Performance Modelling 2

Budapest University of Technology and Economics
Fault Tolerant Systems Research Group

Reminder

- **Stable state:**
  - Calculating with average values
  - $\lambda = X$ (arrival rate = throughput)
- **Maximum throughput (X^{max}):**
  - Upper bound of the reachable throughput
  - $X^{max} = \frac{K}{T}$ (in case of K resource instances)
- **Utilization (U):**
  - Ratio of the actual and maximum throughputs
  - $U = \frac{X}{K} \times T$ (in case of K resource instances)

Visitation number

Little's law

Zip’s law

Changes in Workload

CONTENT

VISITATION NUMBER

How do we do dimensioning?

- In general, resources are assigned to the activities
  - The (average) execution time of the activity is given
    → X^{max} of the activity can be computed
- E.g. in the Neptun system registering for a lecture utilize the DB server for 100 ms
  - $\mathrm{T} = 100 \text{ ms}$
  - $X^{max} = \frac{1}{T} = 10 \frac{registration}{sec}$

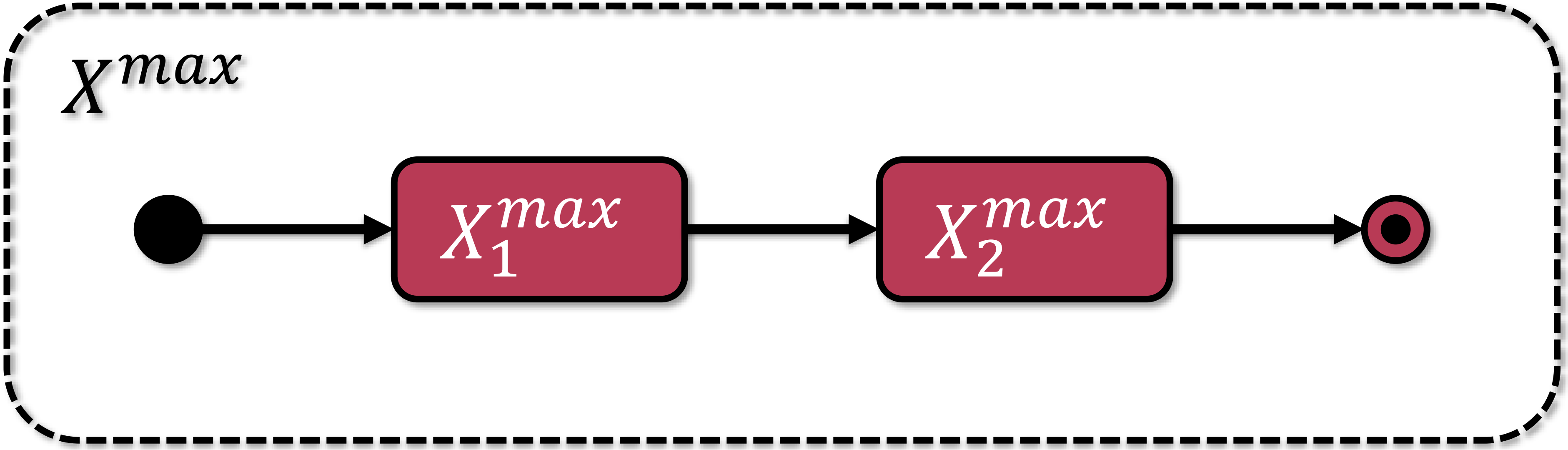
Knowing the process model (e.g. the user behaviour), what is the maximum throughput of the whole system?

Definition: Visitation Number

The **visitation number** indicates the average number of times a given activity/subprocess runs in a single execution of the whole process.

- How many times the process visits/repeats the given activity during a single execution?
- During a single execution of a process one of its activities can run not at all, or once, or several times. (Decisions, loops!)
  - If a choice between different outputs is described with probabilities, then these probabilities also play a role in determining the visitation numbers.

Sequential Composition

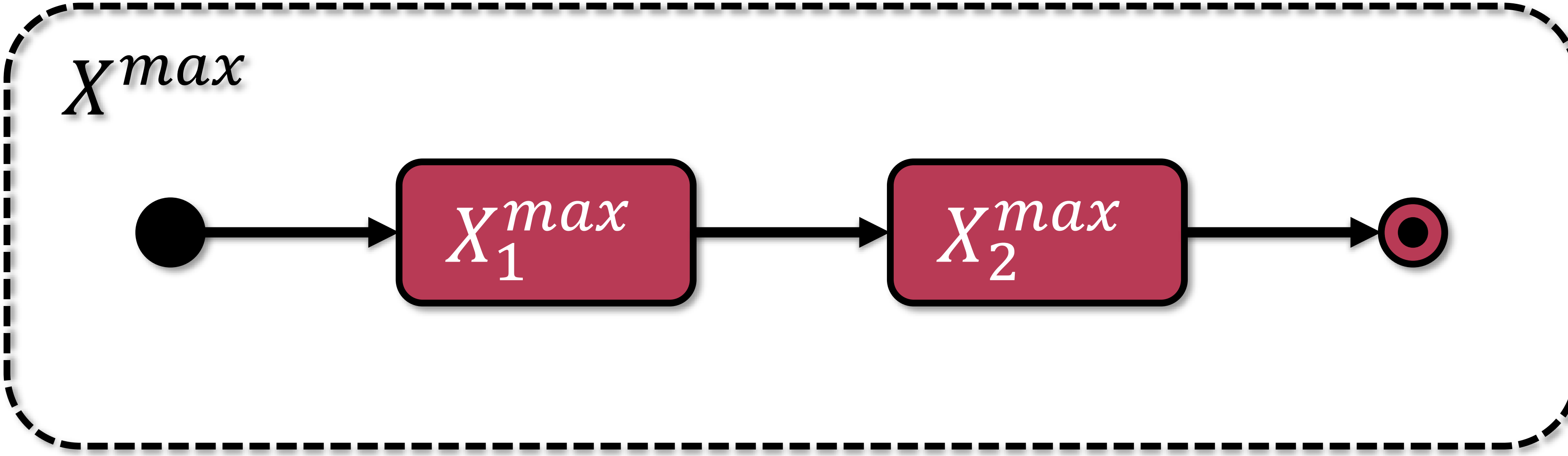


Each activity will be visited once.

It is no use if the one activity is fast, tokens will pile up at the other one.

E.g. in a citizen centre taking a ticket (300/hour), administration work (2/hour)

Sequential Composition

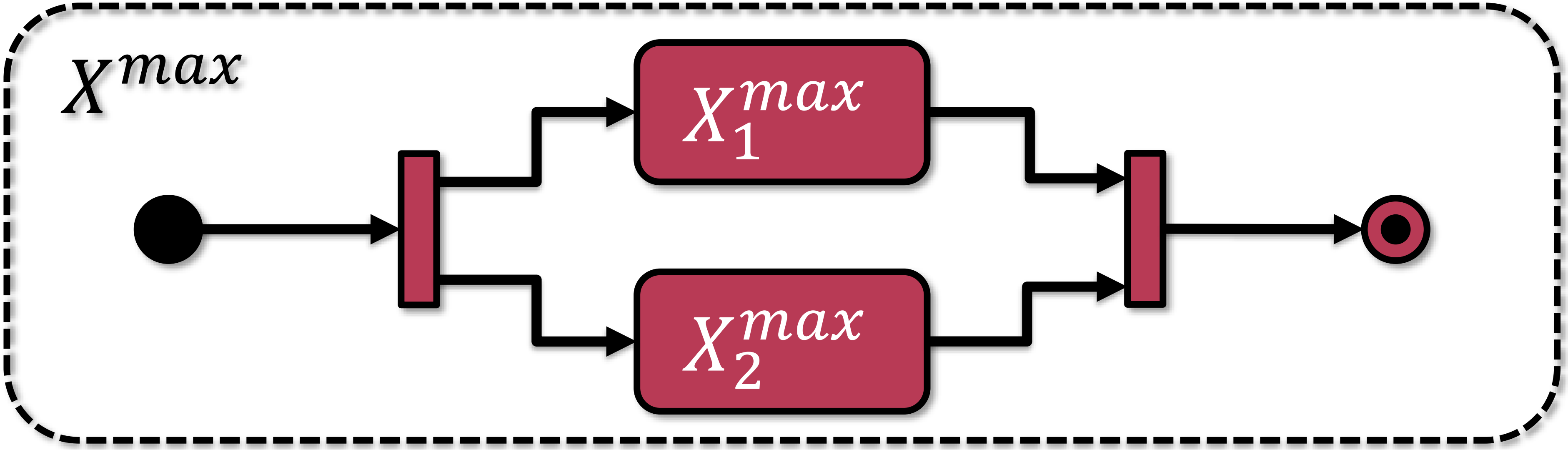


$$X^{max} = min(X_1^{max}, X_2^{max})$$

Bottleneck:

The component with the minimum throughput (or the corresponding resource).

Parallel Composition

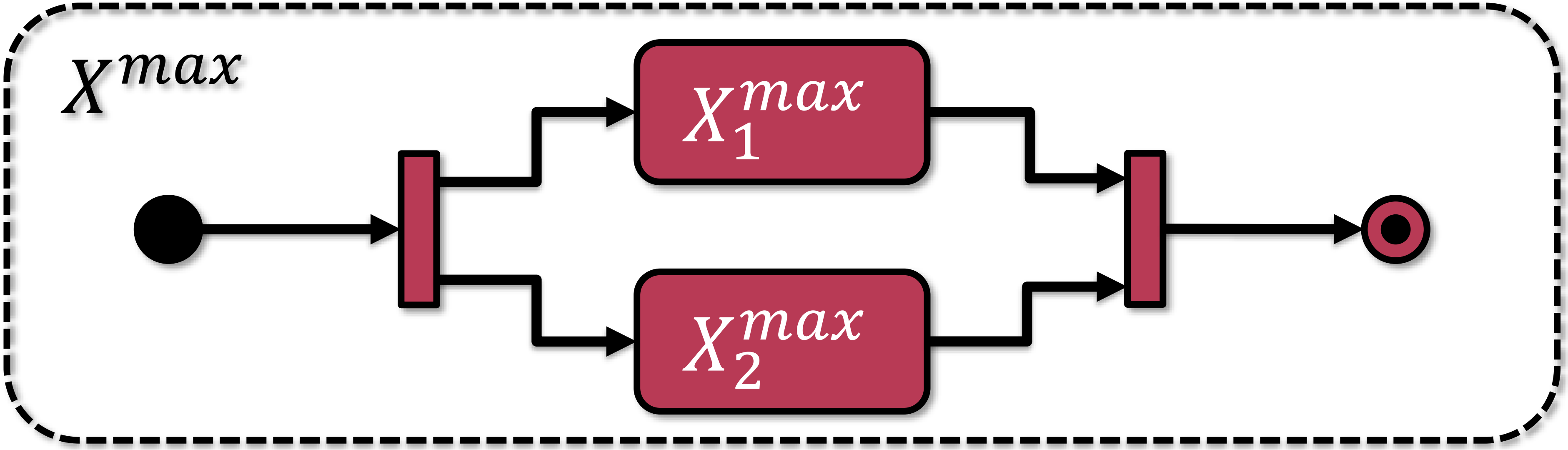


Each activity will be visited once.

It is no use if the one activity is fast, tokens will pile up at the other one.

