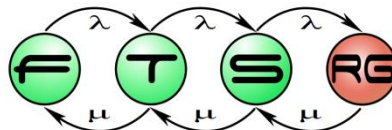


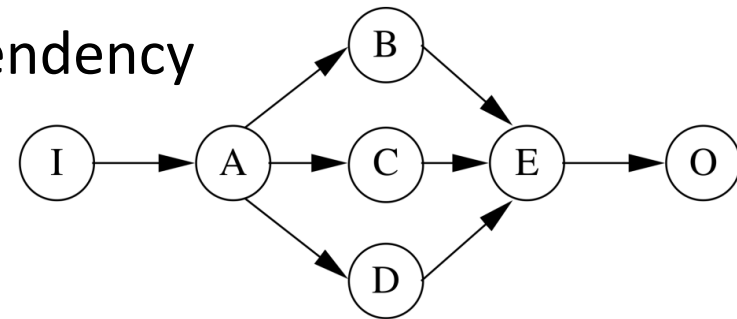
Fault Tolerant Allocation and Schedule

**Budapesti Műszaki és Gazdaságtudományi Egyetem
Hibatűrő Rendszerek Kutatócsoport**



Assumptions

- Given a set of hardware components ($p^1 \dots p^N$)
 - Multiple software can be allocated to P
 - Only one software can run at the same time
 - If a hardware fails, it cannot operate anymore.
- Given a set of software components ($s^1 \dots s^M$)
 - Processes can be described by *data-flow graphs*
 - Each vertex is a software component
 - Each directed edge is a data-dependency between software components.

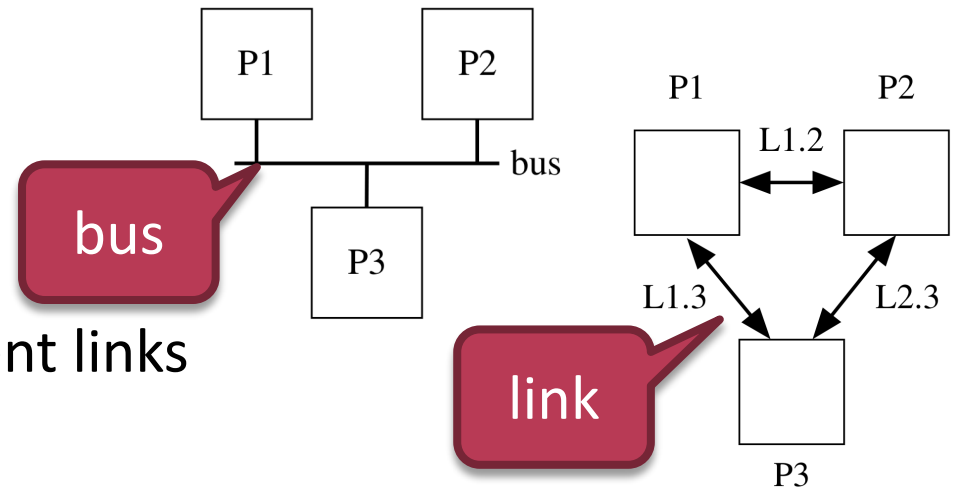


Goal

- Allocate and schedule the software components to hardware components to tolerate K hardware failures, where $K < N$. (N is the number of processors)
 - If K number of hardware components fail, the process should still provide output.
 - If K number of hardware components fail, the tasks should not be rescheduled to provide output.

