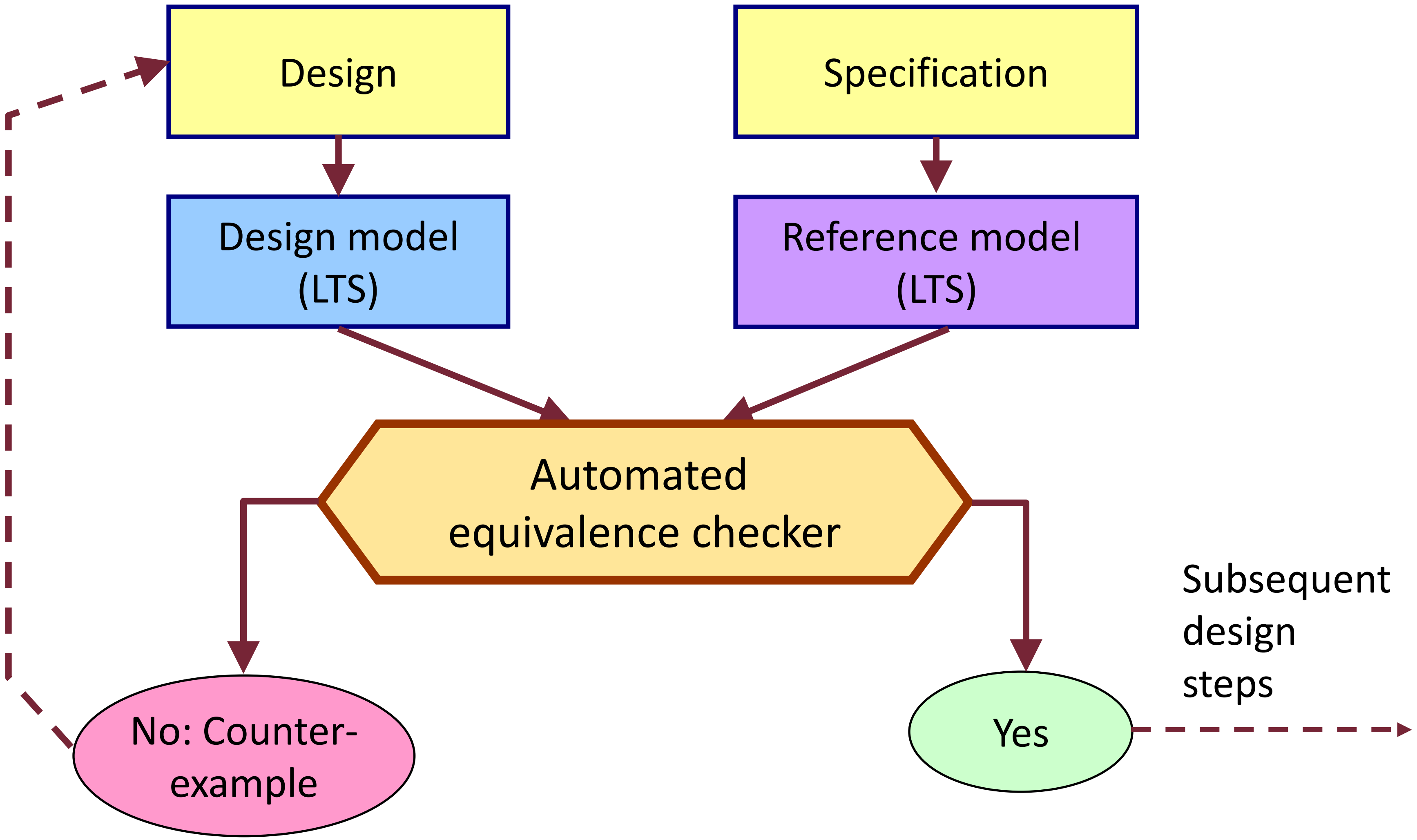
- Equivalence relations, denoted in general by =
  - Reflexive, transitive, symmetric

  Some equivalence relations are congruence:
  - If T1=T2, then for all C[ ] contexts C[T1]=C[T2]
  - The same context preserves the equivalence
  - Dependent on the formalism: how to embed T in C[ ]

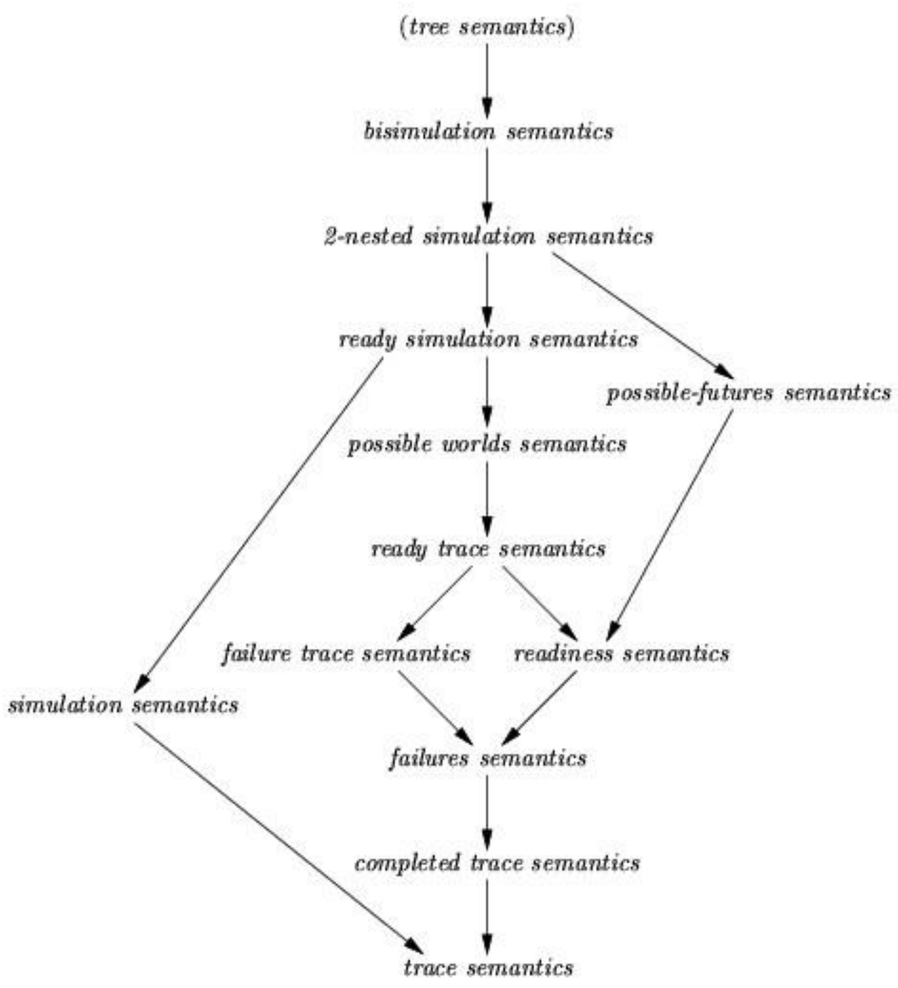
- Refinement relations, denoted by $\leq$
  - Reflexive, transitive, anti-symmetric ($\rightarrow$ partial order)

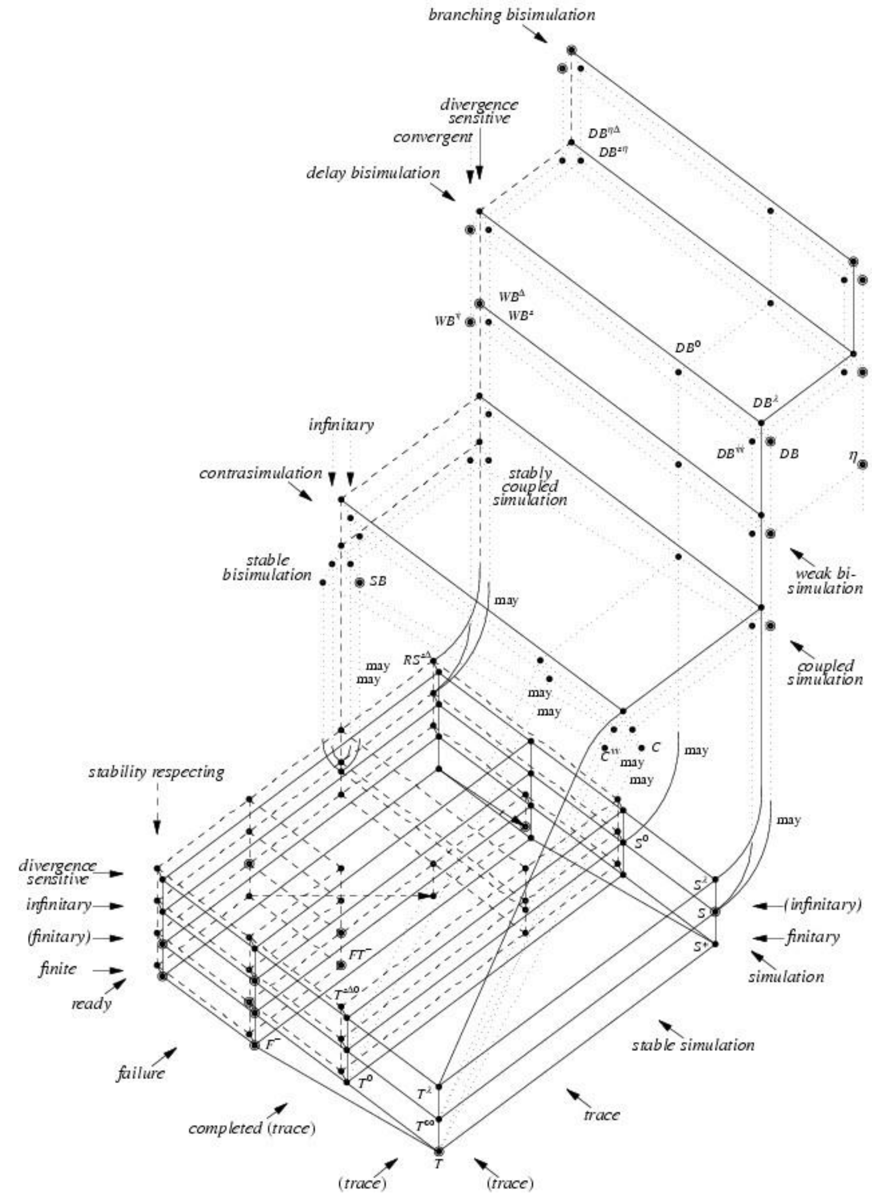
  Precongruence relation:
  - If T1$\leq$T2, then for all C[ ] contexts C[T1] $\leq$ C[T2]
  - The same context preserves the refinement

