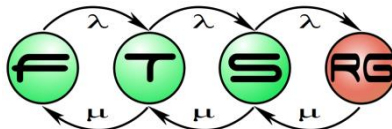


# Modelling and simulation of physical systems

Rebeka Farkas



# Motivation

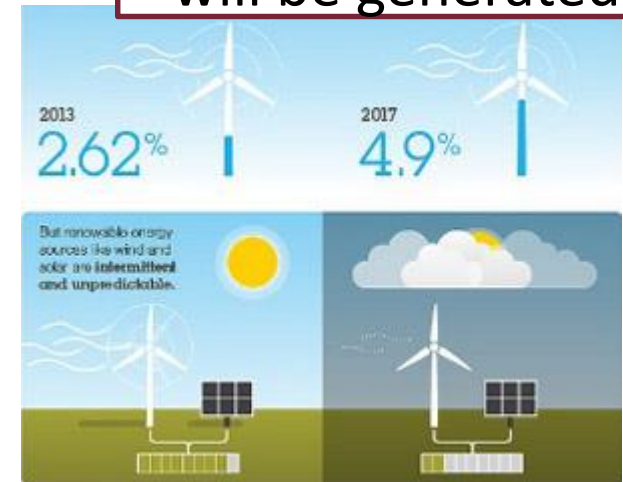
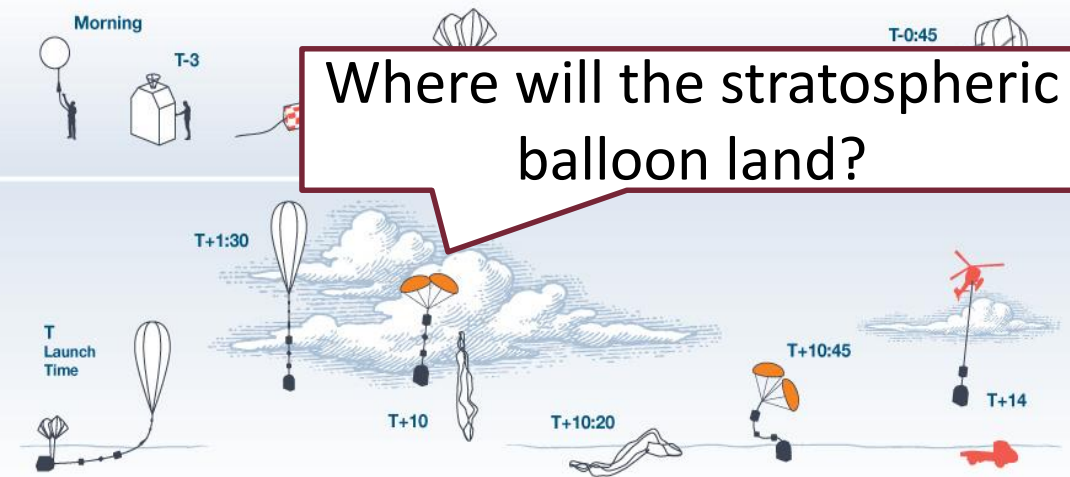
## How to predict the weather?



Should I take my umbrella today?