Communication Time

- Intra-communication on the same P
 - Output of a task is sent to another task on the same P
 - For an intra-communication, the time is always **0**
- Inter-Communication between different Ps
 - Output of a task is sent to another task on different P
 - Bus
 - Always the same time
 - Links
 - Different time on different links



Notations

- ***Candidates^{itr}***: The list of candidate S components, after the itr^{th} iteration of the algorithm.
- ***Scheduled^{itr}***: The list of scheduled S components, after the itr^{th} iteration of the algorithm.
- ***Pred(s_i)***: The set of predecessors of s_i component.
- ***Succ(s_i)***: The set of successors of s_i component.
- ***S^{itr}(s_i, p_t)***: The earliest time at which component s_i can start execution on processor p_t , taking into account all the predecessors replicas except if s_{i-1} and s_i are allocated to the same p_t
- **$\Delta(s_i, p_t)$** : is the execution duration of s_i on p_t

Algorithm

Itr⁰, initialize the list of candidates and scheduled sw components

$$Scheduled^0 = \emptyset, Candidates^1 = \{s \in SW \mid Pred(s) \in Scheduled^0\}$$

Itrⁿ, while $Candidates^n \neq \emptyset$ do

(1) Compute the scheduling pressure (σ) for each $s \in Candidates^n$ on all the hardware components and keep the first $K+1$ results for each s . $\sigma(s_i, p_t) = S^n(s_i, p_t) + \Delta(s_i, p_t)$

(2) Select the worst candidate s_i according to the remaining σ values, and schedule it to the first $K+1$ hardware components.

(3) Update the list of candidates and scheduled sw components

$$Scheduled^n = Scheduled^{n-1} \cup \{s_i\},$$

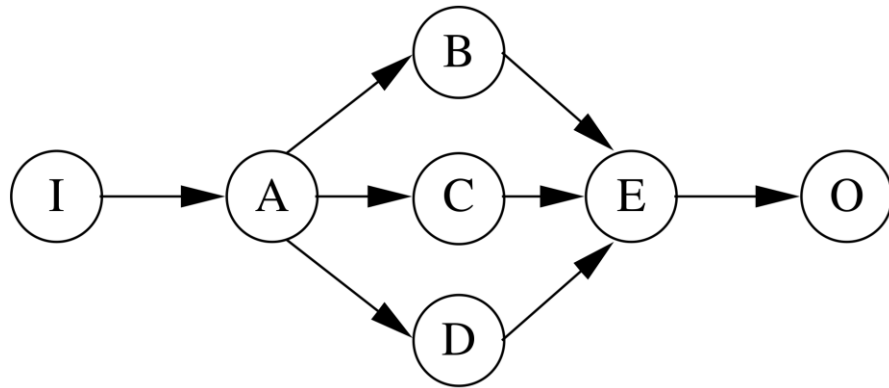
$$Candidates^{n+1} = Candidates^n / \{s_i\} \cup$$

$$\{s_i' \in Succ(s_i) \mid Pred(s_i') \in Scheduled^n\}$$

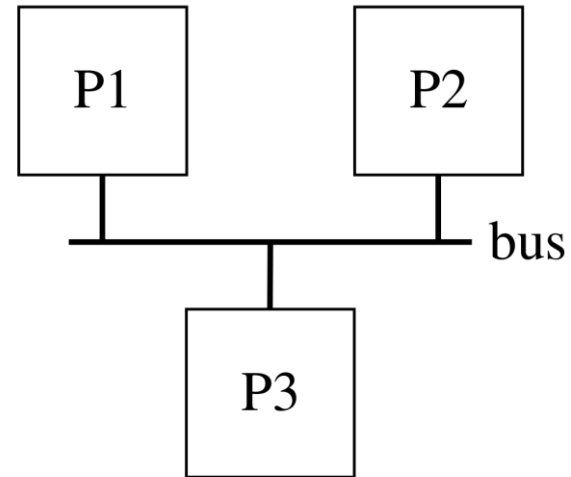
Add a successor only if its predicates are scheduled

Example I (Bus, K=1)

Software Architecture



Platform Architecture



Execution

Time (ET)

		operation							
		time units	I	A	B	C	D	E	O
proc.	P1	1	2	3	2	3	1	1.5	
	P2	1	2	1.5	3	1	1	1.5	
	P3	∞	2	1.5	1	1	1	∞	

Data

Transfer (DT)

		data-dependency								
		time units	I \triangleright A	A \triangleright B	A \triangleright C	A \triangleright D	B \triangleright E	C \triangleright E	D \triangleright E	E \triangleright O
link	bus	1.25	0.5	0.5	1	0.5	0.6	0.8	1	

Example from: Girault, Alain, et al. "Fault-tolerant static scheduling for real-time distributed embedded systems." *INT. CONF. ON DIST. COMP. SYS.*, Vol. 21. IEEE Computer Society; 1999, 2001.