E.g. at evaluating the exams entry test (30/hour), exercises (12/hour)

Parallel Composition



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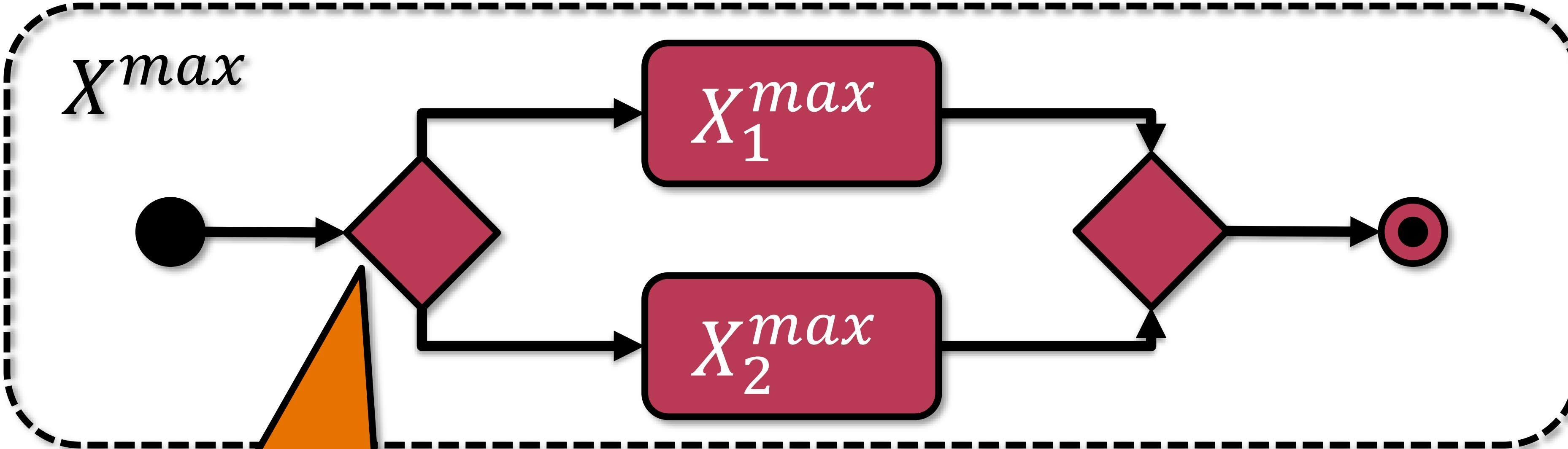
Composition of Free Choice

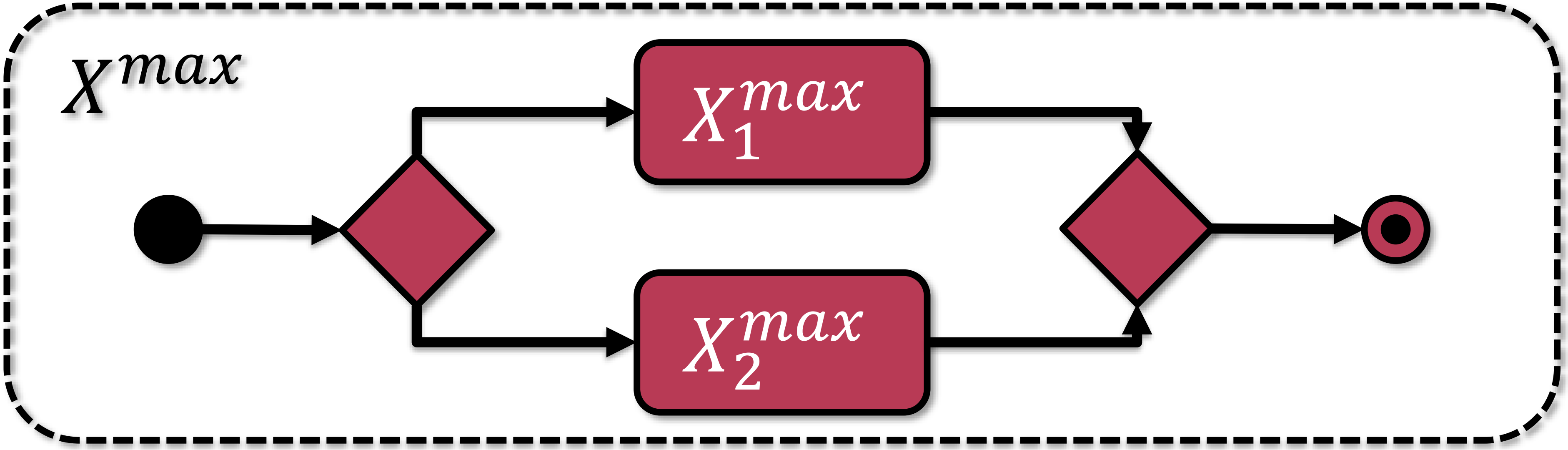


$$X^{max} = X_1^{max} + X_2^{max}$$

The tokens may choose a branch freely. If a branch is saturated, the other one may still be stable.

E.g. in a supermarkt: K cash-desks, 10 customer/h each

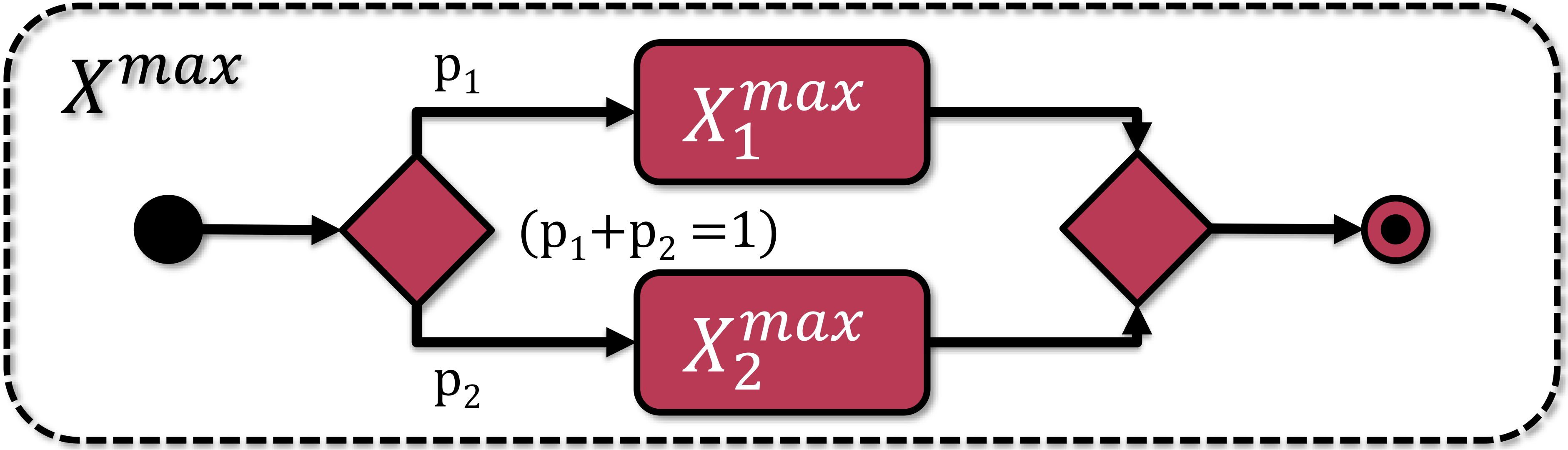
Composition of Free Choice



$$X^{max} = X_1^{max} + X_2^{max}$$

Note: Sometimes we assign the number of resources to the activities, and do not draw them several times explicitely. (see Simulation)
Condition: the resources finish the activity in the same time

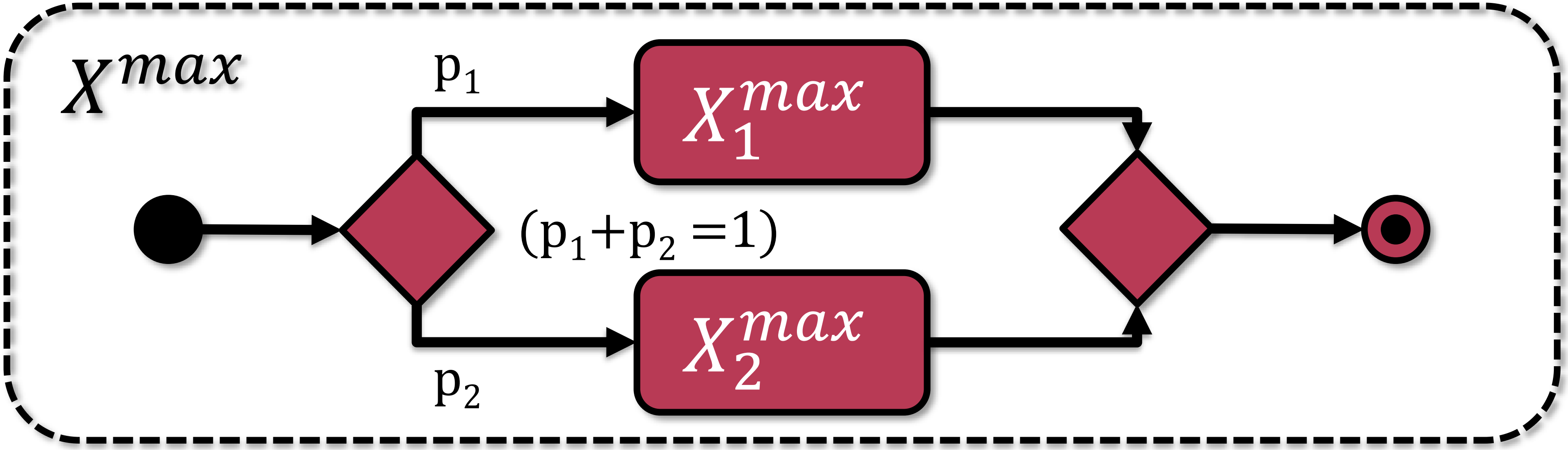
Composition of Stochastic Choice



Activity X_1 will be visited p_1 times in average, activity X_2 will be visited p_2 times in average.

The tokens choose the first or second branch with probabiliy $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$, respectively. In the whole process one token out of $\frac{1}{p_1}$ or $\frac{1}{p_2}$ will visit the activities.

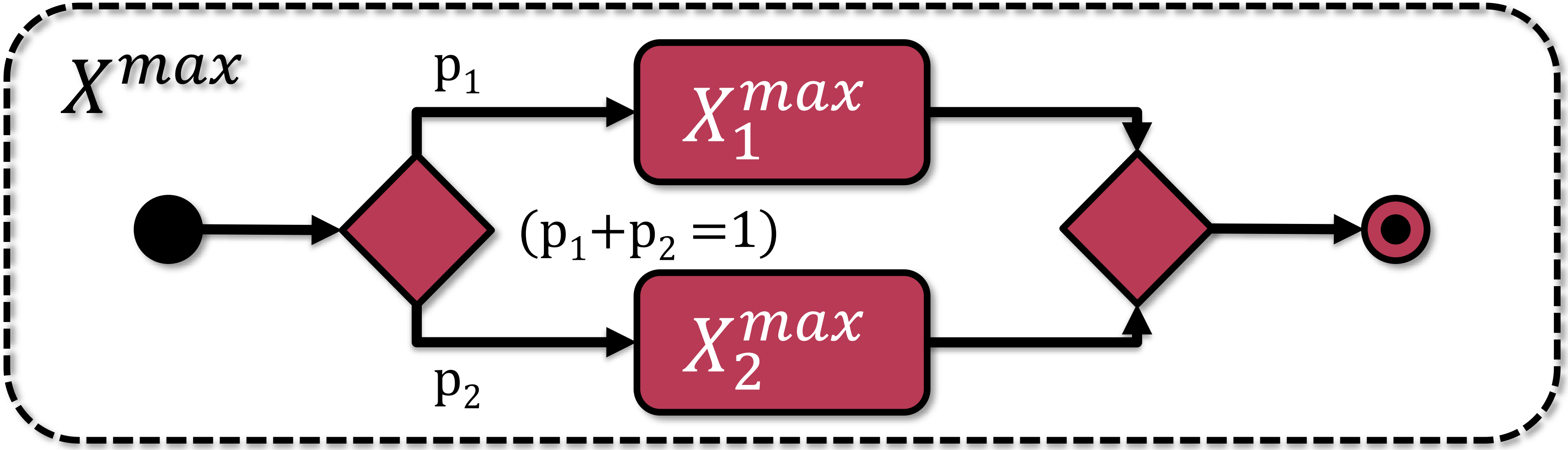
Composition of Stochastic Choice



$$X^{max} = min\left(\frac{1}{p_1} \times X_1^{max}, \frac{1}{p_2} \times X_2^{max}\right)$$

E.g. the user of a web shop:
with 20% chance he buys the good (20/sec),
with 80% chance he cancels it (200/sec)

Composition of Stochastic Choice



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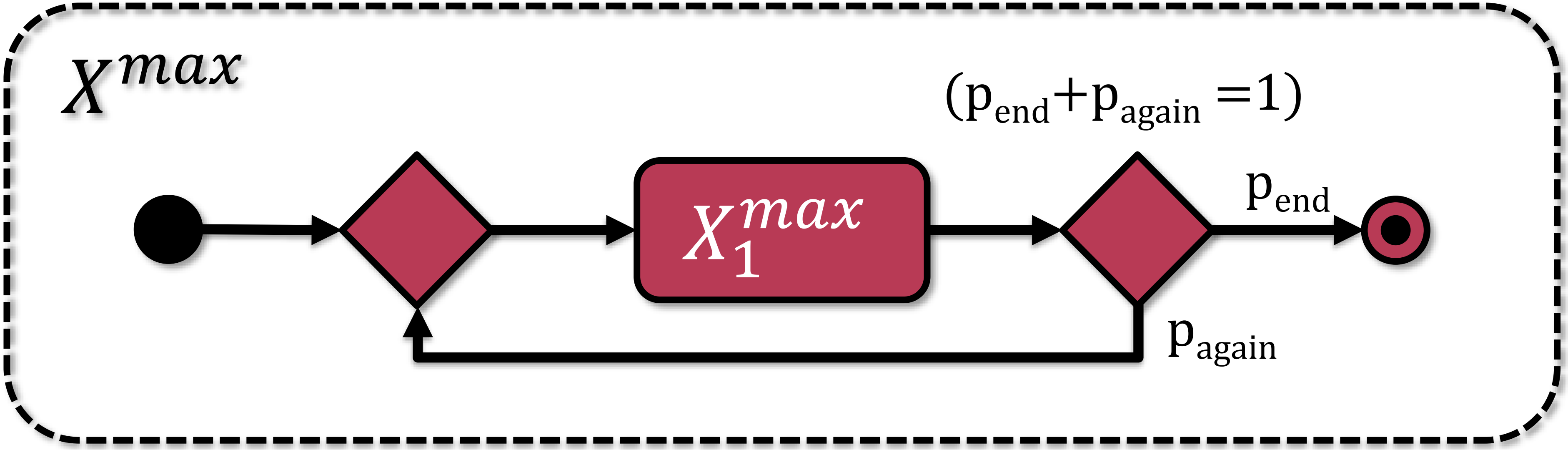
Bottleneck:

The component with the minimum throughput (or the corresponding resource).