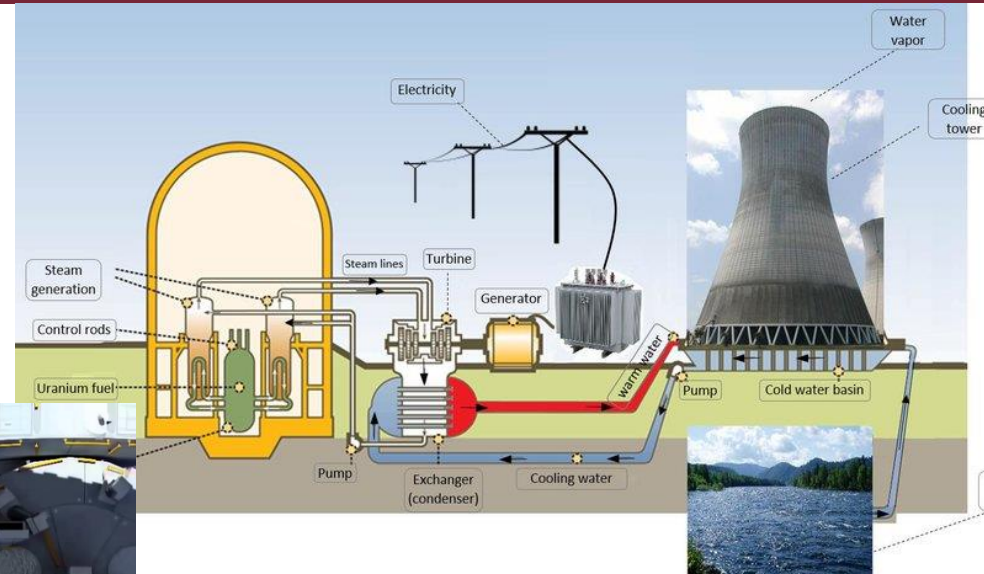
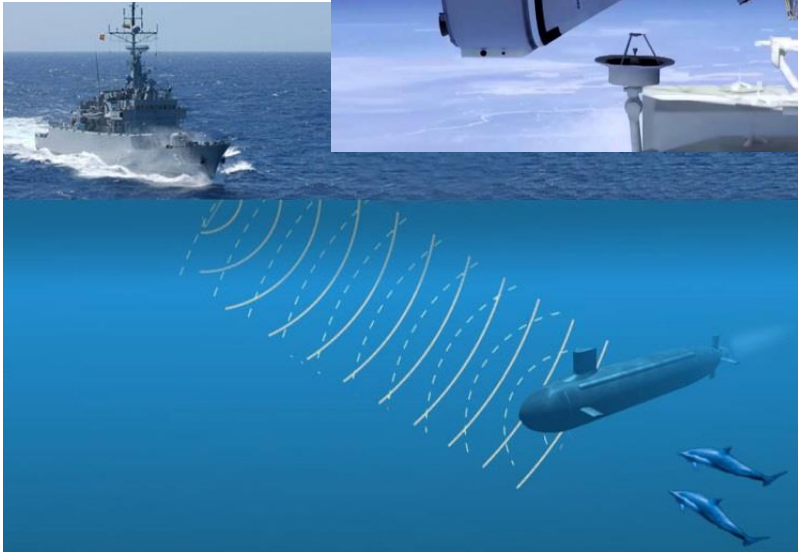


How much energy will be generated?



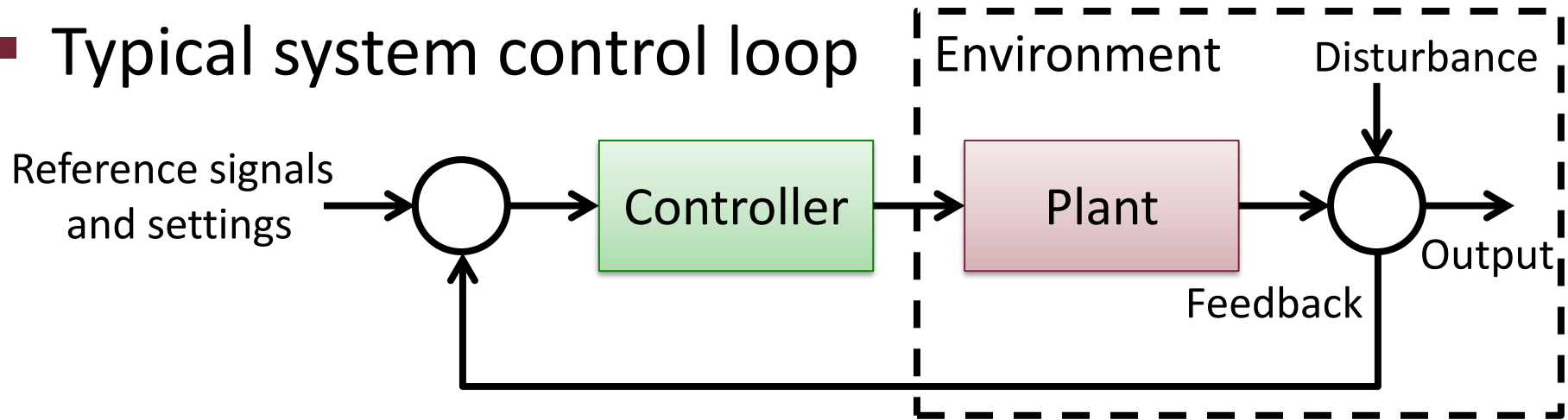
# Motivation

*How to ensure the designed controller (software) works correctly?*



# Controller, Plant, and Environment

## ■ Typical system control loop



## ■ Challenge: validate the design of the controller

- On-site testing and calibration can be
  - Expensive (time + cost)
  - Dangerous
- Instead: run simulation
  - Must model *physical properties* of plant & environment



# Physical systems

## ■ Models up to this point:

- „Controllable” system, model
- System under design
- Modeling goal: facilitate design process, (formal) analysis and verification of internal behaviour

**PRESCRIPTIVE**

## ■ Physical systems:

- Laws of physics can not be changed
- Existing system
- Modeling goal: assure designed system interacts well with environment