Example I (Bus, K=1) – Solution – Itr: 0

Scheduled⁰ = ∅, Candidates¹ = {I}

Example I (Bus, K=1) – Solution – Itr: 1

Candidates¹ ≠ ∅ → do

$$\sigma(I, P1) = 0 + 1 = 1$$

$$\sigma(I, P2) = 0 + 1 = 1$$

$$\sigma(I, P3) = 0 + \text{undefined} = \text{undefined}$$

Example I (Bus, K=1) – Solution – Itr: 1

Candidates¹ ≠ ∅ → do

$$\sigma(I, P1) = 0 + 1 = 1$$

$$\sigma(I, P2) = 0 + 1 = 1$$

$$\sigma(I, P3) = 0 + \text{undefined} = \text{undefined}$$

Select K+1 lowest

Example I (Bus, K=1) – Solution – Itr: 1

Candidates¹ ≠ ∅ → do

$$\sigma(I, P1) = 0 + 1 = 1$$

$$\sigma(I, P2) = 0 + 1 = 1$$

$$\sigma(I, P3) = 0 + \text{undefined} = \text{undefined}$$

Select K+1 lowest

Select highest from
the remaining

Example I (Bus, K=1) – Solution – Itr: 1

Candidates¹ ≠ ∅ → do

$$\sigma(I, P1) = 0 + 1 = 1$$

$$\sigma(I, P2) = 0 + 1 = 1$$

$$\sigma(I, P3) = 0 + \text{undefined} = \text{undefined}$$

Select K+1 lowest

Select highest from the remaining

*allocate I to P1 and P2,
schedule I on P1 from 0,
schedule I on P2 from 0*

Example I (Bus, K=1) – Solution – Itr: 1

Candidates¹ ≠ ∅ → do

$$\sigma(I, P1) = 0 + 1 = 1$$

$$\sigma(I, P2) = 0 + 1 = 1$$

$$\sigma(I, P3) = 0 + \text{undefined} = \text{undefined}$$

Select K+1 lowest

Select highest from
the remaining

*allocate I to P1 and P2,
schedule I on P1 from 0,
schedule I on P2 from 0*

Scheduled¹ = {I}, Candidates² = {A}

Example I (Bus, K=1) – Solution – Itr: 2

Candidates² ≠ ∅ → do

$$\sigma(A, P1) = 1 + 2 = 3$$

$$\sigma(A, P2) = 1 + 2 = 3$$

$$\sigma(A, P3) = 2.25 + 2 = 4.25$$

*allocate A to P1 and P2,
schedule A on P1 from 1,
schedule A on P2 from 1*

- “A” can start immediately after “I” on P1 and P2 → S = “I done”
- “A” has to wait the output of “I” from P1 and P2 on P3 → S = max(“I done”+DT(IA))