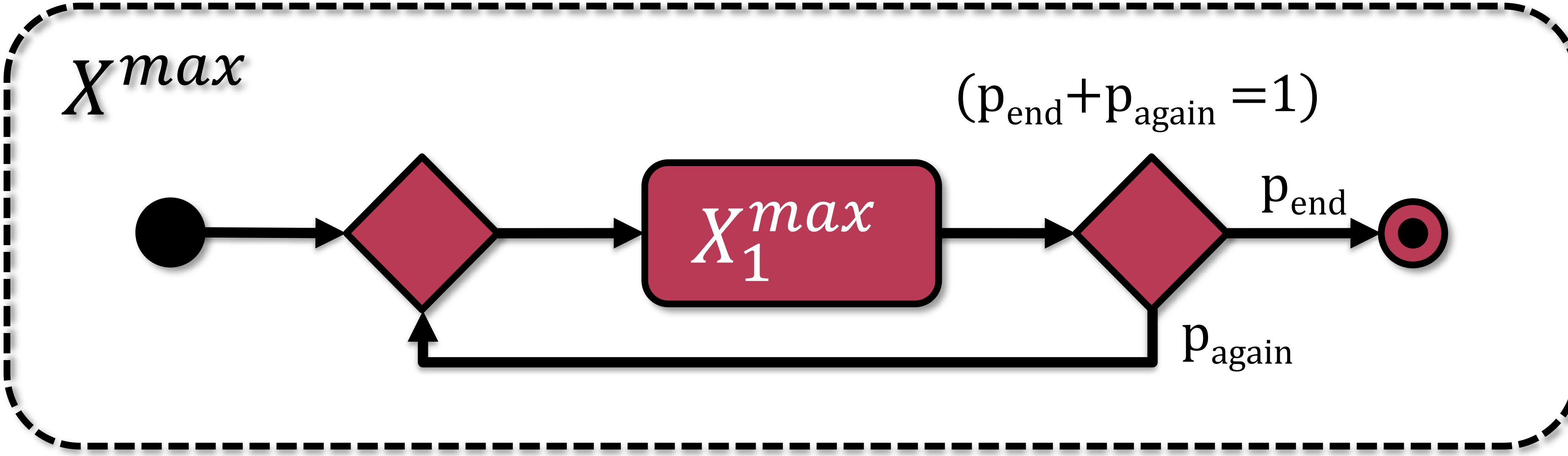
Composition of Loop



Activity X_1 will be visited $\frac{1}{p_{end}}$ times in average.

$\frac{1}{p_{end}}$ is the expected number of repetitions (see in *Probability theory* lecture).

Composition of Loop



$$X^{max} = \frac{1}{\frac{1}{p_{end}}} \times X_1^{max} = p_{end} \times X_1^{max}$$

E.g. the user in a system:
With 10% he leaves, with 90% he asks again.
15 req/s, in avg. 10 req/session

Visitation Number

Choice: $$X^{max} = min(\frac{1}{p_1} \times X_1^{max}, \frac{1}{p_2} \times X_2^{max})$$

Loop: $$X^{max} = \frac{1}{\frac{1}{p_{end}}} \times X_1^{max} = p_{end} \times X_1^{max}$$

- **Visitation number:** indicates the average number of times a given activity/subprocess runs in a single execution of the whole process.
  - In case of choice it is the decision possibility itself
  - In case of cycle it is the expected number of iterations

Visitation Number

Maximum throughput depending on the visitation number:

$$X^{max} = \frac{1}{v} \times X_1^{max}$$

- **Visitation number:** indicates the average number of times a given activity/subprocess runs in a single execution of the whole process.
  - In case of choice it is the decision possibility itself
  - In case of cycle it is the expected number of iterations

Visitation Number

Maximum throughput depending on the visitation number:

$$\frac{1}{X^{max}} = v \times \frac{1}{X_1^{max}}$$

- **Visitation number:** indicates the average number of times a given activity/subprocess runs in a single execution of the whole process.
  - In case of choice it is the decision possibility itself
  - In case of cycle it is the expected number of iterations

Visitation Number

Execution time depending on visitation number:

$$T_{process} = v \times T_{task}$$

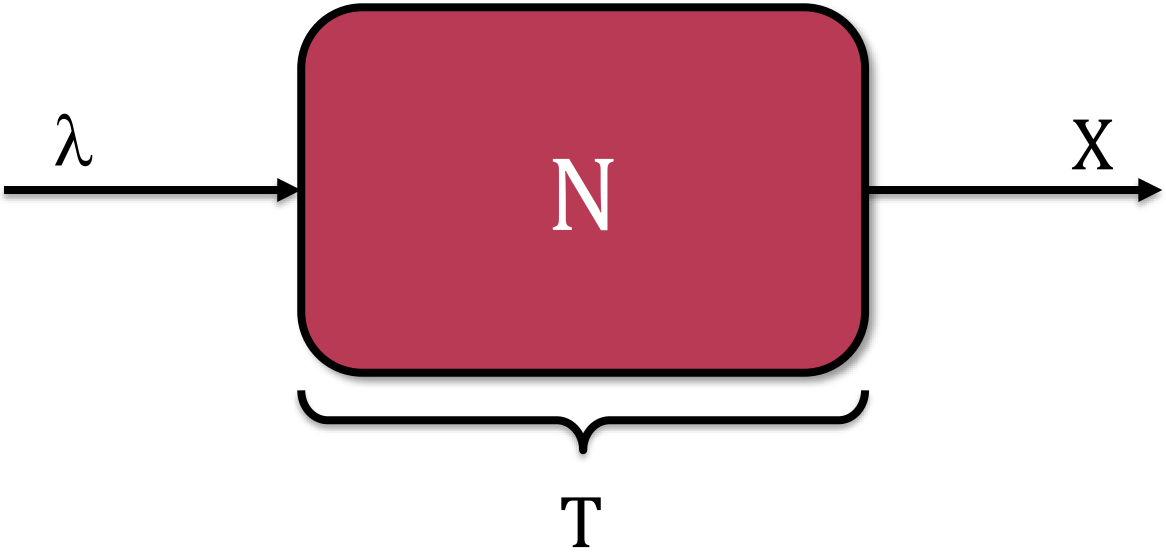
- **Visitation number:** indicates the average number of times a given activity/subprocess runs in a single execution of the whole process.
  - In case of choice it is the decision possibility itself
  - In case of cycle it is the expected number of iterations

LITTLE'S LAW

The basic formula

Little's Law

- λ: arrival rate $\left[\frac{1}{s}\right]$ X: throughput $\left[\frac{1}{s}\right]$
- T: time spent in system $[s]$
- N: number of tokens in system $[1]$



Little's Law

- **In stable state** ($\lambda = X$) Little's law holds:

$$N = X \times T$$

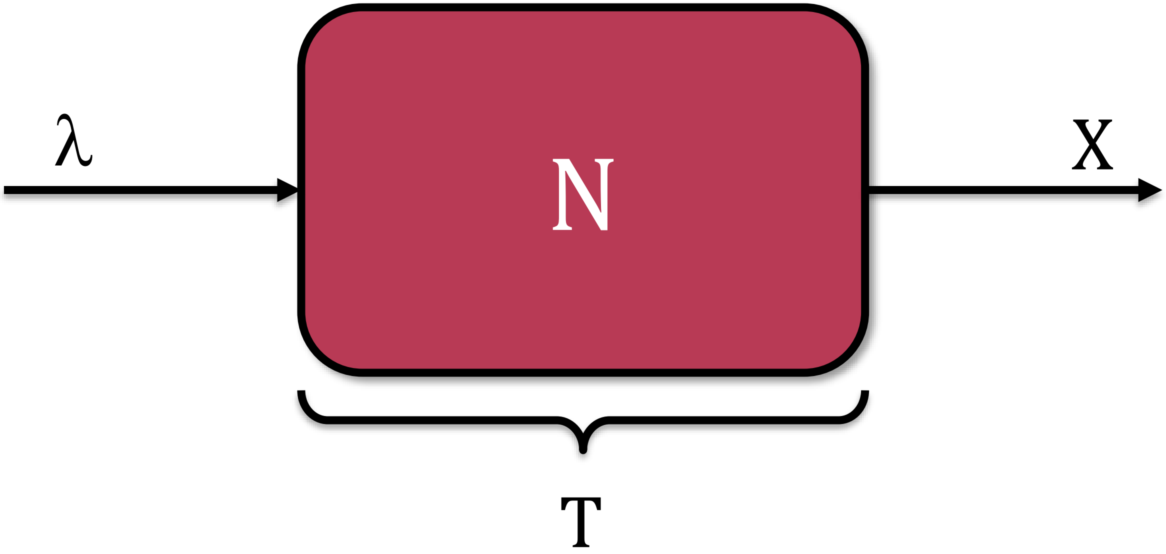
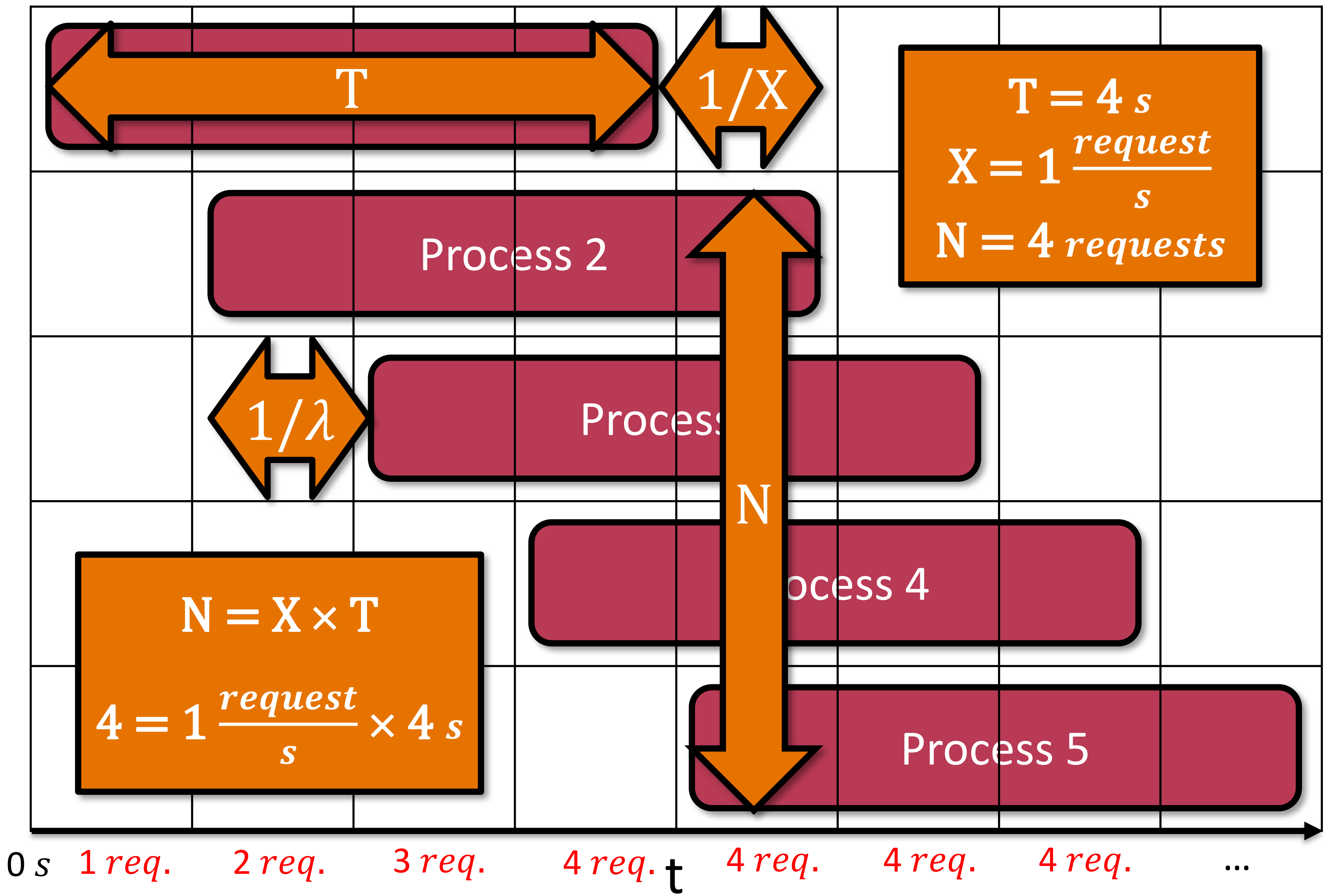