**DESCRIPTIVE**

# Discrete vs continuous models

## Discrete

Reactive, state-based models

Changes described by assignments

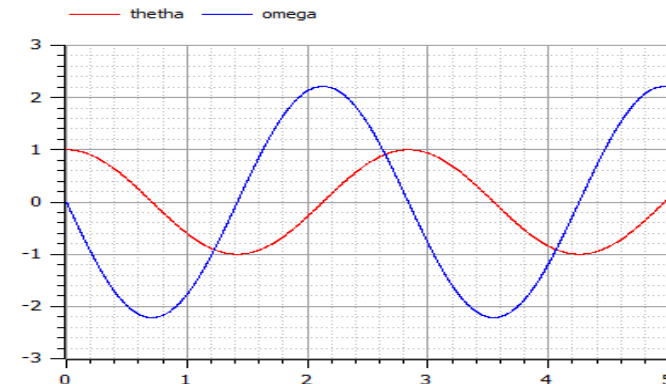
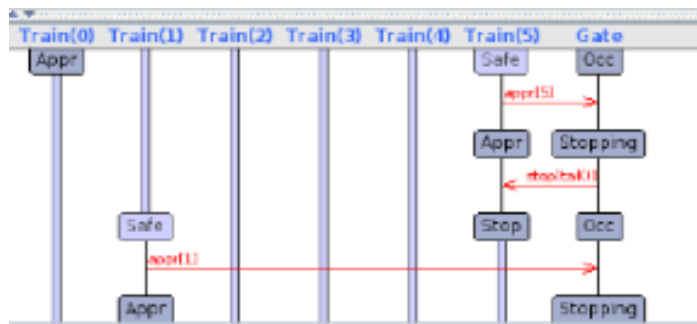
Simulation - logical clocks

## Continuous

Differential equation systems

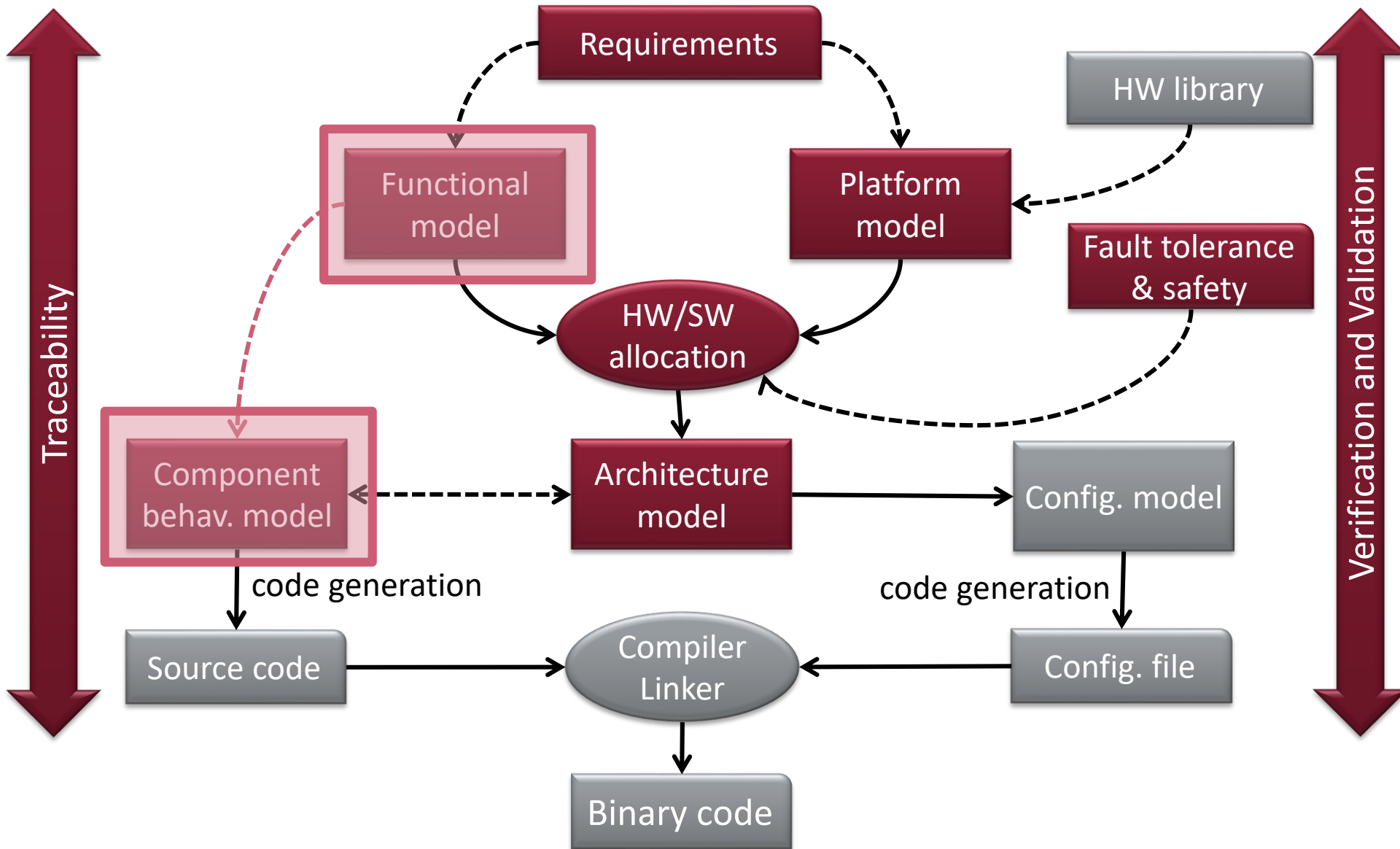
Behaviour described by mathematical equations