DT from all replicas, but in this case these two are equal

Scheduled¹ = {I,A}, Candidates² = {BCD}

Example I (Bus, K=1) – Solution – Itr: 3

Candidates³ ≠ ∅ → do

$$\sigma(B,P1)=3 + 3 = 6$$

$$\sigma(B,P2)=3 + 1.5 = 4.5$$

$$\sigma(B,P3)=3.5 + 1.5 = 5$$

$$\sigma(C,P1)=3 + 2 = 5$$

$$\sigma(C,P2)=3 + 3 = 6$$

$$\sigma(C,P3)=3.5 + 1 = 4.5$$

$$\sigma(D,P1)=3 + 3 = 6$$

$$\sigma(D,P2)=3 + 1 = 4$$

$$\sigma(D,P3)=4 + 1 = 5$$

- **K+1 lowest for B,C,D**

$$\sigma(B,P3)=5$$

$$\sigma(B,P1)=4.5$$

$$\sigma(C,P3)=4.5$$

$$\sigma(C,P1)=5$$

$$\sigma(D,P2)=4$$

$$\sigma(D,P3)=5$$

From 2 equal values,
you can choose randomly

Select the highest
from the remaining:
 $\sigma(B,P3)=5 \rightarrow$ allocate B

Example I (Bus, K=1) – Solution – Itr: 3

Candidates³ ≠ ∅ → do

$$\sigma(B,P1)=3 + 3 = 6$$

$$\sigma(B,P2)=3 + 1.5 = 4.5$$

$$\sigma(B,P3)=3.5 + 1.5 = 5$$

$$\sigma(C,P1)=3 + 2 = 5$$

$$\sigma(C,P2)=3 + 3 = 6$$

$$\sigma(C,P3)=3.5 + 1 = 4.5$$

$$\sigma(D,P1)=3 + 3 = 6$$

$$\sigma(D,P2)=3 + 1 = 4$$

$$\sigma(D,P3)=4 + 1 = 5$$

- **K+1 lowest for B,C,D**

$$\sigma(B,P3)=5$$

$$\sigma(B,P2)=4.5$$

$$\sigma(C,P3)=4.5$$

$$\sigma(C,P1)=5$$

$$\sigma(D,P2)=4$$

$$\sigma(D,P3)=5$$

*allocate B to P2 and P3,
schedule B on P2 from 3,
schedule B on P3 from 3.5*

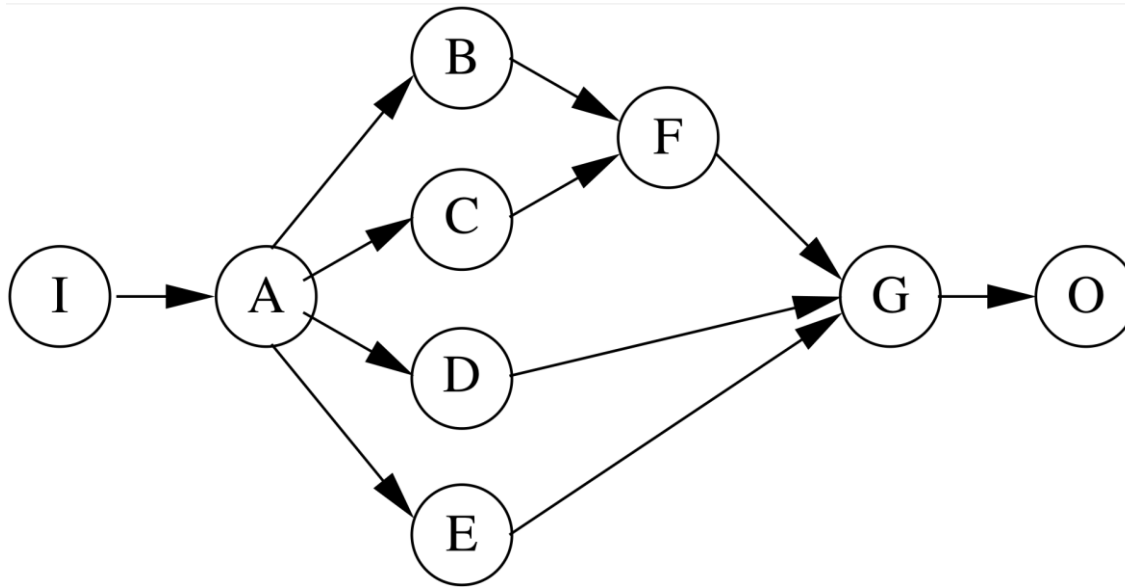
*Scheduled² = {I,A,B},
Candidates³ = {CD}*