Equivalence checking using an equivalence relation



Hierarchy of relations proposed in the literature





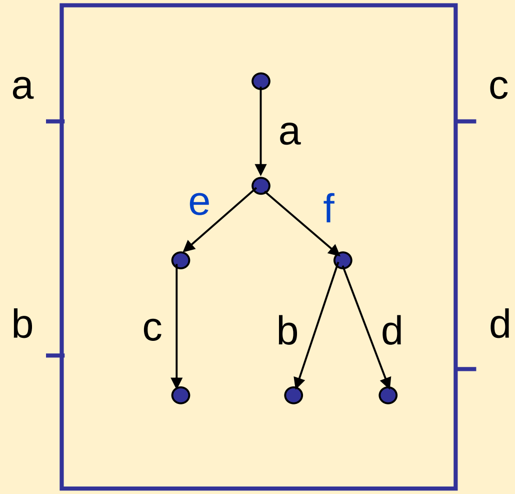
Why do exist so many relations?

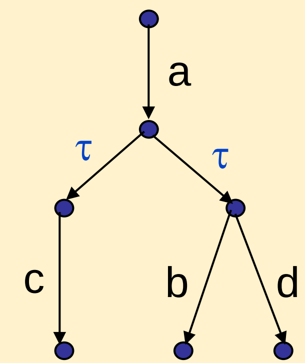
Properties that characterize the relations (1)

- Distinguishing observable and internal actions:
  - Observable actions: Appear on the external interface ("ports") of the modeled component, relevant for the environment
    - Representing: method call, sent or received message, provided service etc.
  - Unobservable (internal) actions: Do not appear on the external interface ("ports") of the modeled component, not relevant for the environment
    - Representing: internal activities, internal calls etc.
    - Effects can be observed only through the consequences (subsequent actions)
    - Notation: τ (or sometimes i)

Example:

- Left: Internal actions e and f
- Right: Observable behavior of the component: e and f are mapped to τ





Properties that characterize the relations (2)

- Distinguishing observable and internal actions:
  - Observable actions: Appear on the external interface (“ports”) of the modeled component, relevant for the environment
    - Representing: method call, sent or received message, provided service etc.
  - Unobservable (internal) actions: Do not appear on the external interface (“ports”) of the modeled component, not relevant for the environment
    - Representing: internal activities, internal calls etc.
    - Effects can be observed only through the consequences (subsequent actions)
    - Notation: τ (or sometimes i)
- Allowing nondeterminism:
  - From a state, many transitions are labeled with the same action
    - ”Image finite system”: their number is finite
  - Typically used in abstract models, resolved during refinement
- Semantics of concurrent component models:
  - Interleaving (one action at a time)
  - True concurrency (several actions at a time)

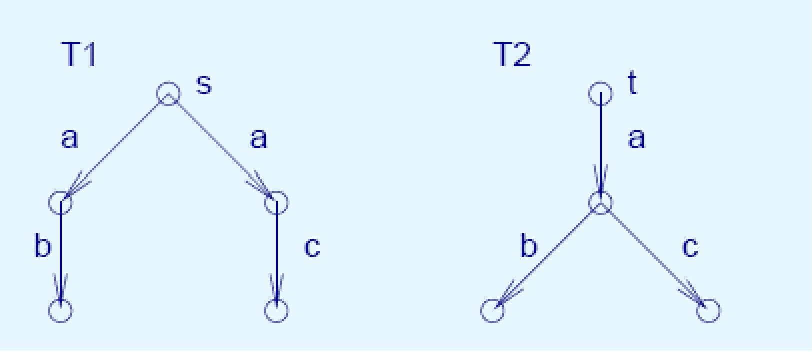
The notion of “test” and “deadlock”

- “Test” in LTS based behavior checking:
  - Test: A sequence of actions that is expected (from the initial state)
    - Analogy: actions represent interactions on ports during testing (e.g., sending or receiving messages, raising or processing events etc.)
  - Test outcome:
    - Fails: The sequence of actions cannot be provided by the LTS
    - Test must be successful: The sequence of actions is always possible
    - Test may be successful: Providing the sequence depends on the non-determinism
- “Deadlock” in LTS based behavior checking:
  - A given action cannot be provided by the system in an expected action sequence (test)
    - Analogy: no interaction is possible on a port (e.g., it is not possible to send or receive message, process an event etc.)
    - The deadlock is given by the action that is not possible
  - Failure of a test: The action that cannot be provided (gives deadlock)
  - Classic example: Piano with keys that are unlocked by the actions of the LTS
    - Successful test is a tune that can be played

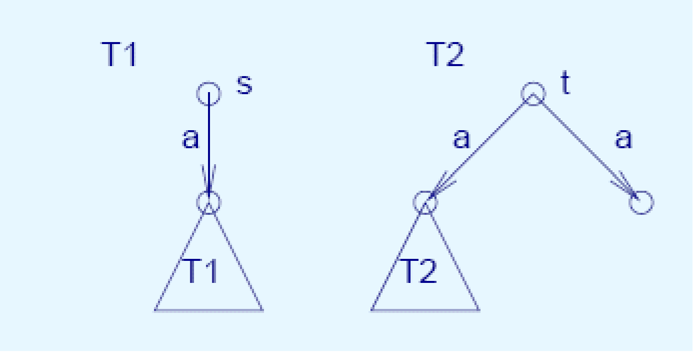
Examples for deadlocks

- What is a potential deadlock after action a?

Act={a, b, c}

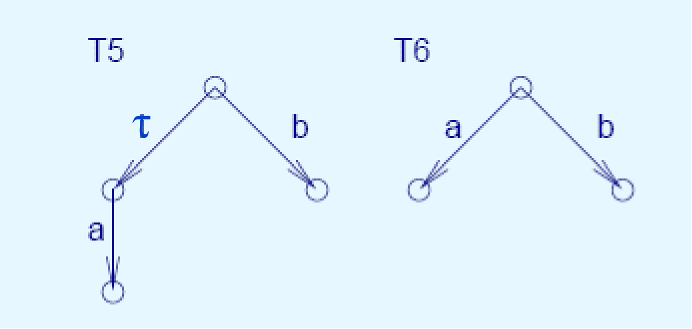


„Recursive” LTS models, Act={a}

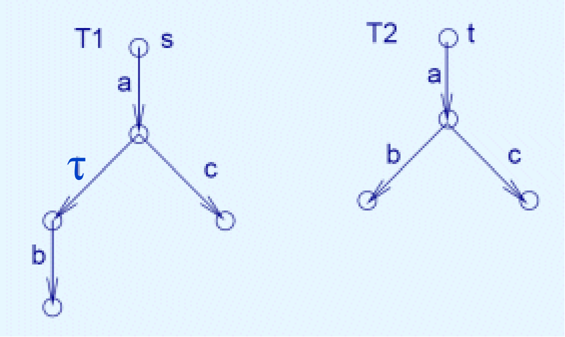


- How internal actions influence deadlock?