Simulation – numerically solving equation systems

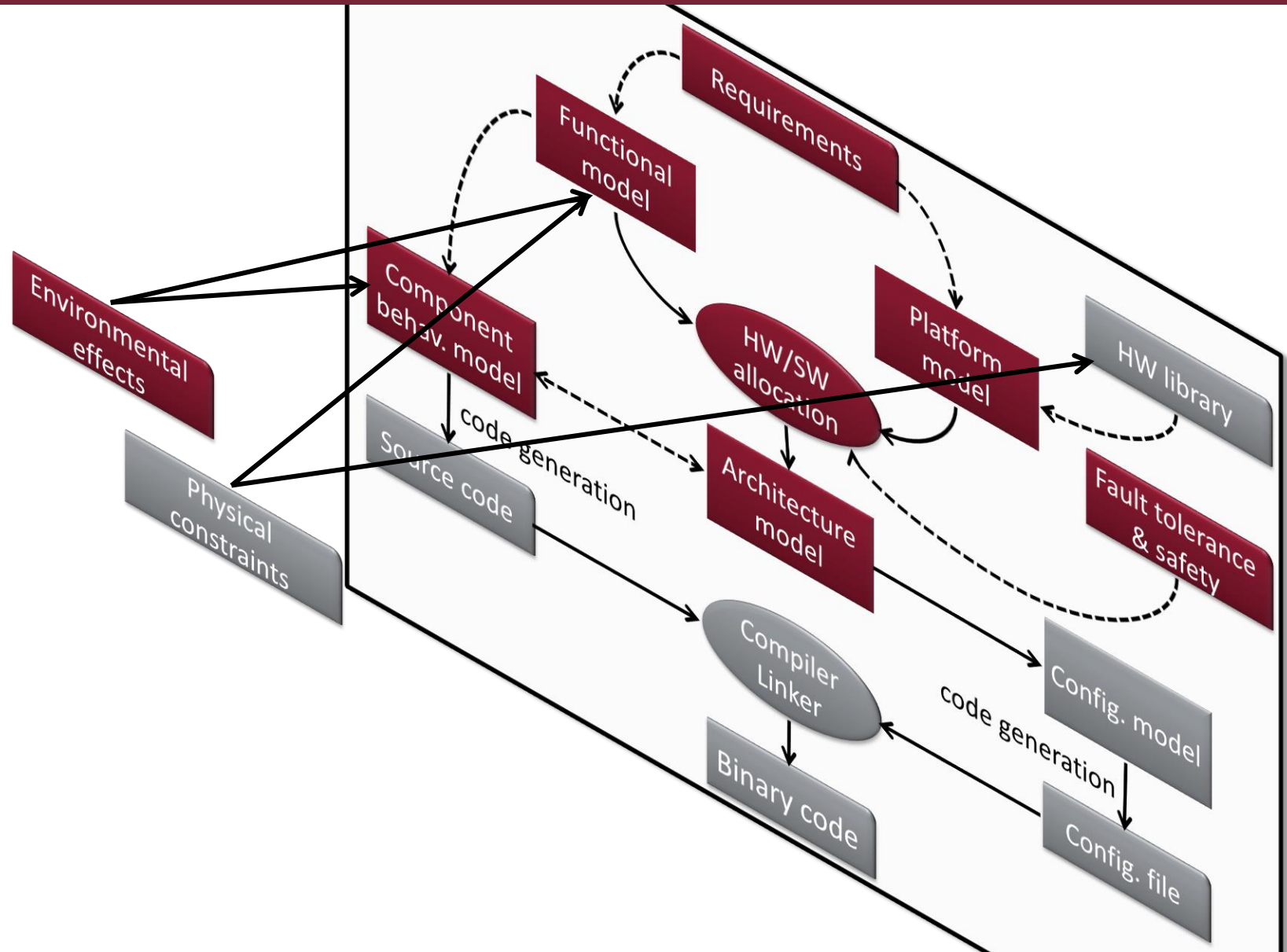


- Reality: Practically all real systems are hybrid
  - Reactive components mixed with continuous changes