Illustration of Little's Law



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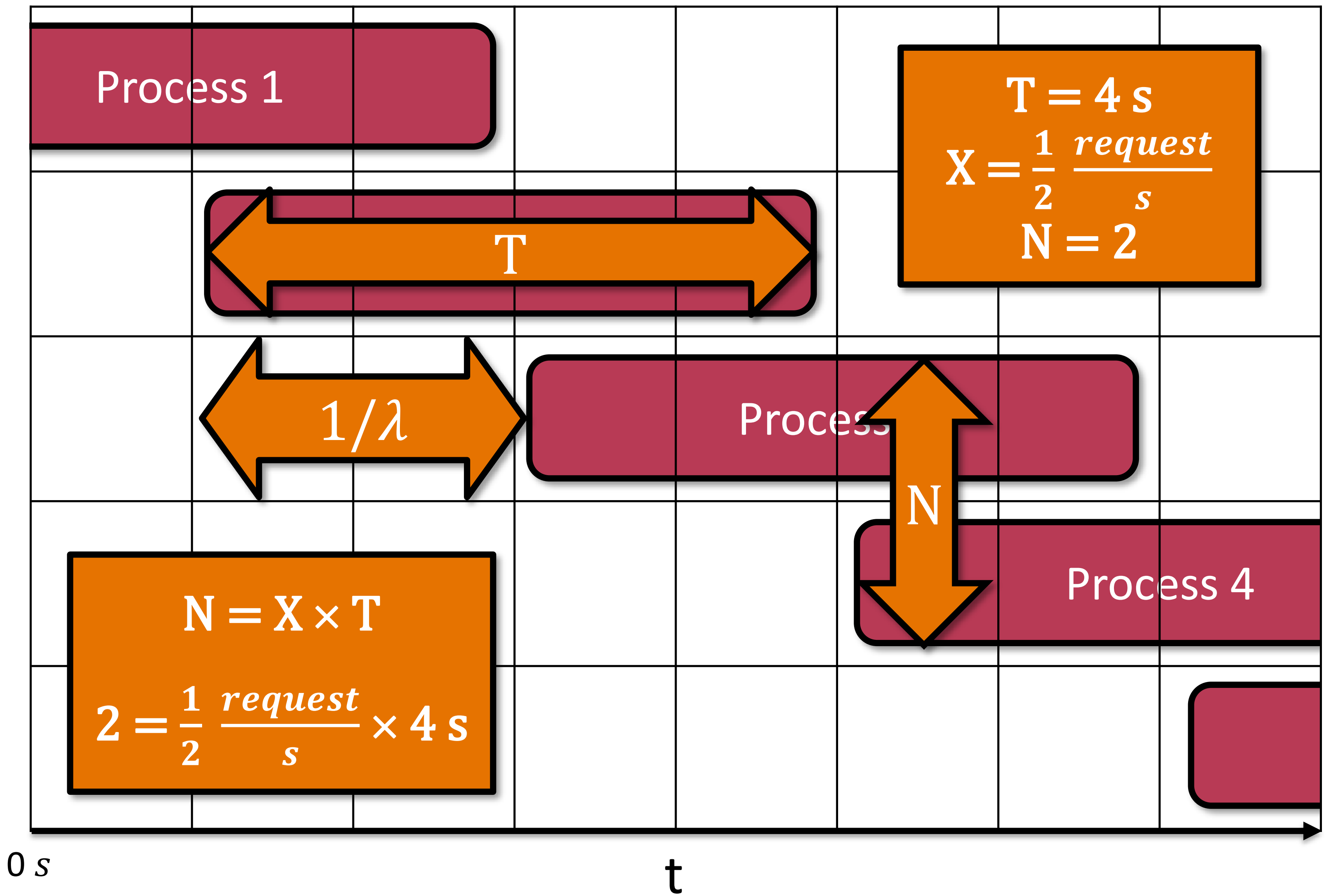


Utilization and Little's Law

- K resource instances: maximum K process instances under execution at the same time
- Little's law:
  number of process instances under execution (N)
- Average utilization can be derived as follows:

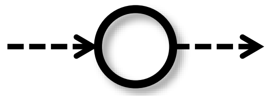
$$U = \frac{X}{K} \times T = \frac{X \times T}{K} = \frac{N}{K}$$

Utilization for K resource instances

Little's law ($N = X \times T$)

LITTLE'S LAW:
PRACTICAL EXAMPLES

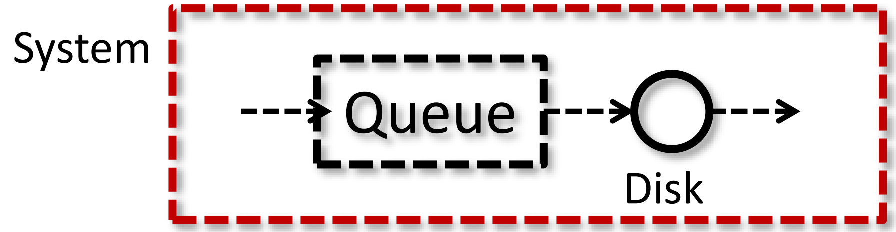
Little’s Law – Example



- Resource: disk
- Serves 40 requests per second (no overlap)
- Serving 1 request takes up 0,0225 seconds on average
- How much is the utilization?

$$U = X \times T_{disk} = 40 \frac{request}{s} \times 0{,}0225\ s = 0{,}9 = 90\%$$

Little's Law – Example

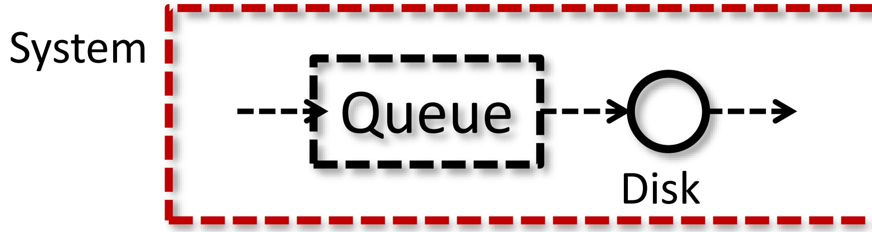


- Queuing before disk
- Disk: 40 $request/s$
- Average requests in system: 4

Average time a request spends in the system? (T_{system})

Average queuing time? ($T_{waiting}$)

Little's Law – Example



- Queuing before disk
- Disk: 40 $request/s$
- Average requests in system: 4

Queuing plus disk serving time

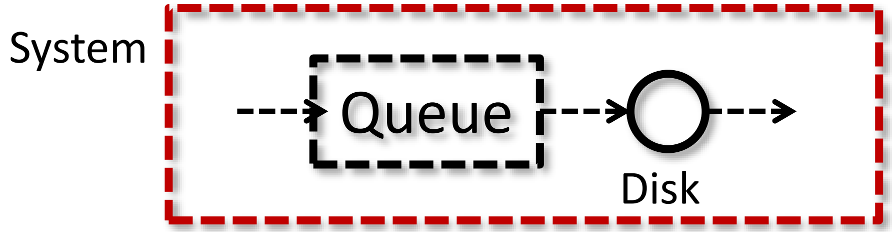
System

$$N = X \times T \rightarrow T_{system} = 4\ requests / 40\ \frac{request}{s} = 0{,}1\ s$$

Average queuing time

$$(T_{system} - T_{disk}) = (0{,}1\ s - 0{,}0225\ s) = 0{,}0775\ s$$

Little’s Law – Example



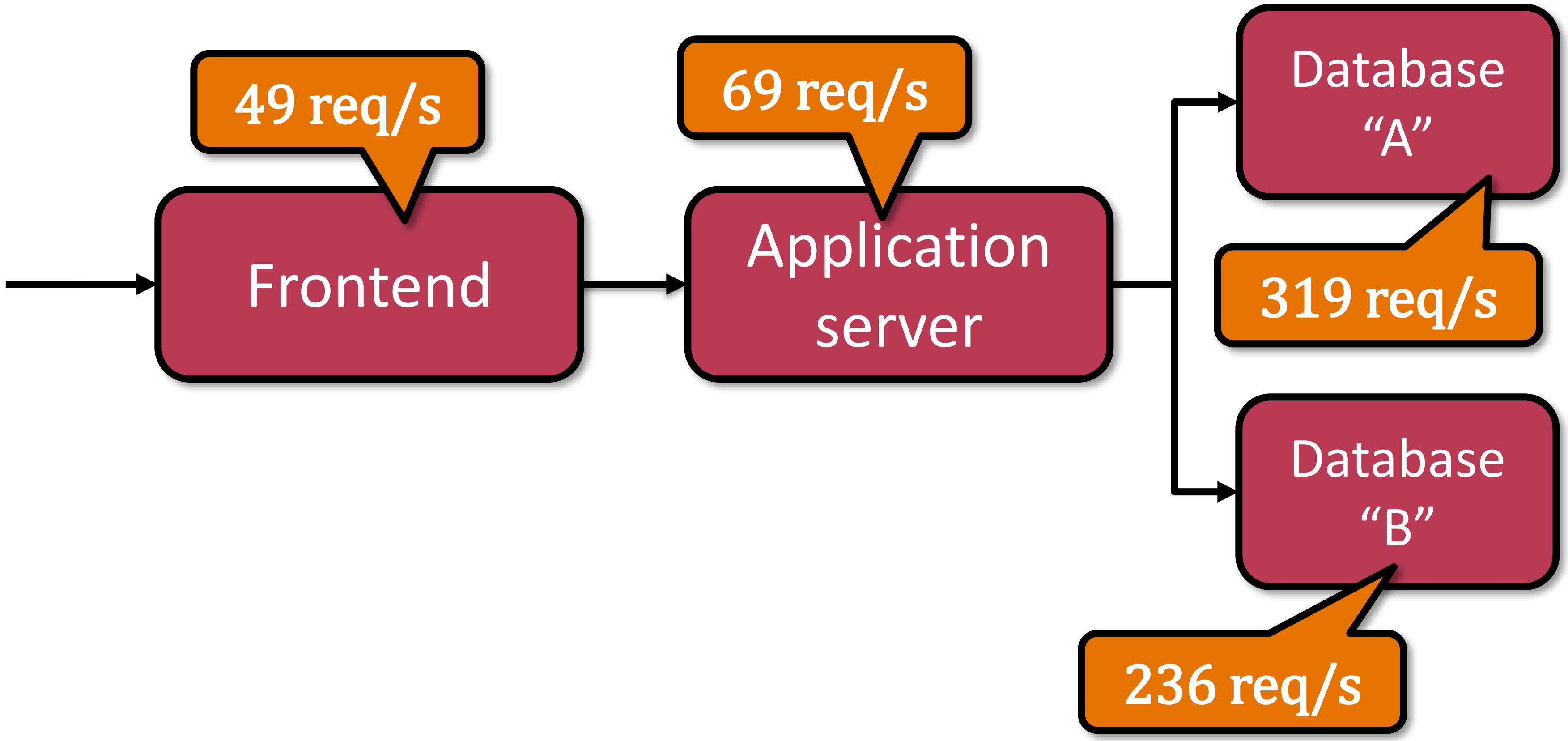
- Queuing before disk
- Disk: 40 $request/s$. In average 0,9 request
- Average requests in system: 4

Average number of requests in queue?
(N_{system} – N_{disk})
4 requests – 0,9 request = 3,1 requests