Example I (Bus, K=1) – Solution – Itr: final

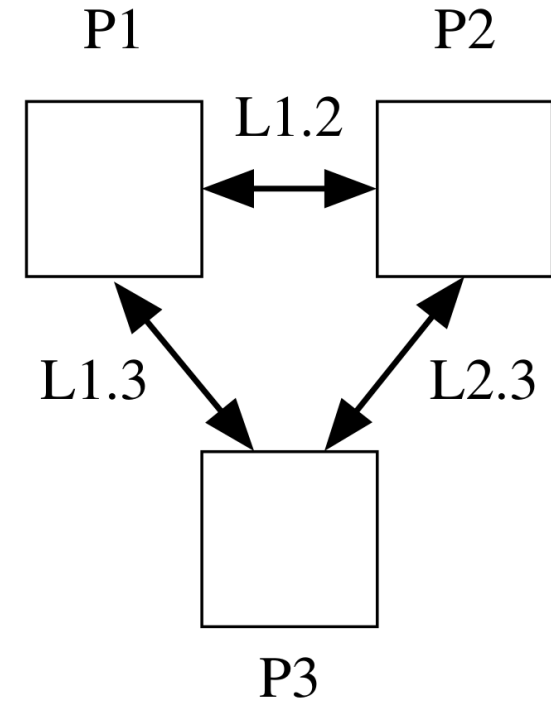
	P1		Bus (1↔2)			P2		Bus (2↔3)			P3		Bus (3↔1)		
	Start	End	Data	Start	End	Start	End	Data	Start	End	Start	End	Data	Start	End
I	0	1				0	1								
A	1	3				1	3	A>B	3	3.5			A>B	3	3.5
B						3	4.5				3.5	5			
C	3	5	C>E	5	5.6			C>E	6	6.6	5	6			
D						4.5	5.5				6	7			
E			E>O	7.6	8.6	6.6	7.6				7	8	E>O	8	9
O	9	10.5				7.6	9.1								

Example II (Links, K=1)

Software Architecture



Platform Architecture



Example from: Girault, Alain, et al. "An algorithm for automatically obtaining distributed and fault-tolerant static schedules." Proceeding of International Conference on Dependable Systems and Networks. IEEE, 2003.

Example II (Links, K=1)

time	operation								
proc.	I	A	B	C	D	E	F	G	O
P1	1	2	3	2	3	1	2	1.4	1.4
P2	1.3	1.5	1	3	1.7	1.2	2.5	1	∞
P3	∞	1	1.5	1	3	2	1	1.5	1.8

time	data-dependency					
link	I \triangleright A	A \triangleright B	A \triangleright C	A \triangleright D	A \triangleright E	B \triangleright F
L1.2	1.75	1	1	1.5	1	1
L2.3	1.25	0.5	0.5	1	0.5	0.5
L1.3	1.25	0.5	0.5	1	0.5	0.5

time	data-dependency				
link	C \triangleright F	D \triangleright G	E \triangleright G	F \triangleright G	G \triangleright O
L1.2	1.3	1.9	1.3	1	1.1
L2.3	0.8	1.4	0.8	0.5	0.6
L1.3	0.8	1.4	0.8	0.5	0.6

Example II (Links, K=1) - Solution

	P1		L1.2			P2		L2.3			P3		L1.3		
	Start	End	Data	Start	End	Start	End	Data	Start	End	Start	End	Data	Start	End
I	0	1				0	1.3								
A	1	3				1.3	2.8	A>C	2.8	3.3			A>C	3	3.5
D	3	6				2.8	4.5	A>B	2.8	3.3			A>B	3	3.5
C			C>F	7.5	8.8	4.5	7.5				3.5	4.5	C>F	4.5	5.3
B						7.5	8.5				4.5	6			
F	8.8	10.8									6	7			
E			E>G	9.7	11	8.5	9.7				7	9	E>G	9	9.8
G	11	12.4									9	10.5			
O	12.4	13.8									10.5	12.3			

Note

- This algorithm is a simplified version of
Alain G., Hamoudi K., Mihaela S., Yves S.:
**An Algorithm for Automatically Obtaining
Distributed and Fault-Tolerant Static Schedules.**
DSN 2003: 159-168