# Platform-based systems design



# Platform-based systems design





# Learning Objectives

## Modeling physical parameters and constraints

Describe continuous behaviour of physical systems

Include rules constraining physical properties

Capture properties and constraints using the SysML language

Use the Modelica language to describe physical systems

## Simulation of discrete and continuous models

- Work with systems of discrete and continuous states
- Capture both continuous-time and discrete time properties
- Perform discrete event and continuous time simulation
- Understand challenges of simulation in industrial settings

# Outline

**Modeling physical systems**



**Simulation basics**



**Simulation of discrete and continuous systems**



**Motivating examples and case studies**

# Modeling physical systems



# Physical models

## Software models

Usually discrete

Dissected – system is built by integration of components

Understandable, maintainable, usable

Any engineer can create the model

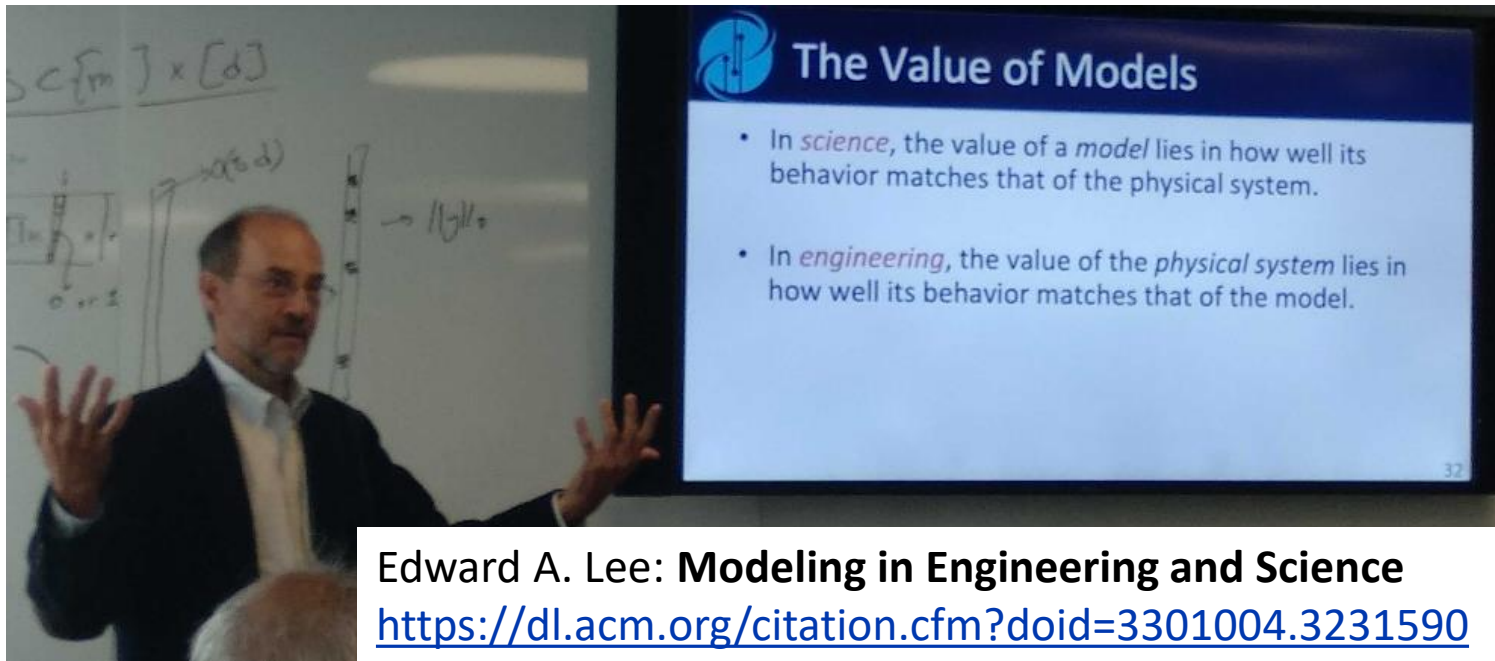
## Physical models

Usually continuous

In many cases everything has an impact on everything (e.g. weather – temperature)