Little’s Law in Practice

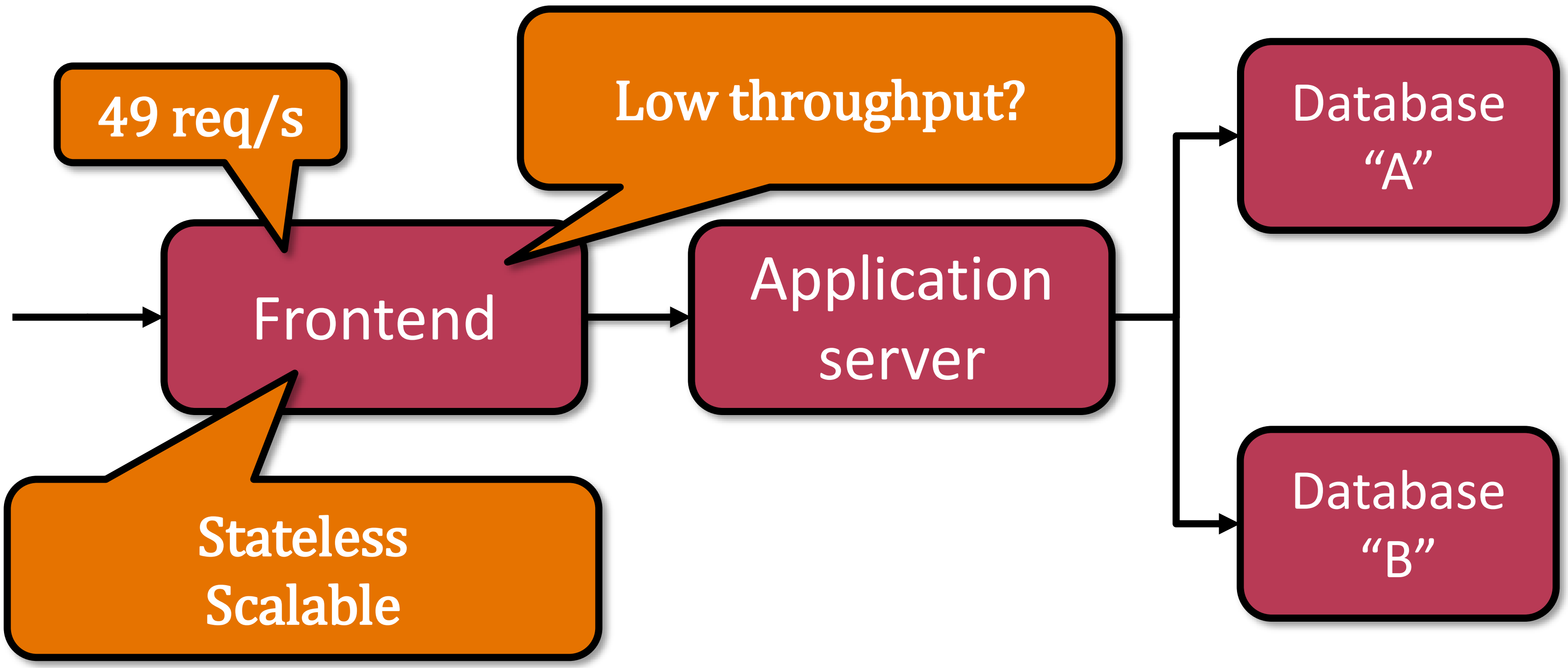
- Simulation
  - Dobson&Shumsky
  - https://youtu.be/UjzXQPGBaNA
- Why it is taught
  - http://pubsonline.informs.org/doi/pdf/10.1287/ited.7.1.106
- Examples
  - http://web.mit.edu/sgraves/www/papers/Little's%20Law-Published.pdf
    - E.g.: How long do the wine bottles stay in the cellar?
      - The cellar is filled up to ⅔ in average. (~160 bottles)
      - We bought 8 bottles per month in the last one year.
      - According to Little’s law, the bottles stay in average $T=N/X$, that is $160/8=20$ months in the cellar.

Performance of 3-tier Architecture



These metrics indicate the incoming load of the complete system!
For instance „Database A" becomes the bottleneck if 319 requests arrive to the <u>system</u> each second.

Performance of 3-tier Architecture



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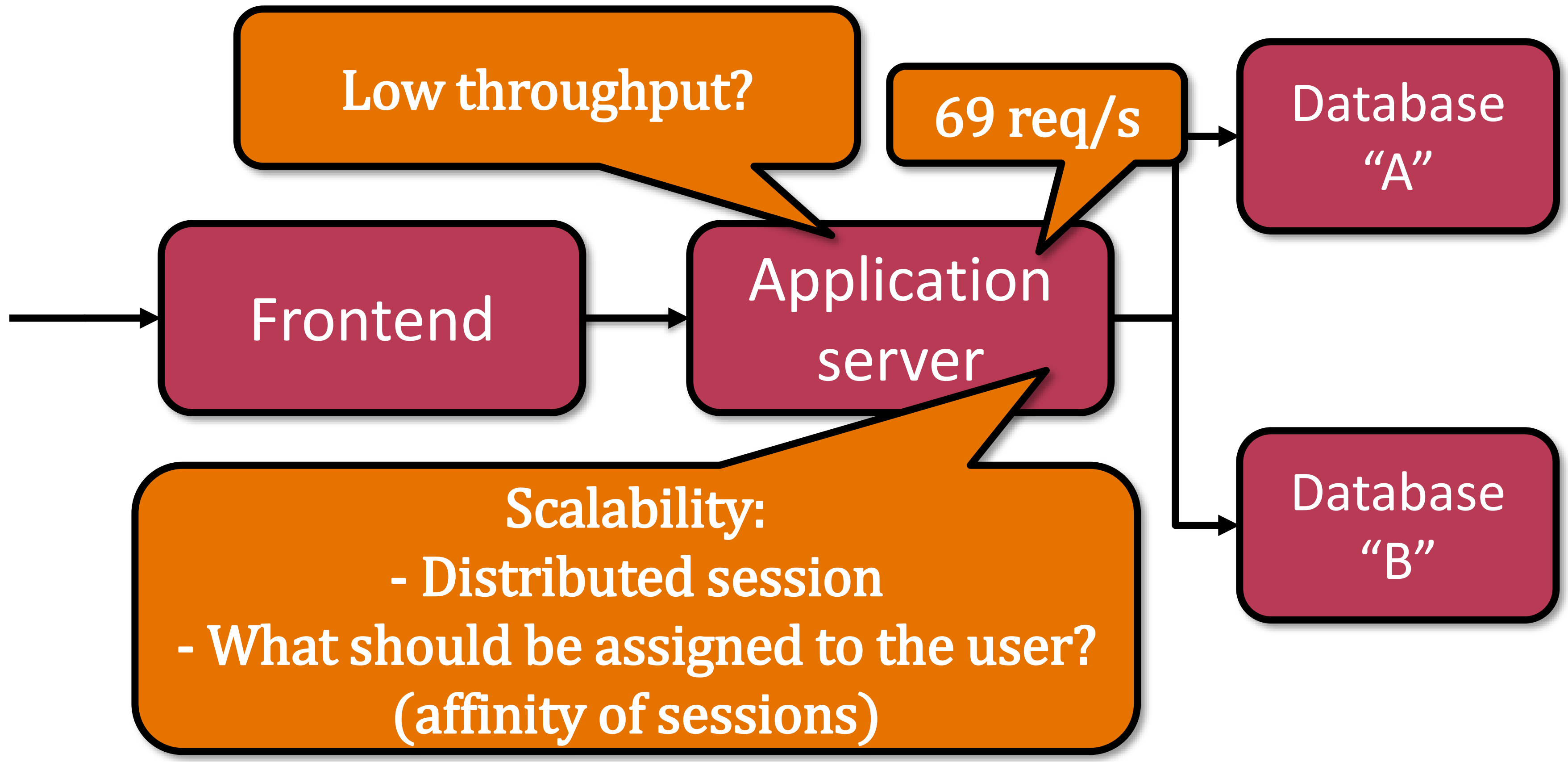

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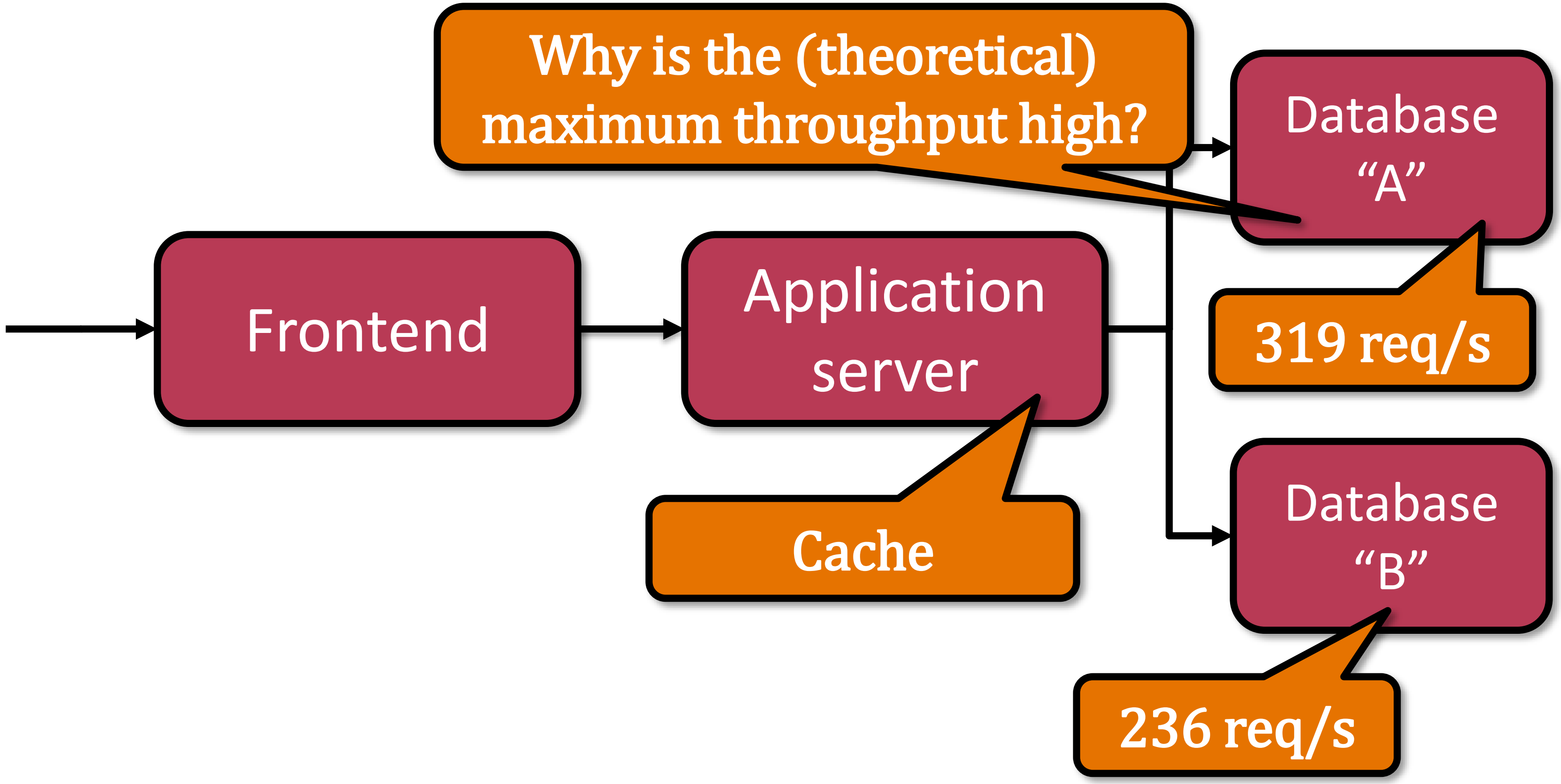