Act={a, b, c}



Act={a, b, c}



Trace equivalence: Notation

- Analogy: Automata on finite words

$$A_1 = A_2 \text{ if } L(A_1) = L(A_2)$$

  - Applying this analogy in case of LTS:
    - Each state is an "accepting state"
    - "Language": Each possible action sequence (trace)

- Notation:

$$\alpha = a_1 a_2 a_3 a_4 \ldots a_n \in Act^* \text{ finite action sequence } (\varepsilon \text{ is empty})$$

$$s \xrightarrow{\alpha} s' \text{ if } \exists \mathrm{s}_0 \mathrm{s}_1 \ldots \mathrm{s}_\mathrm{n} \text{ state sequence where } \mathrm{s}_0 = s,\ \mathrm{s}_\mathrm{n} = s',\ s_i \xrightarrow{a_{i+1}} s_{i+1}$$

$$\alpha(s) \text{ is a trace from s, if } \exists s' : s \xrightarrow{\alpha} s'$$

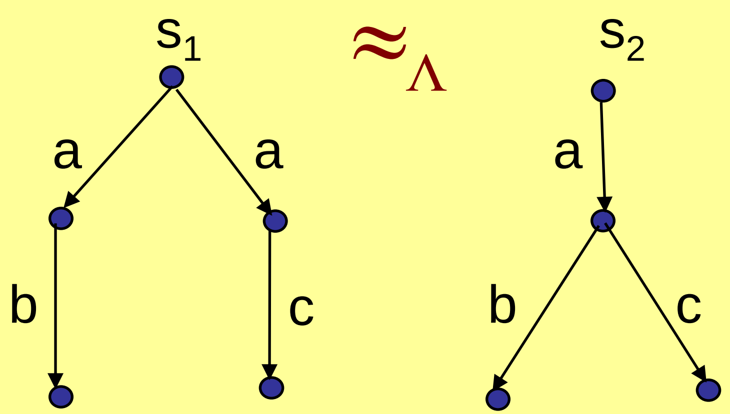
$$\Lambda(s) \text{ is the set of traces from s: } \Lambda(s) = \left\{ \alpha \mid \exists s' : s \xrightarrow{\alpha} s' \right\}$$

Trace equivalence: Definition and examples

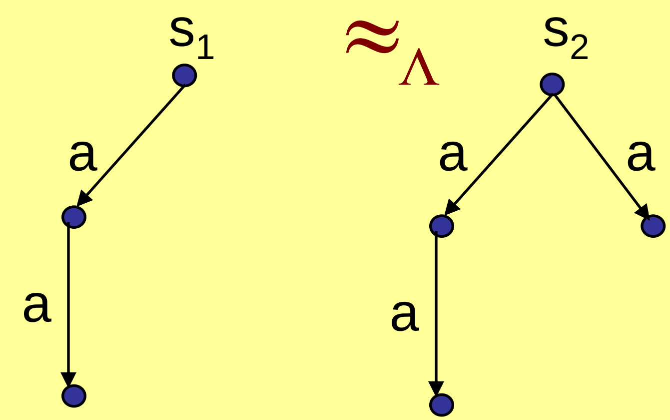
- Definition of trace equivalence $\approx_\Lambda$
  for T_1 and T_2 LTSs, with s_1 and s_2 initial states:

$$T_1 \approx_\Lambda T_2 \quad \text{iff.} \quad \Lambda(s_1)=\Lambda(s_2)$$

- Examples:



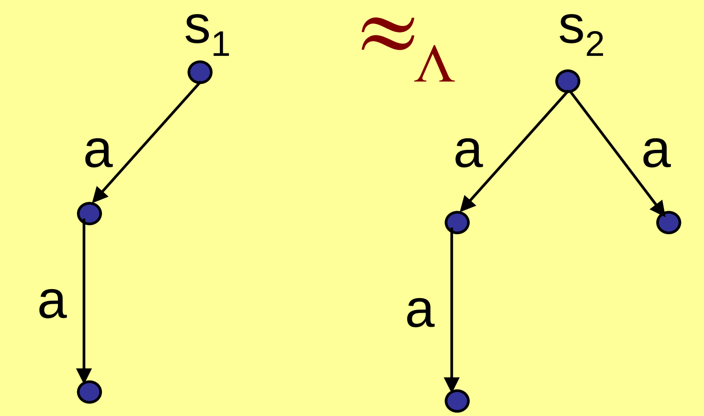
$$\Lambda(s_1)=\Lambda(s_2)=\{\varepsilon, a, ab, ac\}$$



$$\Lambda(s_1)=\Lambda(s_2)=\{\varepsilon, a, aa\}$$

Trace equivalence: Disadvantages

- (In)sensitivity to deadlock
  - Equivalent LTSs may have different deadlock behavior
  - Caused by nondeterminism or internal actions



$$\Lambda(s_1) = \Lambda(s_2) = \{\varepsilon, a, aa\}$$

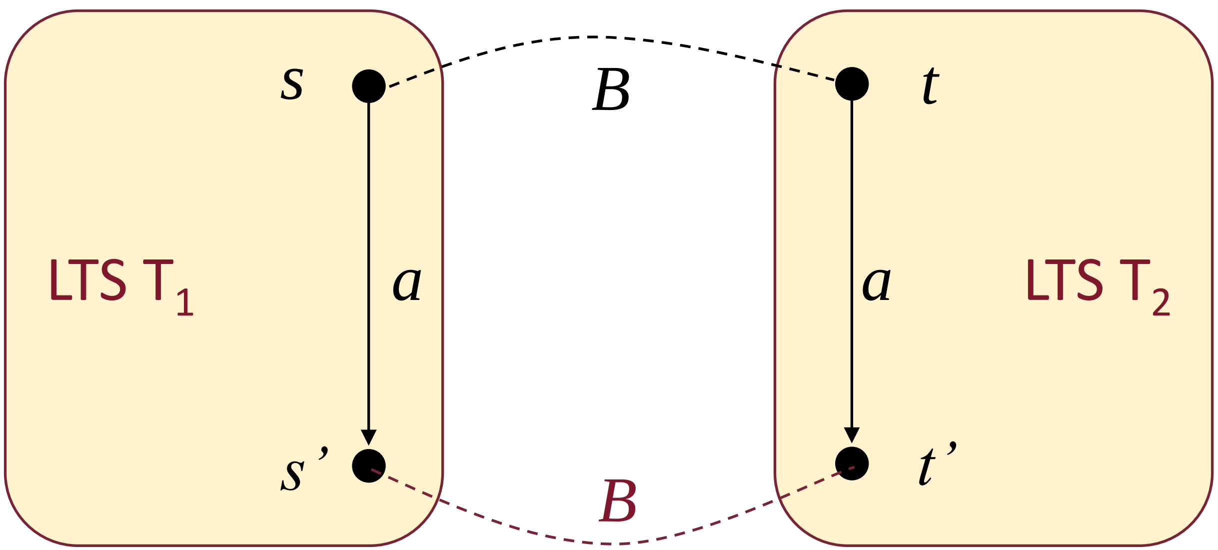
- Solution:
  - It has to be checked whether the states reached by the same trace allow the same continuation of the trace

Strong bisimulation relation: Definition

- Definition of the strong bisimulation relation B:

  $B \subseteq S \times S$ is a bisimulation, if for all $(s,t) \in B$ and any $a \in Act,\ s',t' \in S$ it holds:

  - if $s \xrightarrow{a} s'$ then $\exists t' : t \xrightarrow{a} t'$ and $(s',t') \in B$
  - if $t \xrightarrow{a} t'$ then $\exists s' : s \xrightarrow{a} s'$ and $(s',t') \in B$

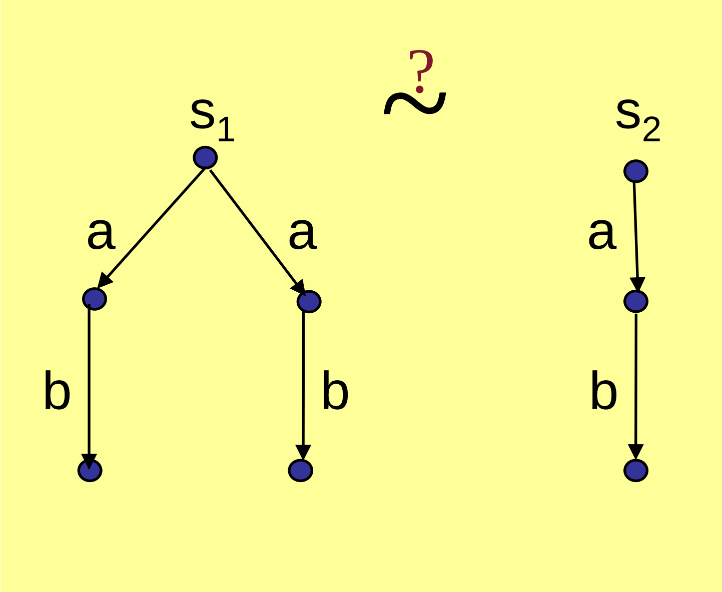


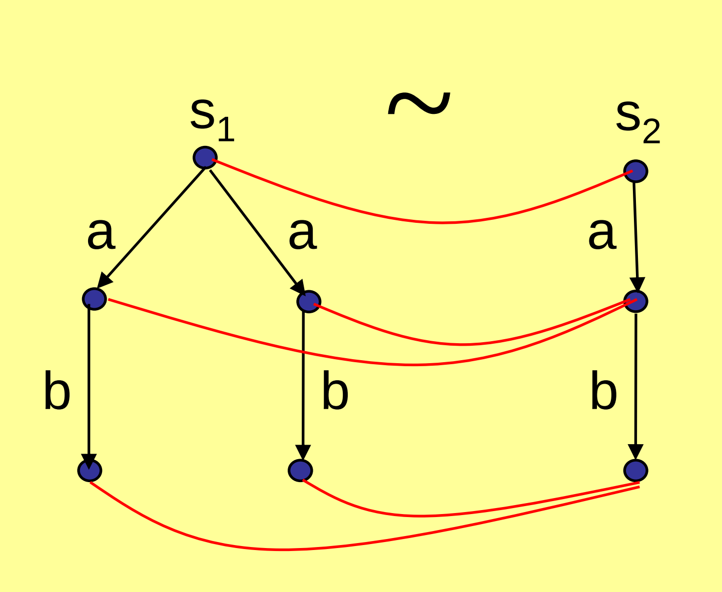
Strong bisimulation equivalence: Definition