„God doesn't build in straight lines”

Good model requires domain expertise



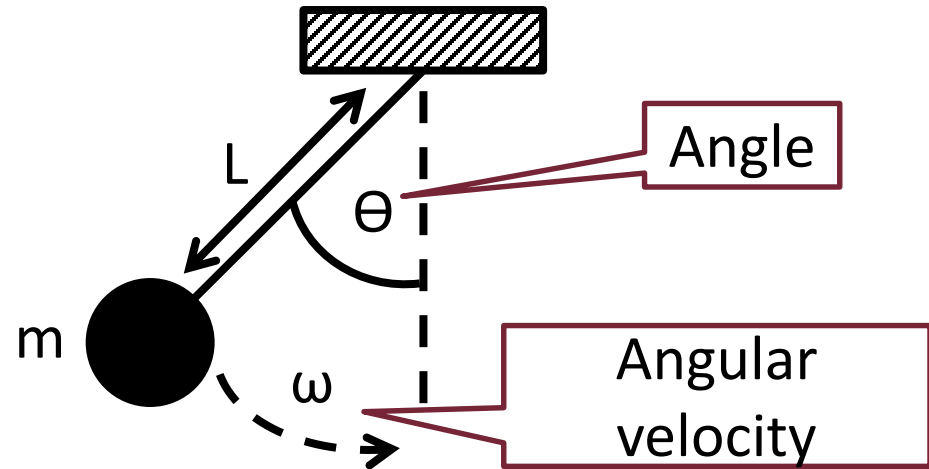
**The Value of Models**

- In *science*, the value of a *model* lies in how well its behavior matches that of the physical system.
- In *engineering*, the value of the *physical system* lies in how well its behavior matches that of the model.

Edward A. Lee: **Modeling in Engineering and Science**  
<https://dl.acm.org/citation.cfm?doid=3301004.3231590>

# Example: modeling a simple pendulum

- Simple pendulum



- Behavior of the pendulum as a function of time:

Angular acceleration

$$\begin{pmatrix} \dot{\theta}(t) \\ \dot{\omega}(t) \end{pmatrix} = \begin{pmatrix} \omega(t) \\ -\frac{g}{L} \sin \theta(t) \end{pmatrix}$$



# Assignments and equations

- **Causal** connection  $\approx$  assignment in programming language

$$y := x + 3$$

- Right-hand-side value determines left-hands-side variable
- Typical use: to implement controller

- **Acausal** connection  $\approx$  mathematical equation

$$y = x + 3 \Leftrightarrow y - 3 - x = 0$$

- Always holds; if any of the variables has a new value, it enforces that the other variables change accordingly
- Typical use: to model behaviour of plant / environment