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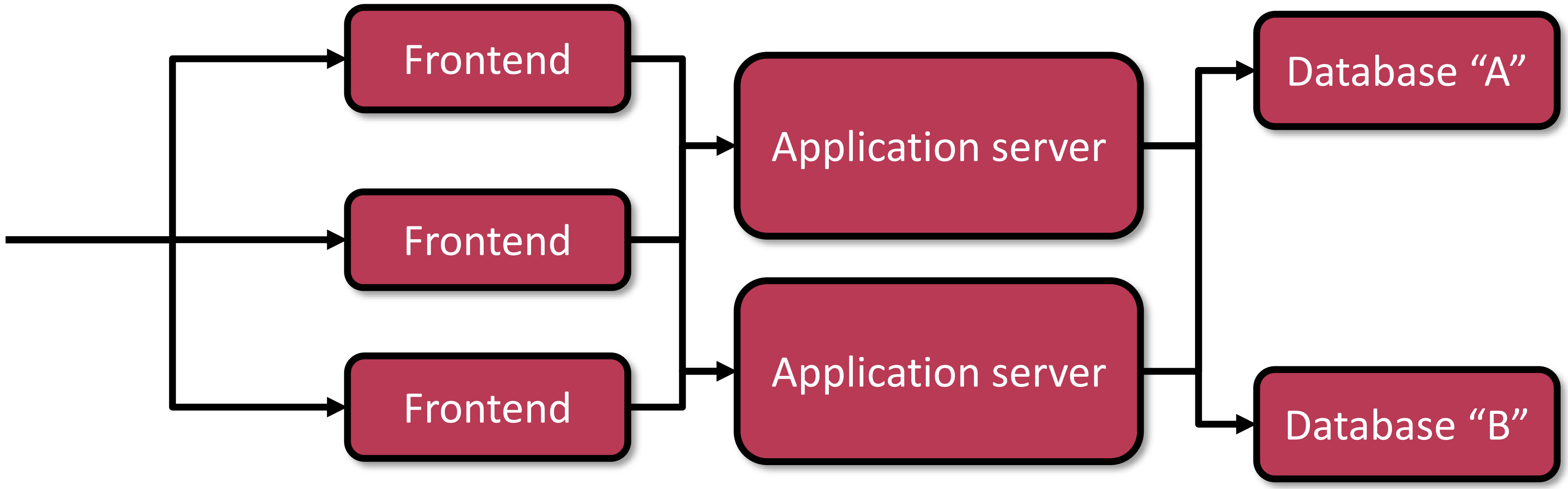
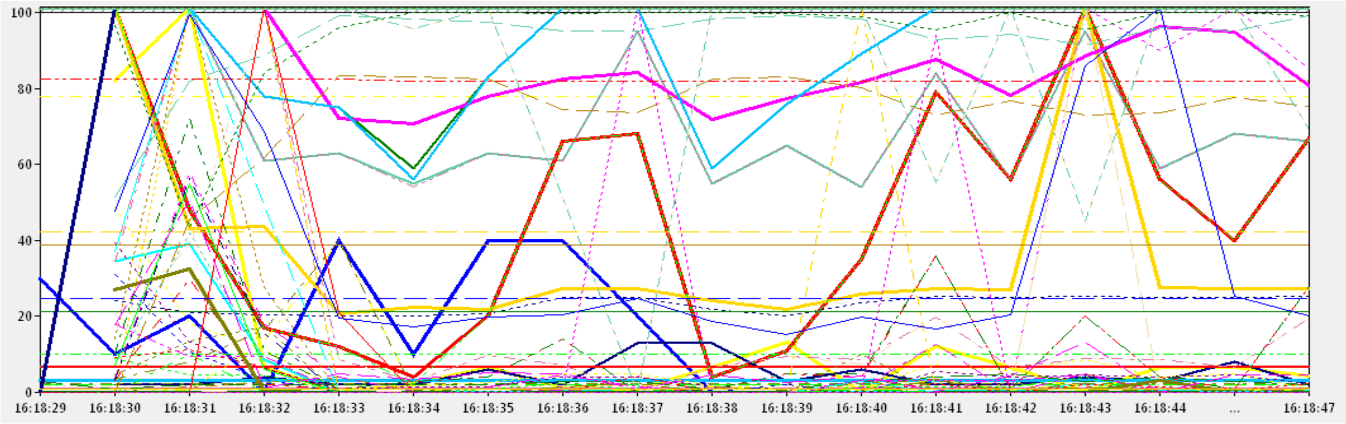
3-tier Architecture in Reality



(Example: technological background for the interested)
http://www.projectclearwater.org/wp-content/uploads/2013/05/Clearwater-Deployment-Sizing-10-Apr-13.xlsx
http://www.projectclearwater.org/technical/clearwater-performance/

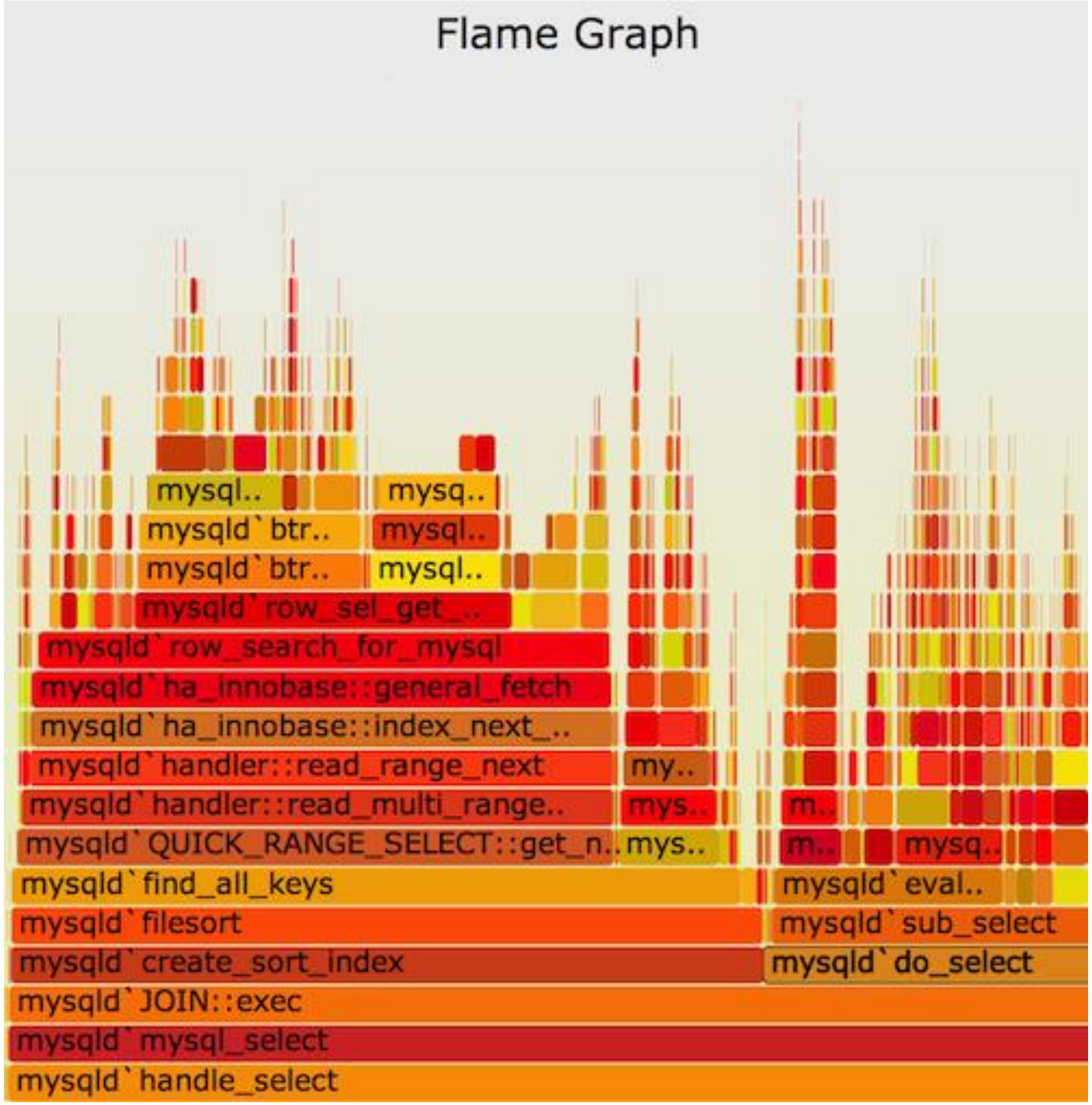
What to Measure? / What is Important?

- Metrics “in small”
  - E.g. Task manager, Resource monitor, the same on server-side…



- Metrics “in big”
  - E.g. virtualized systems
- Which metrics are interesting?

E.g.: What Takes so Much to Compute?



http://www.brendangregg.com/flamegraphs.html

Where Do We Approximate?

“Watch my hands, because …”
(picture: wikipedia)

- In practice the values are difficult to measure
  - (e.g. response time fluctuation, …)
- Applications compete
  - $(2 * \lambda \neq \lambda + \lambda)$
- Decision between resources
  - → Load balancer is also critical
  - → E.g. requests of the same user to the same server
- We ignored the actual order/pattern of arrival
  - Advantage of Little’s law
- Execution of a task may be data-dependent
- Structure/parameters of the system may change

Where Do We Approximate?

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  - (e … n, …)
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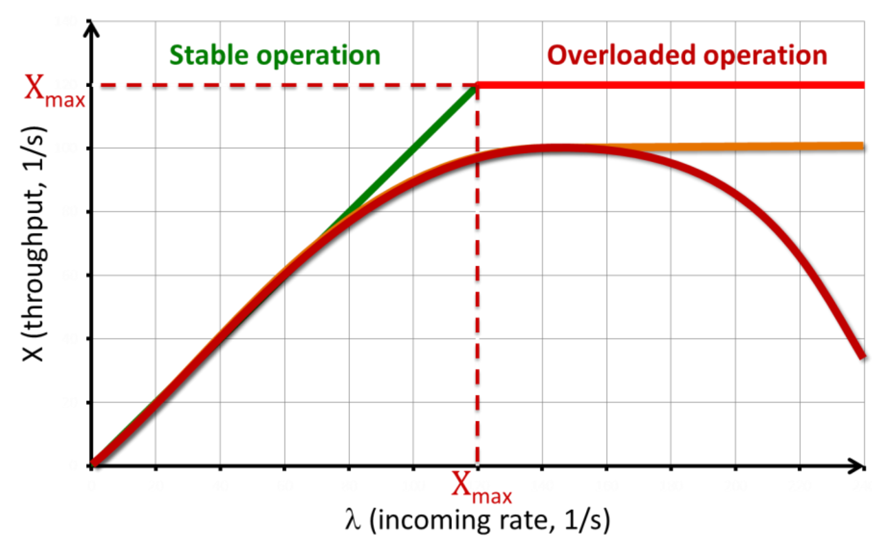
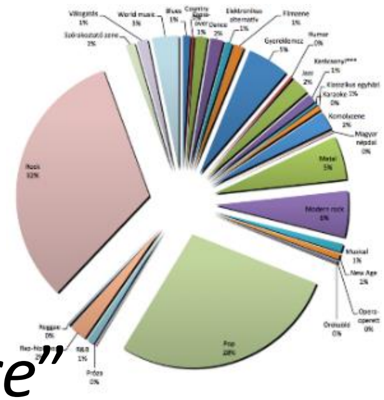
Everything is „lies and deception” only.

…ands, because …”
…edia)



LOAD MODELS: ZIPF’S LAW

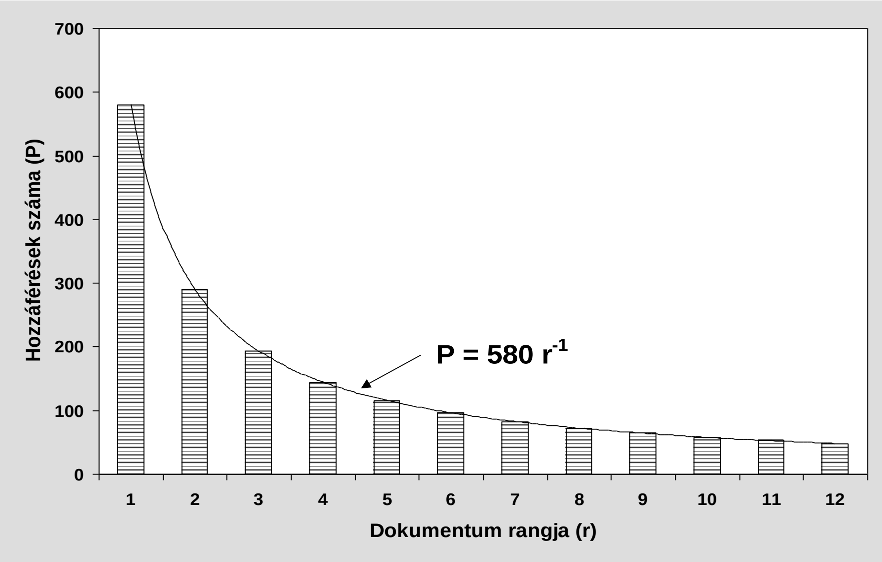
What is the Content of the Requests?