- Strong bisimulation equivalence ~:

$$T_1 \sim T_2 \;\text{ iff }\; \exists B : (s_1, s_2) \in B, \text{ also denoted as } s_1 \sim s_2$$

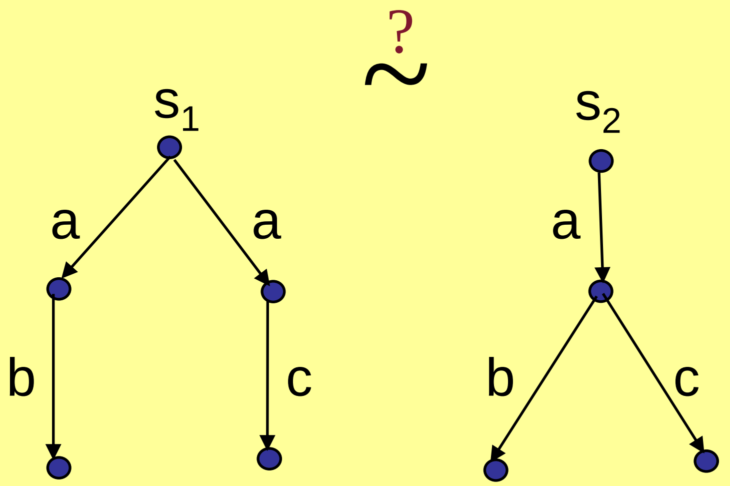
- Intuition: Equivalent systems can "simulate" each other
  - Matching transitions with actions in equivalent states
  - The same traces are possible through equivalent states
- Examples:



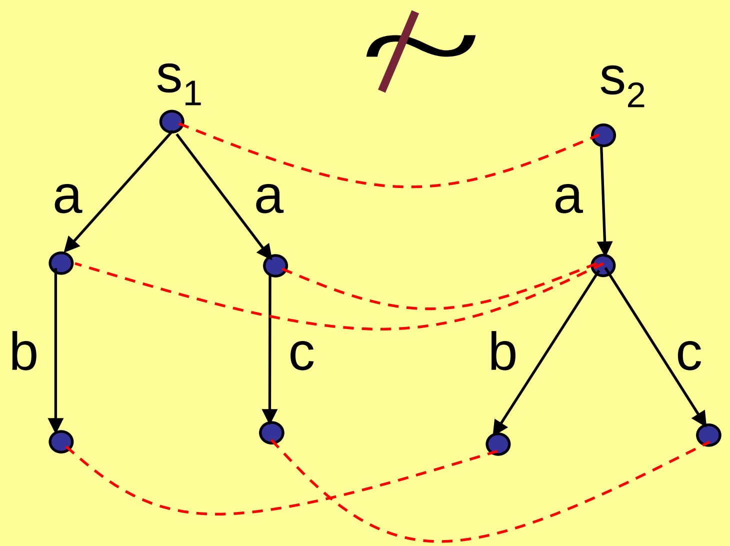


Strong bisimulation equivalence: Example

- Strong bisimulation equivalence between LTSs:



$$\Lambda(s_1) = \Lambda(s_2) = \{\varepsilon, a, ab, ac\}$$



$$\Lambda(s_1) = \Lambda(s_2) = \{\varepsilon, a, ab, ac\}$$

Strong bisimulation equivalence: Advantages

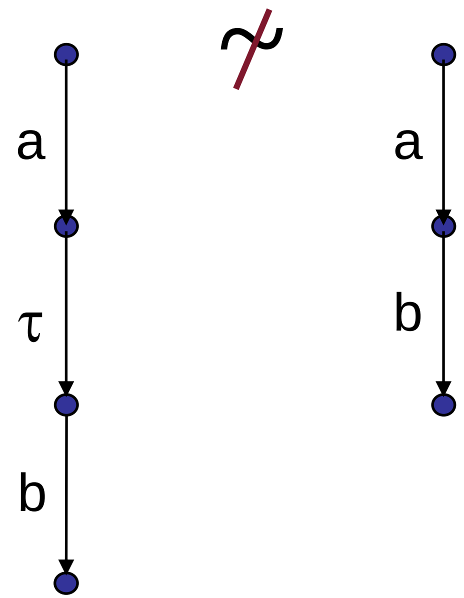
- Strong bisimulation implies trace equivalence
- Strong bisimulation equivalent systems provide the same deadlock behavior
  - T1 ~ T2 means: if deadlock is possible in LTS T_1 then the same deadlock is possible in LTS T_2
- It is congruence for specific “CCS-like” LTS
  - Recap: An equivalence relation is congruence if the same context preserves the equivalence:
    - Here in case of T1 ~ T2, for all C[ ] context C[T1] ~ C[T2]
  - “CCS-like” LTS and embedding in a context:
    - LTS has a tree structure
    - Embedding an LTS: merging initial state of the embedded LTS T_i with any state of the context LTS C[ ] to get $C[T_i]$

Strong bisimulation equivalence: Formalizing deadlock

- Possible deadlocks can be expressed using the Hennessy-Milner logic
  - In a given state, deadlock for action a is expressed as [a]false
    - It holds only if there is no transition labeled with a, i.e., a is a deadlock
  - Deadlock for a set of actions $\{a_1, a_2, \dots a_n\}$:
    $\{[a_1]\text{false} \wedge [a_2]\text{false} \wedge \dots \wedge [a_n]\text{false}\}$
  - Deadlock for a set of actions in a state reachable by $\langle b_1\rangle\langle b_2\rangle\dots\langle b_n\rangle$:
    $\langle b_1\rangle\langle b_2\rangle\dots\langle b_n\rangle \{[a_1]\text{false} \wedge [a_2]\text{false} \wedge \dots \wedge [a_n]\text{false}\}$
- Theorem:
  In case of two LTSs, $T_1 \sim T_2$ iff for any HML expression p:
  - either $T_1,s_1 \models p$ and $T_2,s_2 \models p$, (i.e., both satisfy p)
  - or $T_1,s_1 \not\models p$ and $T_2,s_2 \not\models p$ (i.e., do not satisfy p)

Strong bisimulation equivalence: Disadvantages

- Sensitivity to unobservable actions:
  - In some cases there is no observable effect of an internal action, but the relation makes a difference
  - Simple example:



Weak bisimulation equivalence: Notation

- The “weak” variant of strong bisimulation
  - It is not sensitive to internal actions without observable effect
  - Rationale: Have the possibility of the same observable traces through equivalent states
- Notation:

$\alpha \in Act^*$ finite action sequence (ε is empty)

$\hat{\alpha} \in (Act - \tau)^*$ observable action sequence (τ deleted)

here $\hat{\alpha} = \varepsilon$ if $\alpha = \tau$

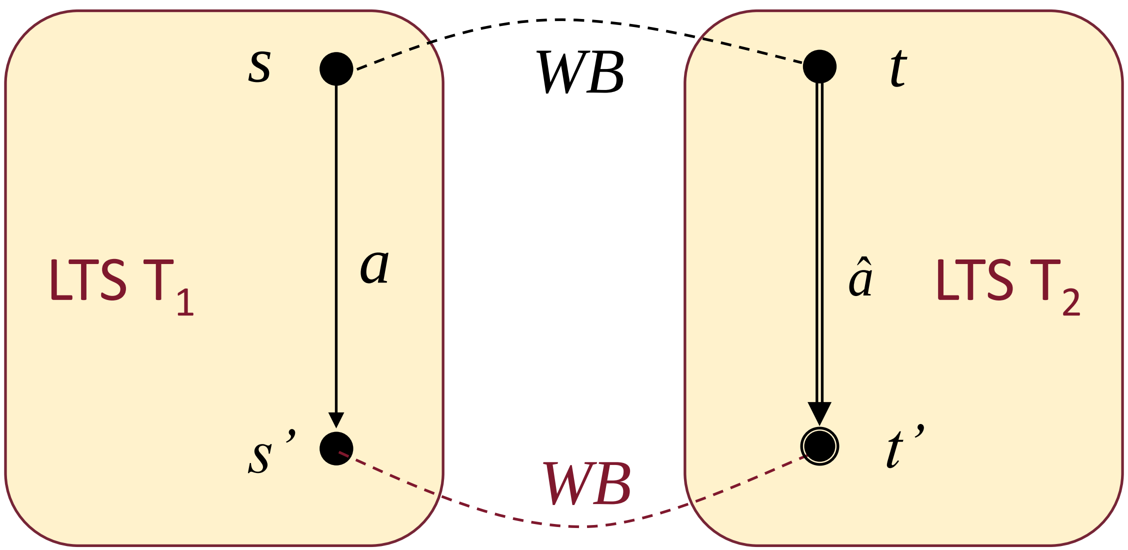
$s \overset{\beta}{\Rightarrow} s'$ if $\exists \alpha : s \overset{\alpha}{\rightarrow} s'$ and $\beta = \hat{\alpha}$