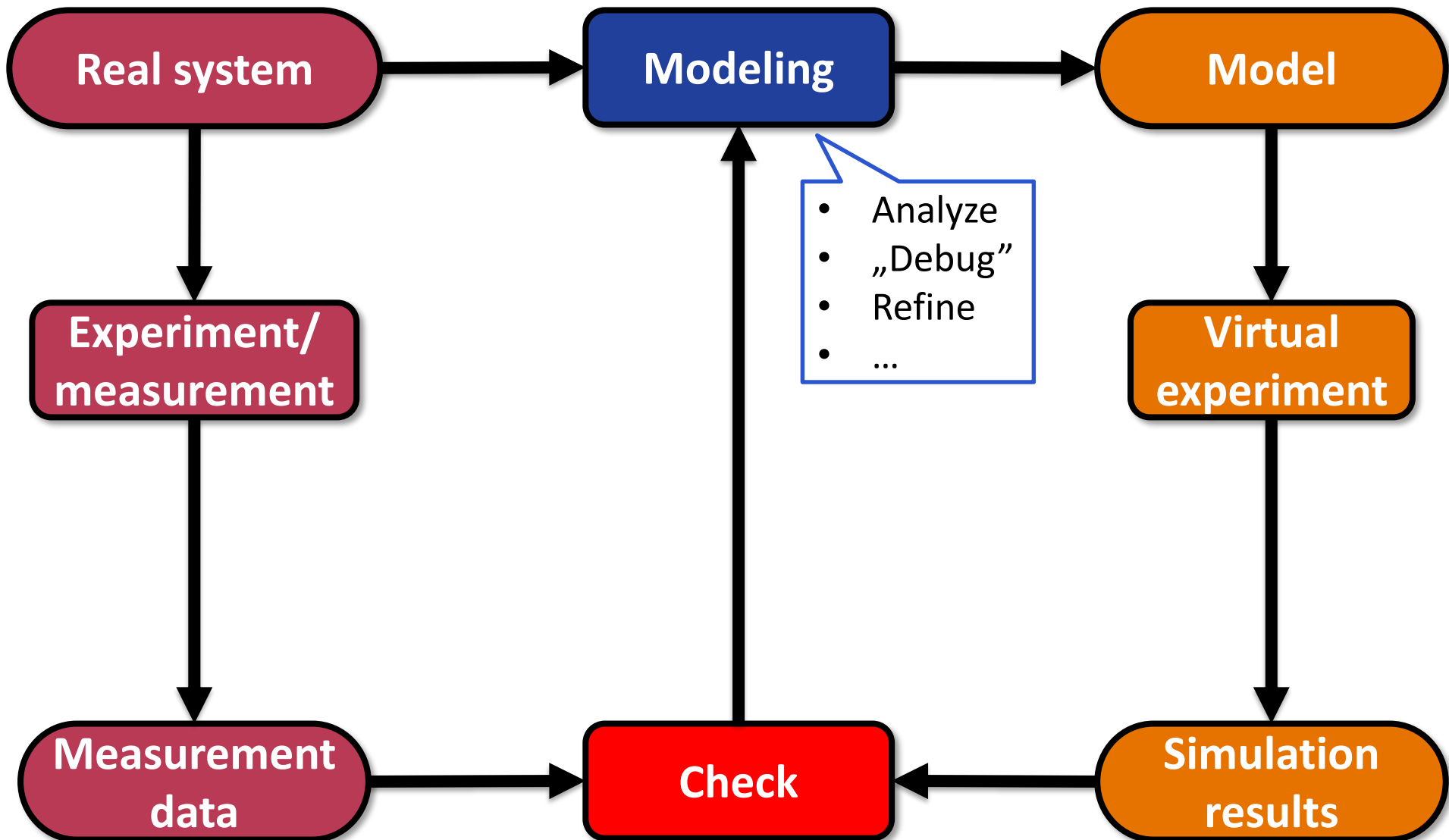
# Requirements

- Model **can be simulated** → Modeling tools
  - Neither over- nor underdetermined equation system (theoretical requirement)
  - Modeling tools have additional constraints
- **Representative** model
  - Obeys physical laws → formulate in model
  - Accurate representation of real world systems
    - compare simulation with real measurements
- General **usability**
  - Maintainable, reusable, etc. → block-based modeling

# How to create a model

1. Decompose the system
2. Customize existing components
  - Better to use components provided by tools  
→ (Just like programming languages)
  - Assign parameter bindings
3. Adjust connections
4. **Check model accuracy**
  - Accurate modeling is difficult
    - Models are created by domain experts
    - There are complete books on modeling
  - Simulation can be used for verification
    - (non-exhaustive, just like testing)