- Up to now: each requests are alike
  - “I need the details of a book”
- Actually: requests have content
  - “I need the details of *Foundation and Empire*”
  - See Pareto principle (80% – 20%)
  - Majority of the requests concerns minority of data
- Essential, because…
  - Has technical effects
    - Cache, pool size, static storage, …
  - Concerns the system model
    - Special handling of frequent requests

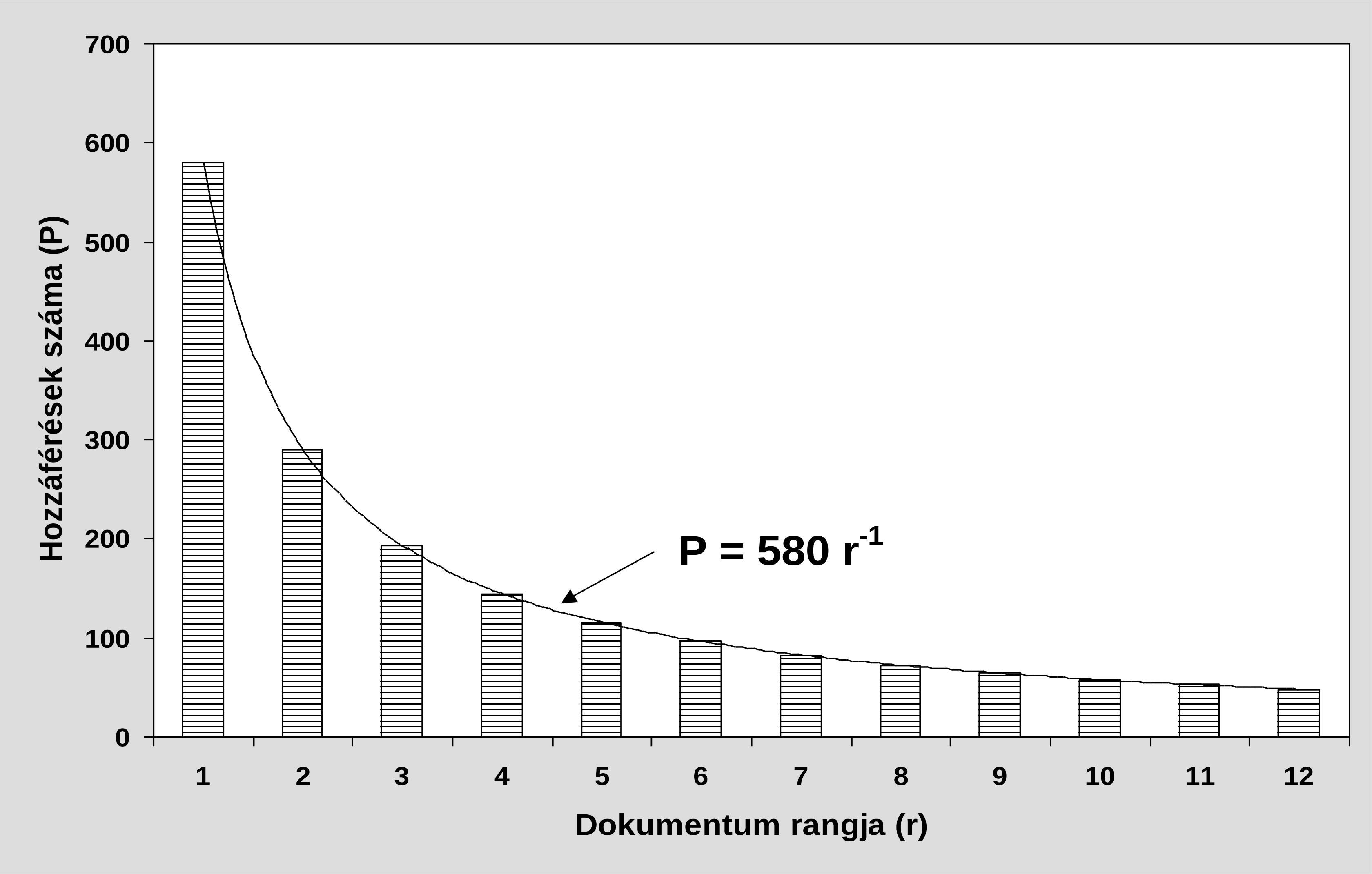
Zipf's Law

- Originally: number and frequency of words in *corpora* shows a characteristic distribution
  - True for not only language texts



George Kingsley Zipf
(1902–1950)
US American linguist
and philologist

Zipf's Law – Example



Zipf’s Law - Examples

- Hit lists
- Population of cities by their ranks
- Characteristics of internet traffic
- Popularity of websites’ subpages
- Evolution of open source systems

Zipf's Law - Formula

$$R_i \sim \frac{1}{i^{\alpha}} \qquad f \sim \frac{1}{p}$$

- R_i – is the incidence of the i^{th} word
- α – a value charasteristic of the corpus
  - close to 1
- Simplified (α = 1):
  - f - frequency
  - p – popularity: rank of the text (decreasing order)

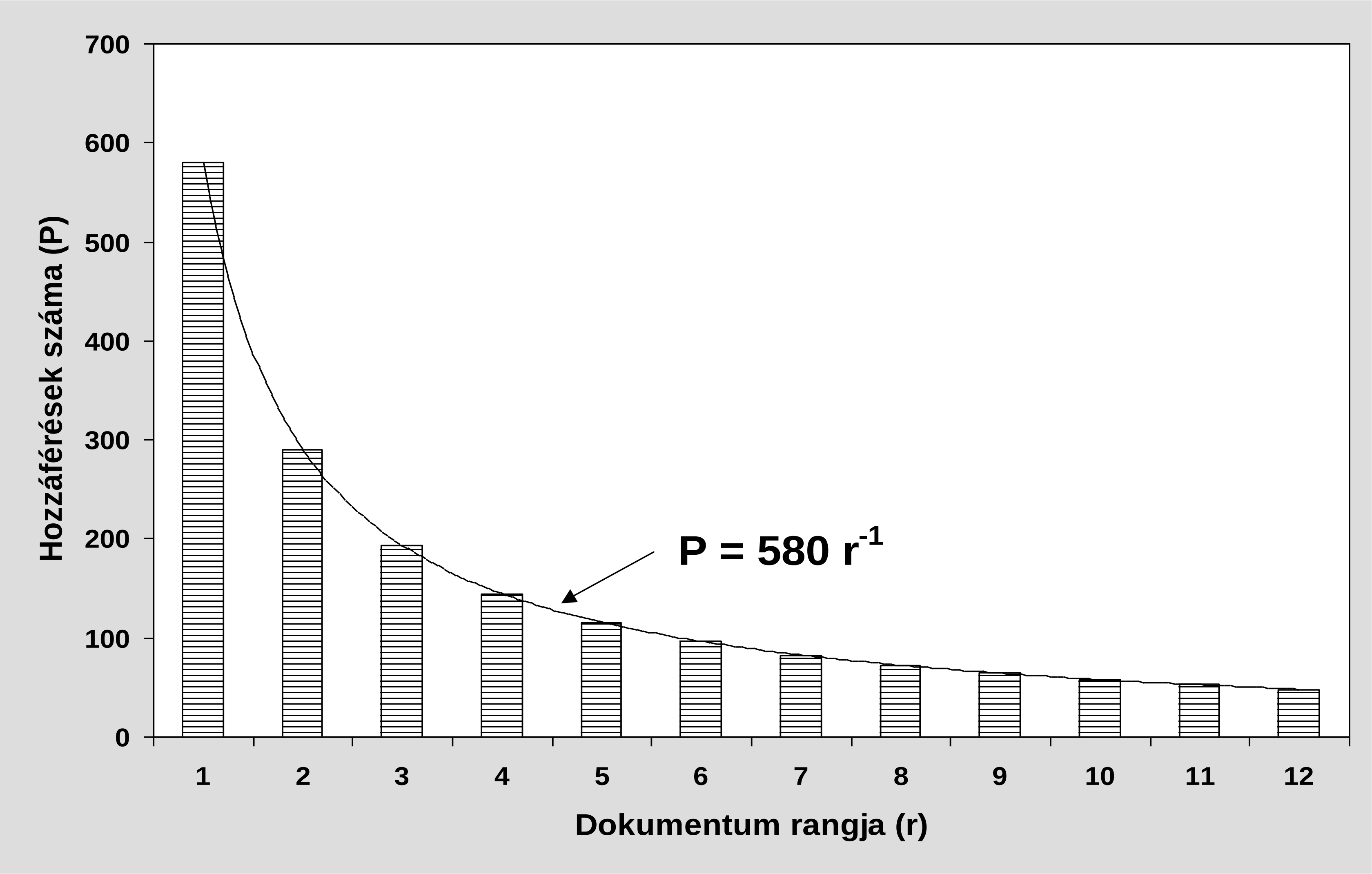
Zipf's Law – Example: Web Documents

$$P = \frac{k}{r}$$

- P – references (hits)
- r – rank (1 = most frequent)
- k – positive constant

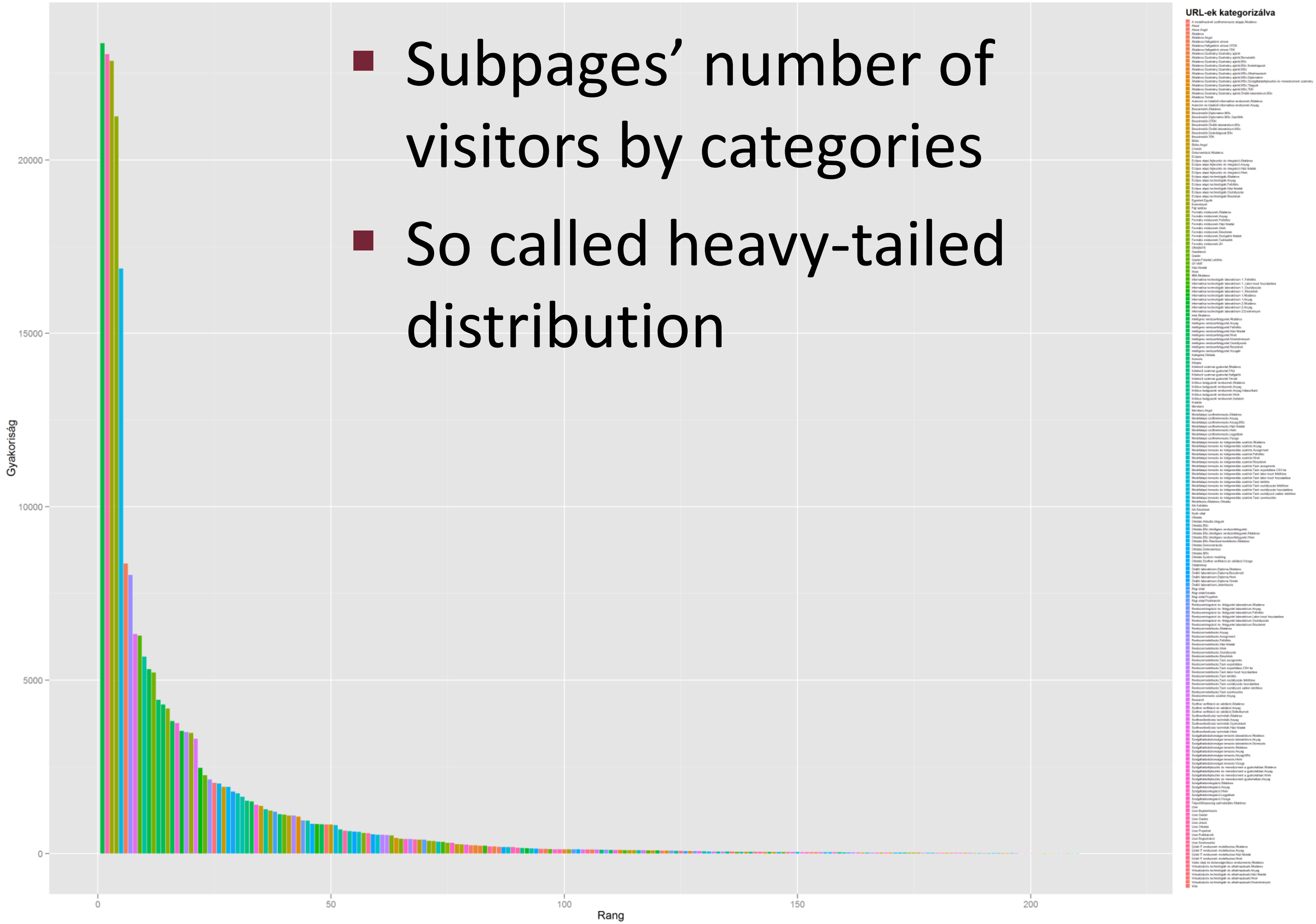
For more information see:
http://www.hpl.hp.com/research/idl/papers/ranking/adamicglottometrics.pdf