Weak bisimulation relation: Definition

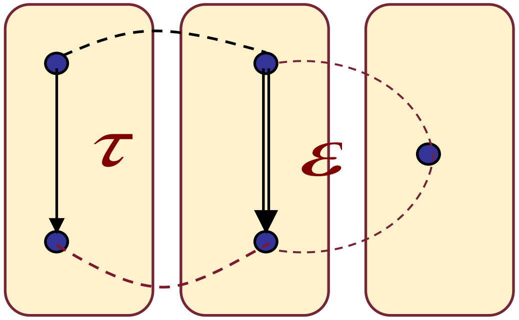
- Definition of weak bisimulation relation WB:

  $WB \subseteq S \times S$ weak bisimulation, if for all $(s,t) \in WB$ and any $a \in Act,\ s',t' \in S$ it holds:

  - if $s \xrightarrow{a} s'$ then $\exists t' : t \xRightarrow{\hat{a}} t'$ and $(s',t') \in WB$
  - if $t \xrightarrow{a} t'$ then $\exists s' : s \xRightarrow{\hat{a}} s'$ and $(s',t') \in WB$



In case of τ :

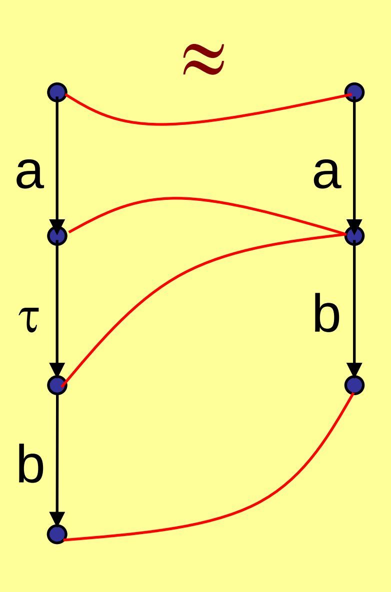


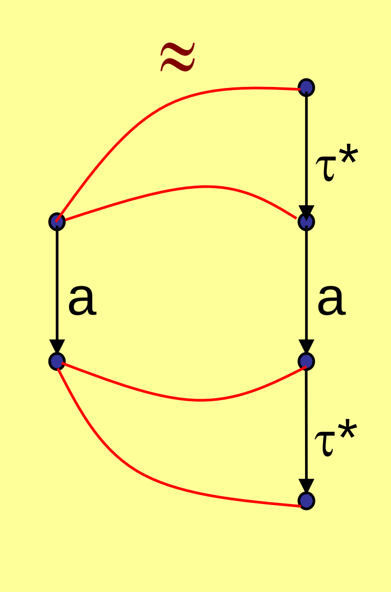
Weak bisimulation equivalence: Definition

- Weak bisimulation equivalence $\approx$
  (also called as Observation equivalence)

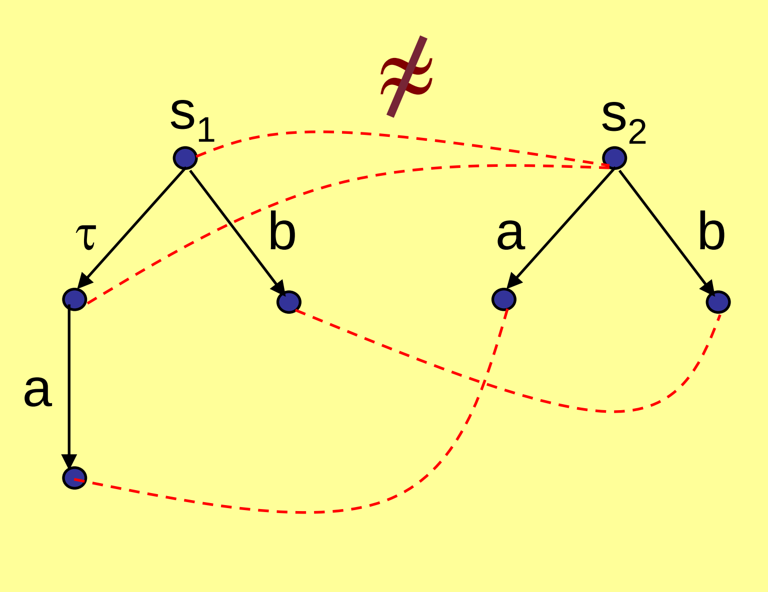
$$T_1 \approx T_2 \quad \text{iff} \quad \exists WB : (s_1, s_2) \in WB, \ \text{also denoted as } s_1 \approx s_2$$

- Examples:





Internal action with effect:



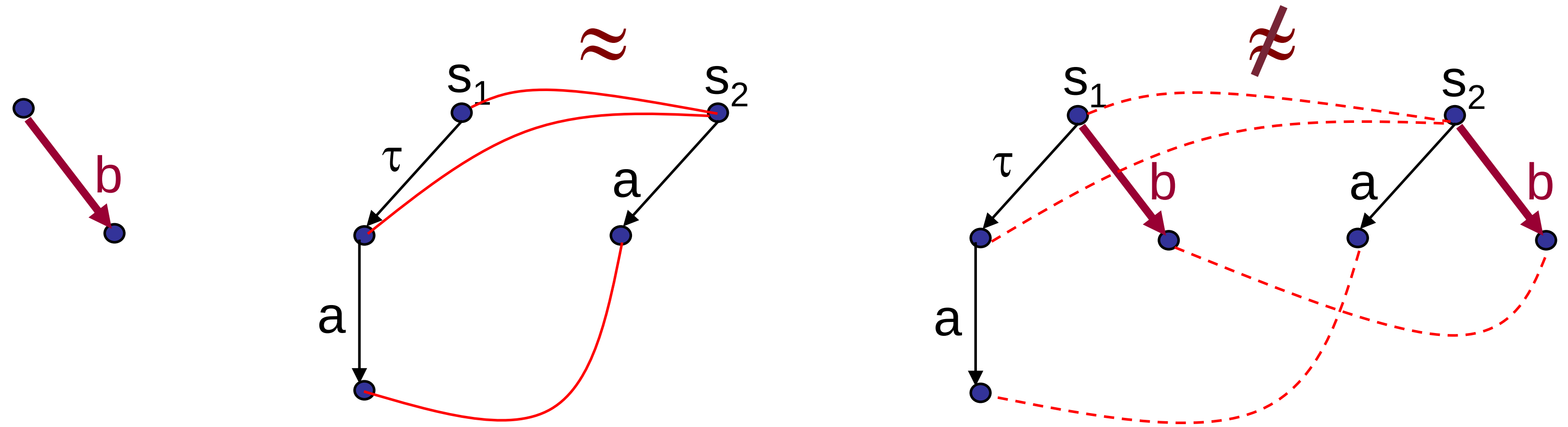
Weak bisimulation equivalence: Formalizing deadlock

- HML variant for observable actions:
  HML* ::= true | false | $p \wedge q$ | $p \vee q$ | [[a]] p | <<a>> p
- Semantics:
  - **H3***: $T,s \models [[a]]p$ iff $\forall s'$ where $s \Rightarrow^a s'$: $s' \models p$
  - **H4***: $T,s \models \langle\langle a \rangle\rangle p$ iff $\exists s'$: $s \Rightarrow^a s'$ and $s' \models p$
- Theorem:
  In case of LTSs, $T_1 \approx T_2$ iff for any HML* expression p:
  - either $T_1,s_1 \models p$ and $T_2,s_2 \models p$
  - or $T_1,s_1 \not\models p$ and $T_2,s_2 \not\models p$

Weak bisimulation equivalence: Properties

- It is not congruence for CCS-like LTSs (there is a counterexample):

Context: T1≈T2: C[T1] $\not\approx$ C[T2]



- Interesting fact: The most permissive congruence relation, that implies weak bisimulation equivalence:

$$s \approx^c t, \text{ if for any } a \in Act,\ s', t' \in S \text{ it holds:}$$

- $\text{if } s \xrightarrow{a} s' \text{ then } \exists t' : t \overset{a}{\Rightarrow} t' \text{ and } s' \approx t'$
- $\text{if } t \xrightarrow{a} t' \text{ then } \exists s' : s \overset{a}{\Rightarrow} s' \text{ and } s' \approx t'$

Computing equivalence relations: Basic idea

Partition refinement algorithm

1. Initially, each pair of states is assumed to be in relation

   They form a single partition (equivalence class)