Zipf's Law – Example



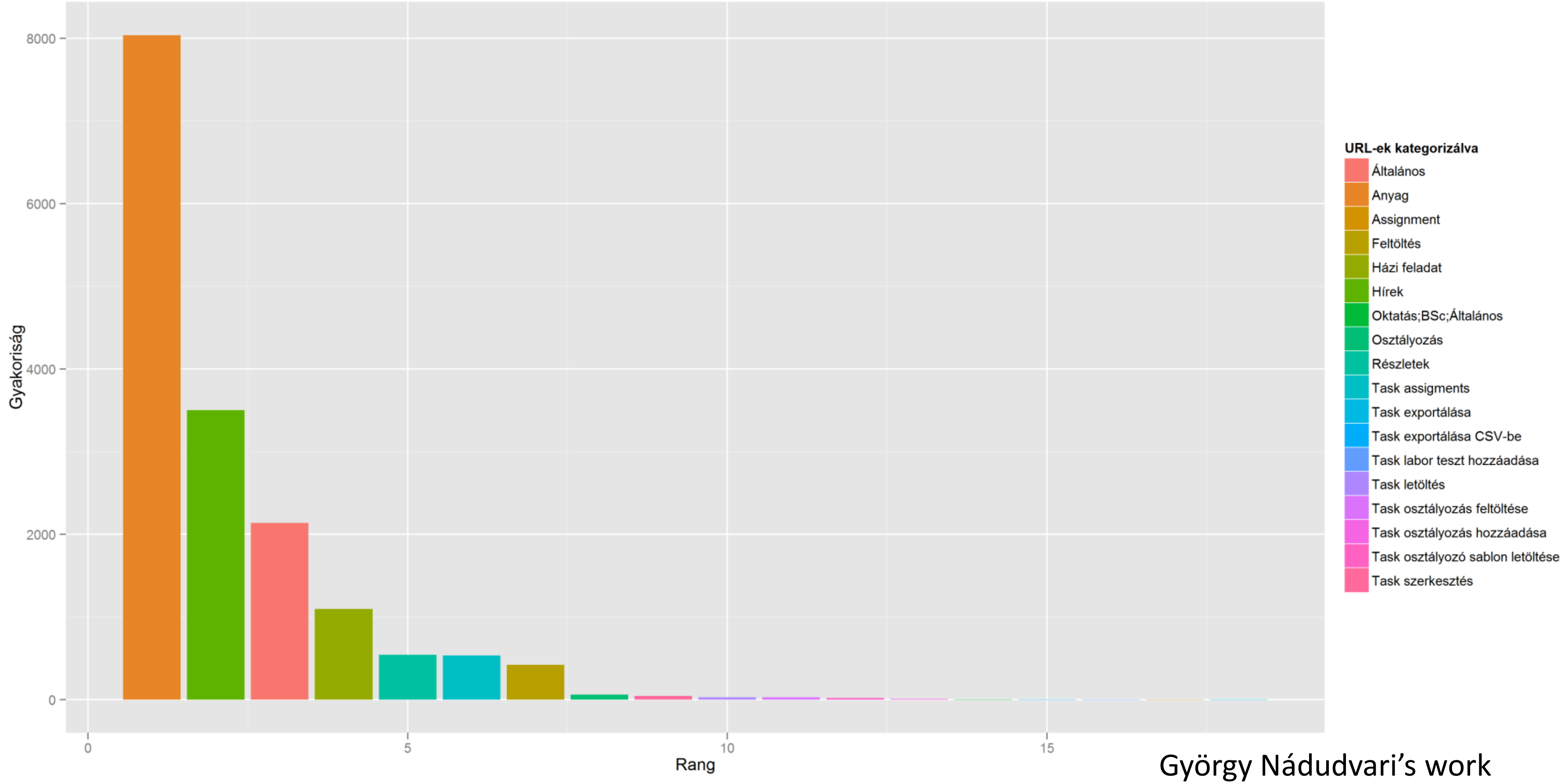
Zipf – Example: Website of our Group

- Subpages' number of visitors by categories
- So called heavy-tailed distribution



Zipf – Example: Website of our Group

- Visitors of the webpages of the System Modelling course



György Nádudvari’s work

Visitation number | Little's law | Zip's law | **Changes in Workload**

CHANGES IN THE WORKLOAD

What kind of workload?

- Up to now:
  - We calculated with average values
  - Regarded the system’s behaviour depending on the *load (intensity)*
  - But: In reality the increase of the load is not necessarily predictable
- In reality
  - The behaviour of the system *changes over time*
  - This has technological effects
    - Switching between tasks, resource reservation, etc. (see: Operating systems)

Changes in the Workload – Example

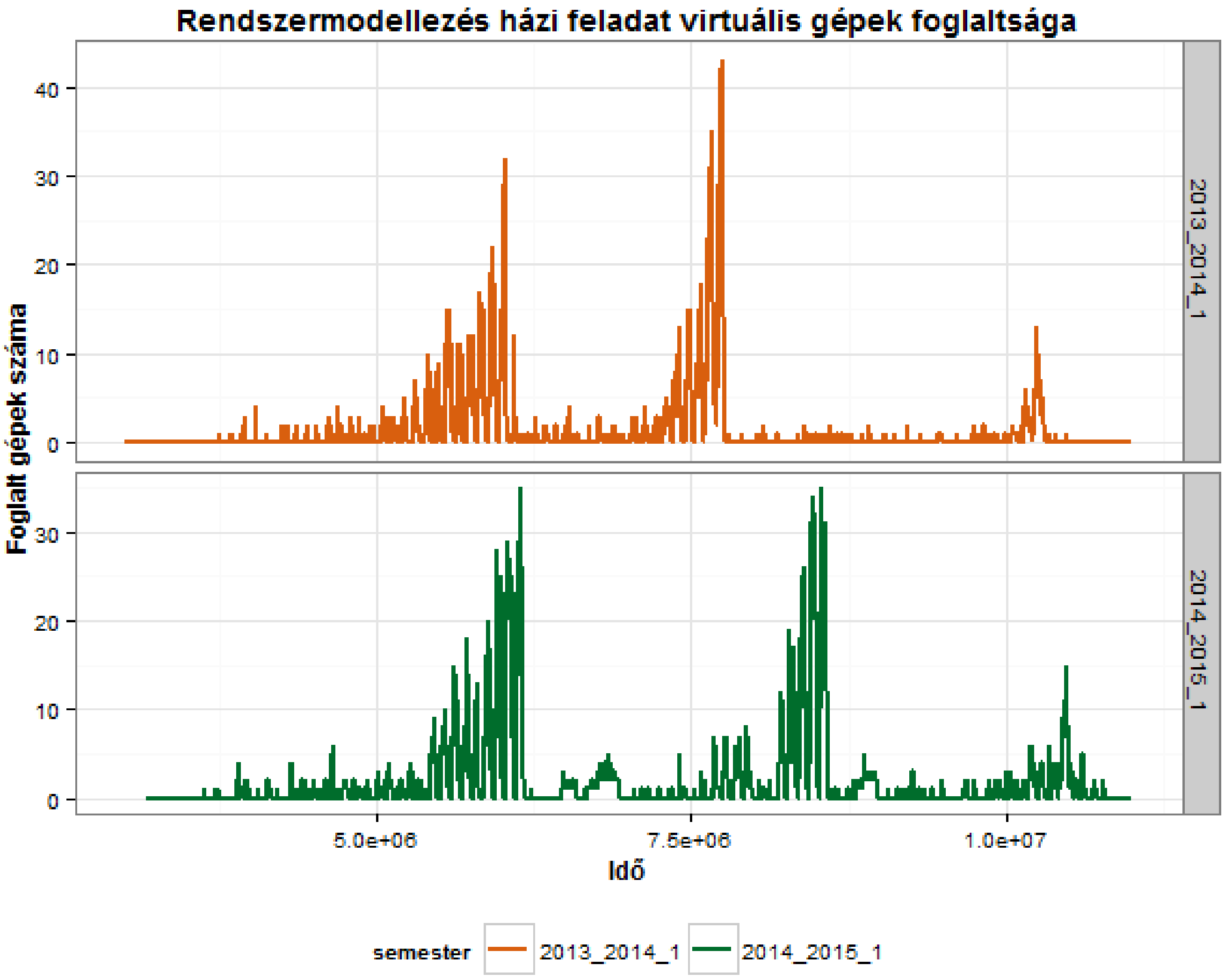
- Dimensioning a systems for producing the (at that time) new identity cards
  - It is predictable how many new cards will be applied for in a year. (expirations, next age group)
  - It is predictable how many hours there are in a year.

    → We have the avg. arrival rate of the applications [$card/h$]

    Can it be used for dimensioning the system?

- Consider two different hours
  1. the 24th December 10-11PM
  2. the 15th June 4-5PM
     (End of working day shortly before the main summer holiday time)

System Modelling (7th semest.) – in the cloud



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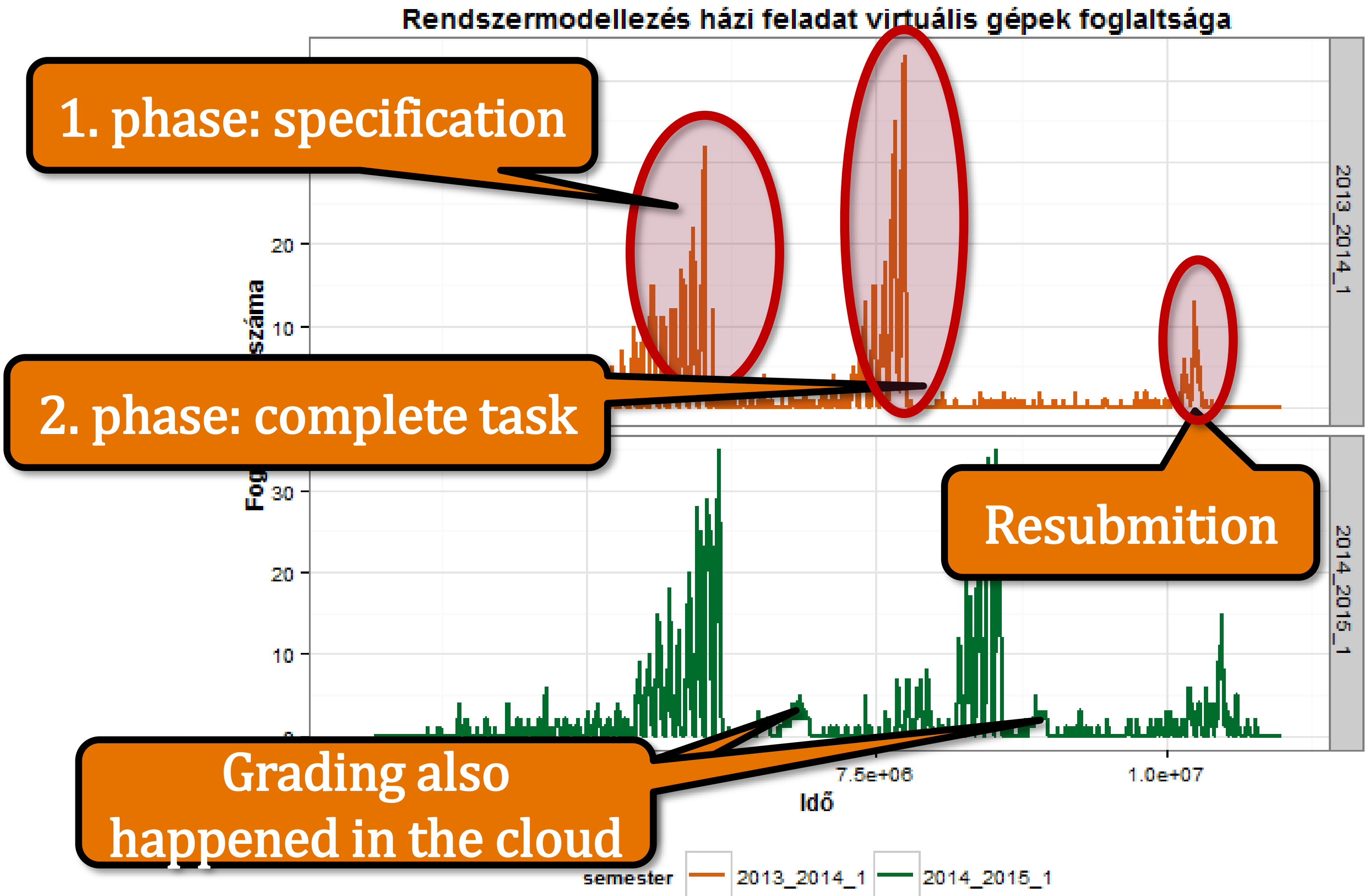

Real (historical) Load Example (iwiw)

Approximations can be applied to sections
- Linear, exponential, logarithmic
- Regression, detailed in Probability Theory

Source: http://www.sg.hu/cikkek/42924/