2. For each pair of states, it is to be checked:

   If there is a labeled transition starting from one of the states

   that cannot be simulated by a labeled transition from the other state, then

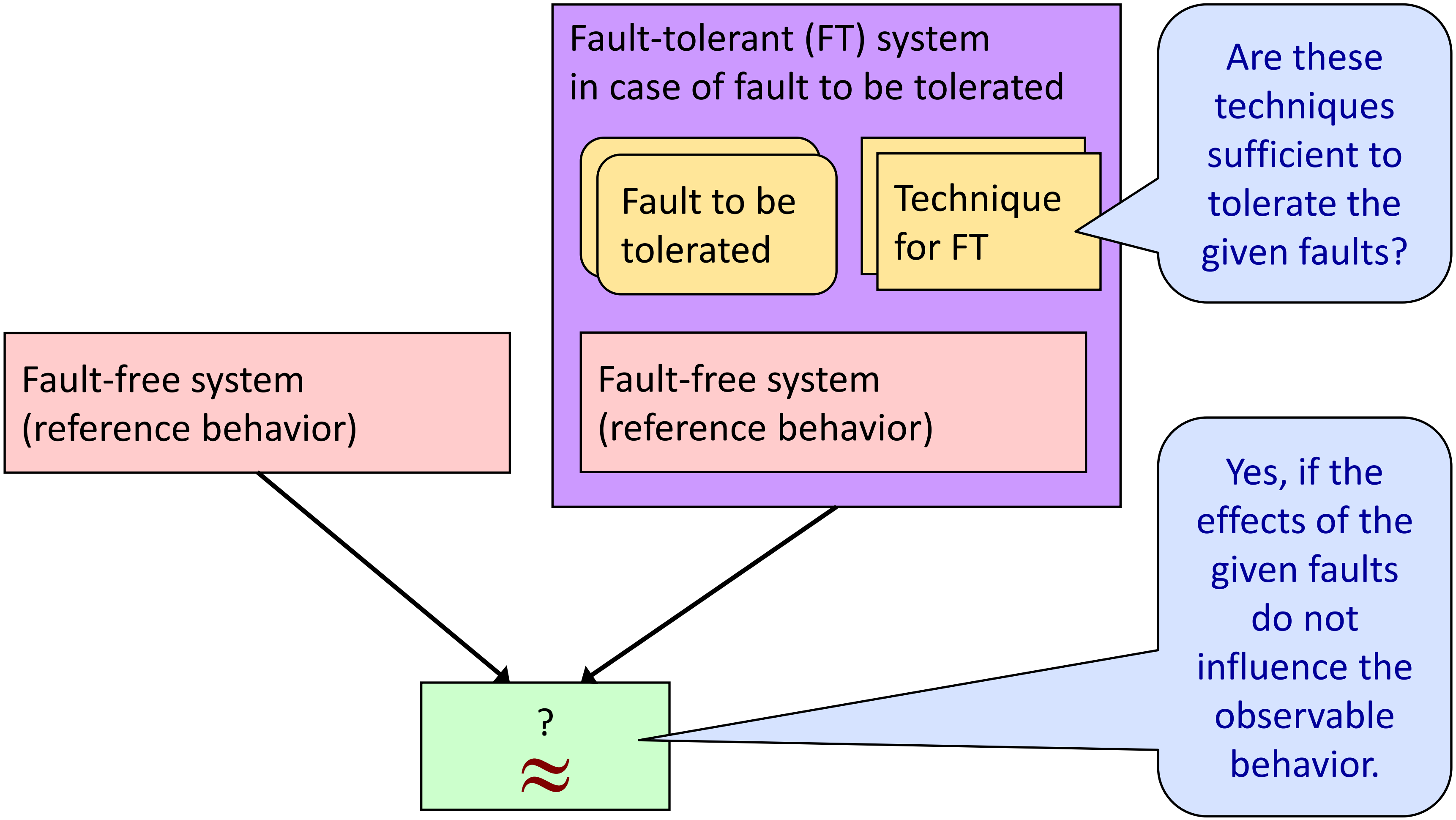
   - Remove that state pair from the partition
   - Apply the consequences of the removal: also remove the state pairs at the sources of matching incoming transitions
     - Since these are not equivalent if the matching transitions lead to non-equivalent states

3. If there are no changes (fix-point is reached): Equivalence classes are found

   - If the initial states are in the same equivalence class then the LTSs are equivalent

Case study: Verification of fault tolerance using observation equivalence

Case study: Verification of fault tolerance



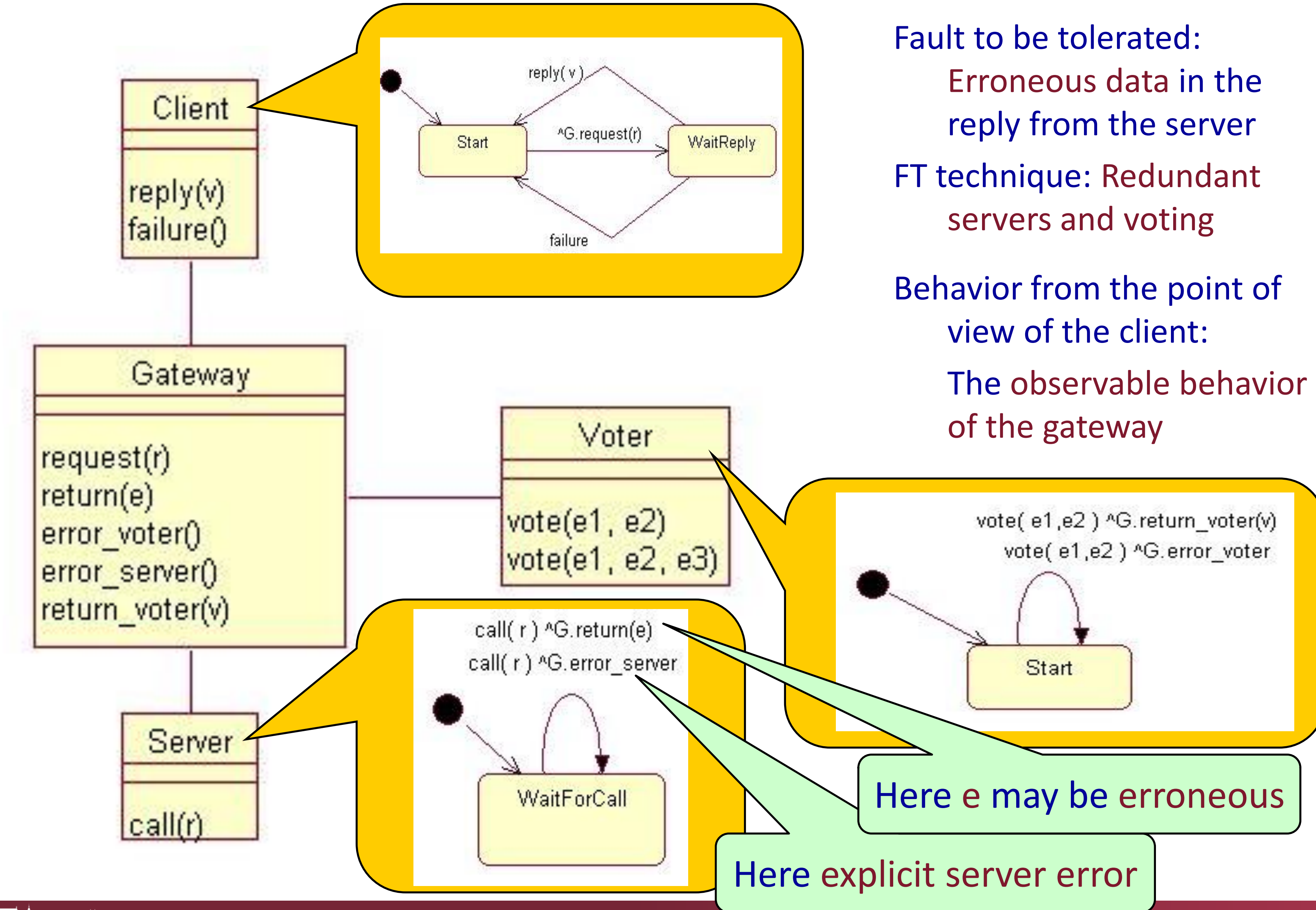
System architecture

Fault to be tolerated:
Erroneous data in the reply from the server

FT technique: Redundant servers and voting

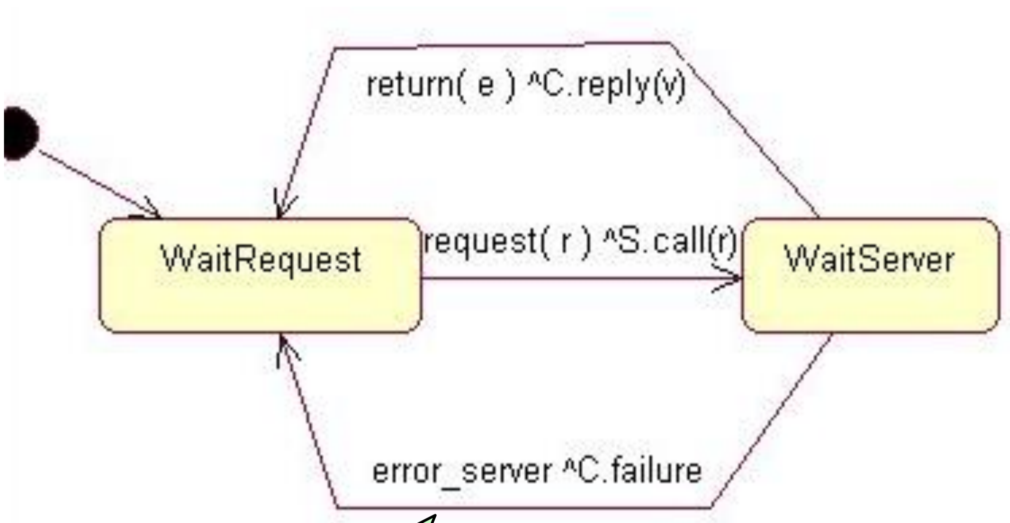
Behavior from the point of view of the client:

The observable behavior of the gateway



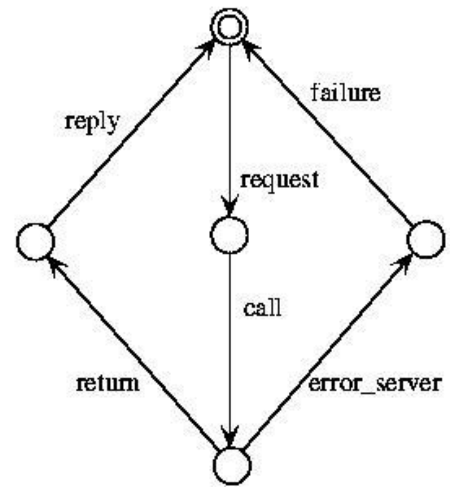
The Gateway component without fault tolerance

- Statechart diagram (reference behavior):



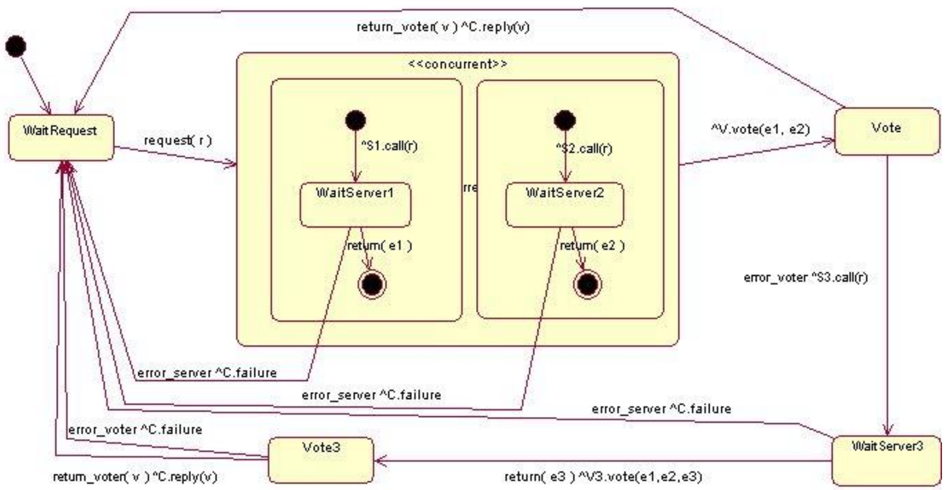
Explicit error signal from the server; not to be tolerated

- LTS representation (reference behavior):

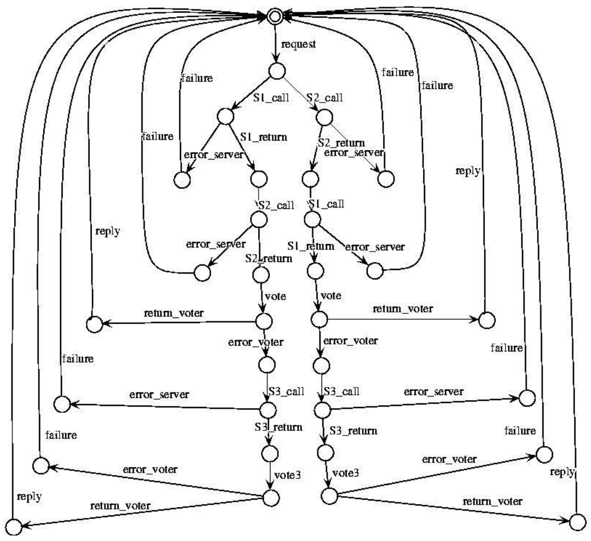


The Gateway component with fault tolerance

- Statechart diagram:

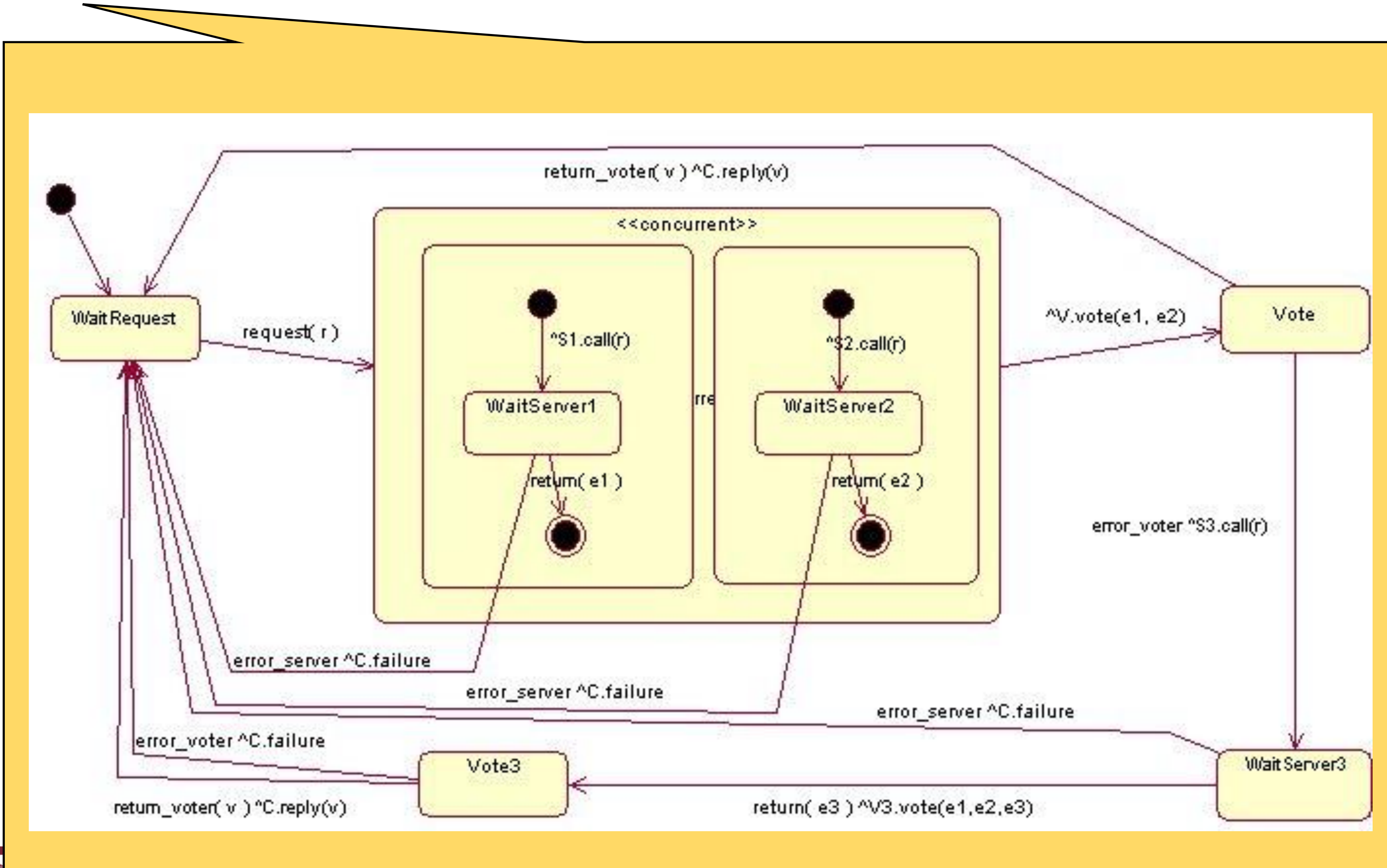


- LTS representation:



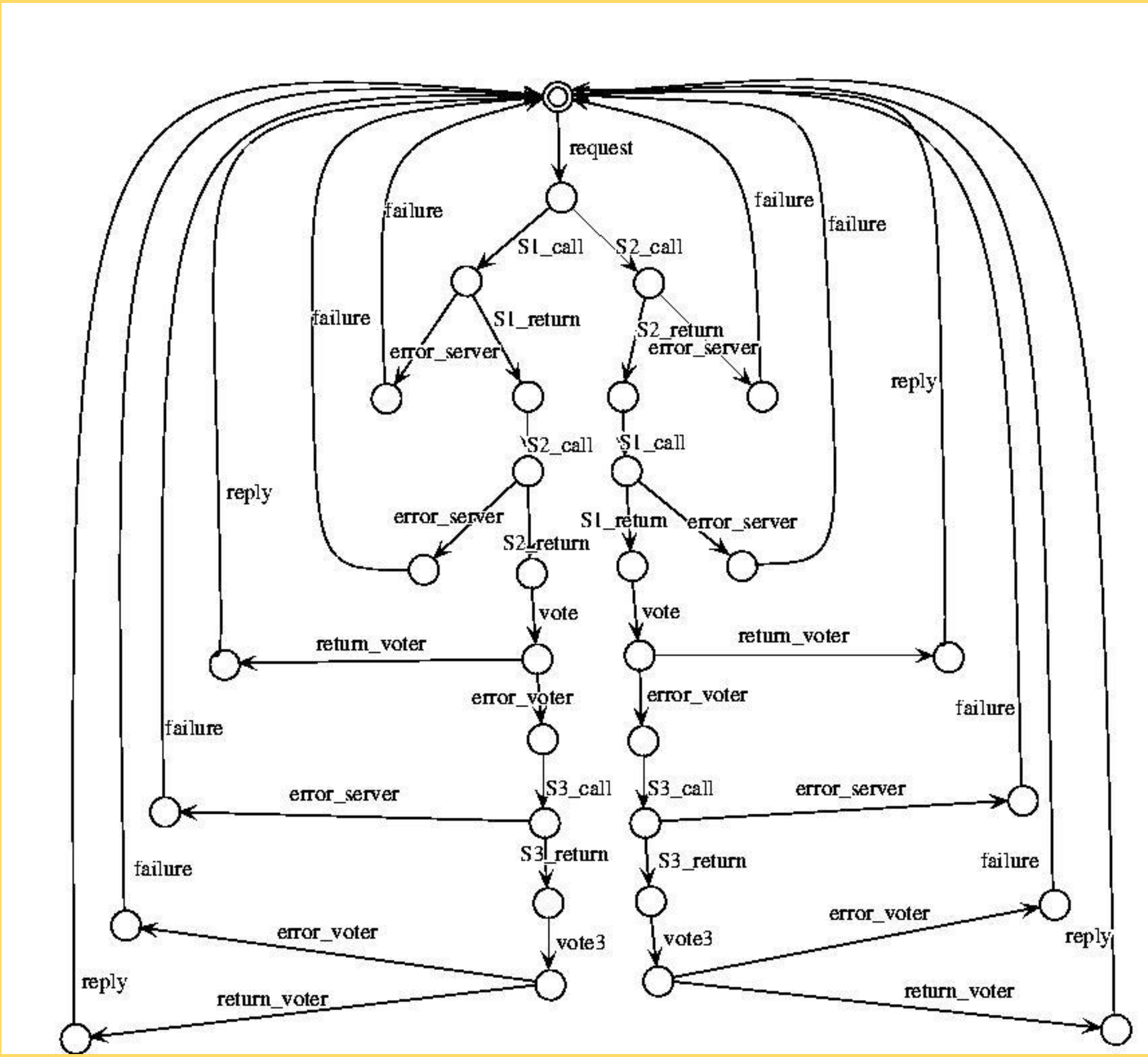
The Gateway component with fault tolerance

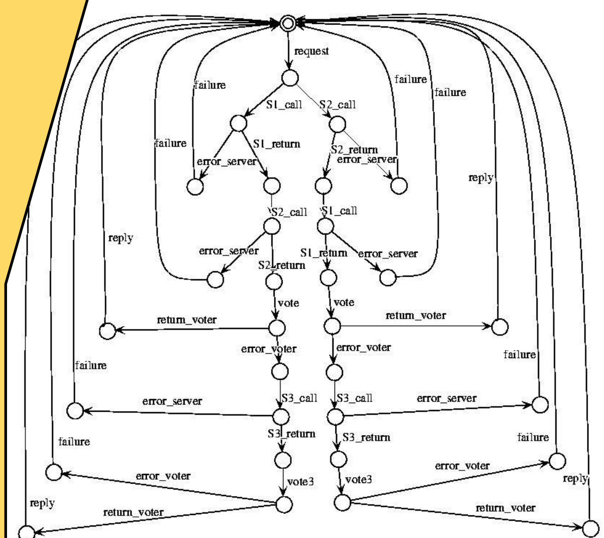
- Statechart diagram:
- LTS representation:



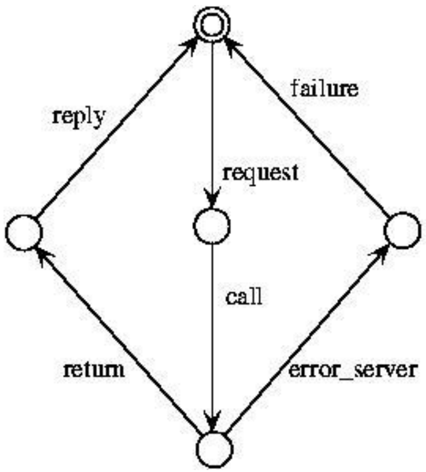
The Gateway component with fault tolerance

LTS representation:





Checking observation equivalence



Reference behavior of the Gateway (without fault tolerance)

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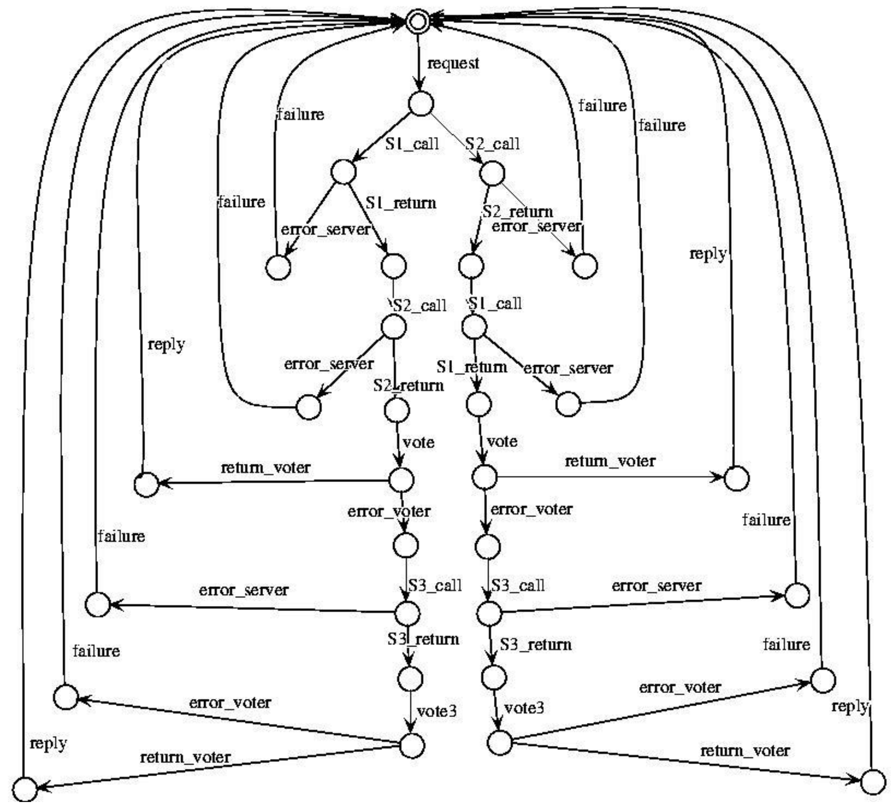
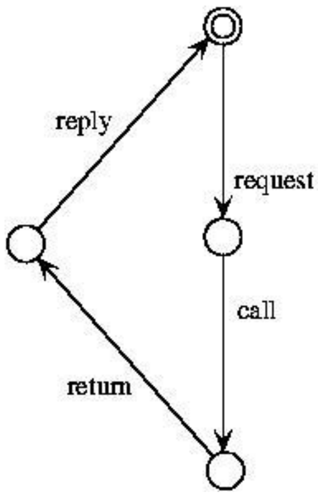
This way it is shown:
for the client
the fault tolerance technique
is transparent



Behavior of the FT Gateway;
here each action that is not observable by the client becomes τ

Checking fault tolerance in case of error from S1



Behavior of the fault-free gateway

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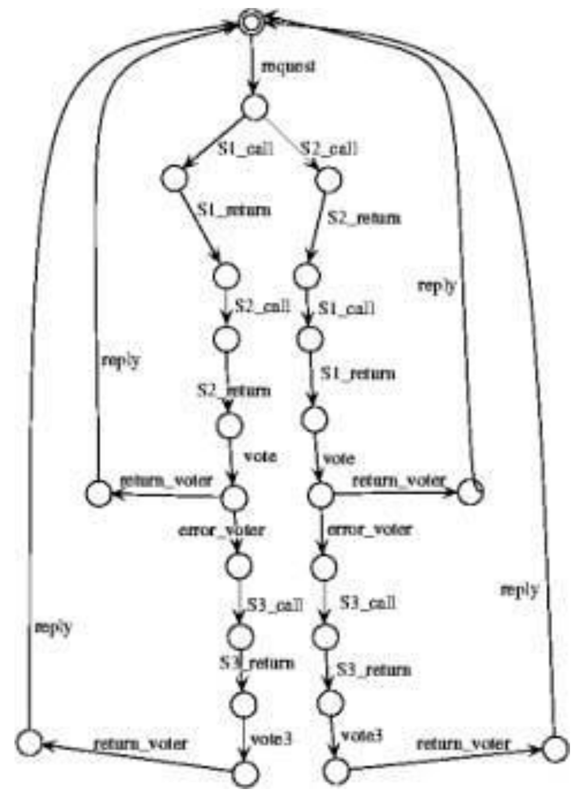
This way it is shown: for the client fault tolerance holds



Behavior of the FT Gateway in case of error from S1 (voting and call of S3); here each action that is not observable by the client becomes τ

Summary

- Motivation and basic ideas
  - The role of behavioral equivalence and refinement
  - Observable and unobservable behavior
  - The notion of testing and deadlock
- Equivalence relations
  - Trace equivalence
  - Strong bisimulation equivalence
  - Weak bisimulation equivalence (observation equivalence)
- Case study
  - Verifying fault tolerance using observation equivalence
- (Refinement relations: See later!)