# Checking physical model accuracy



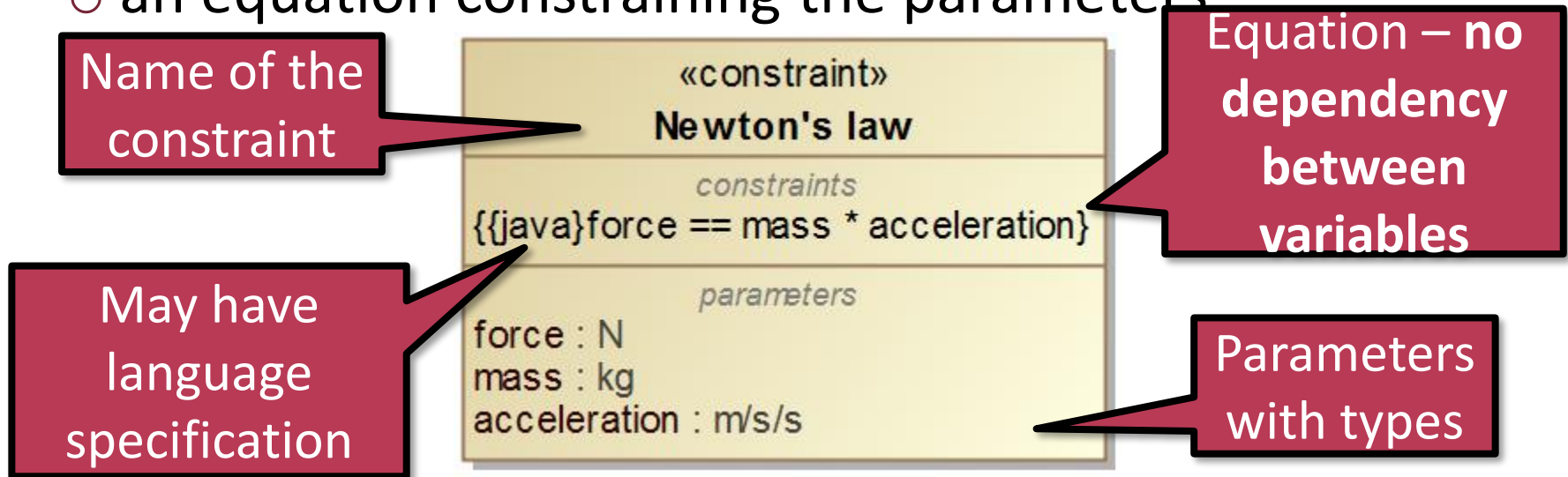
# Modeling physical systems





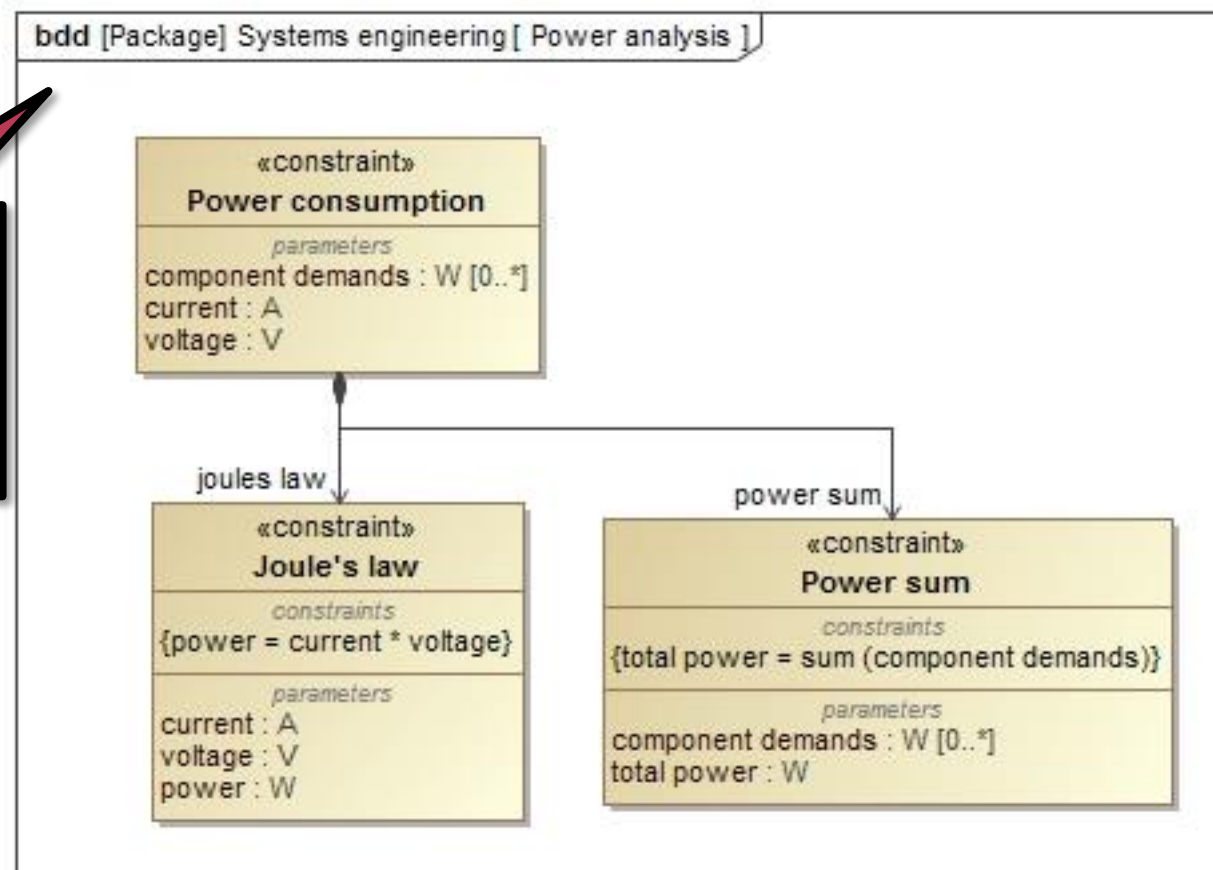
# Constraint blocks

- **Constraint:** equations with parameters bound to the properties of the system
- **Constraint block:** supports the definition and the reuse of constraints. It holds
  - a set of parameters and
  - an equation constraining the parameters



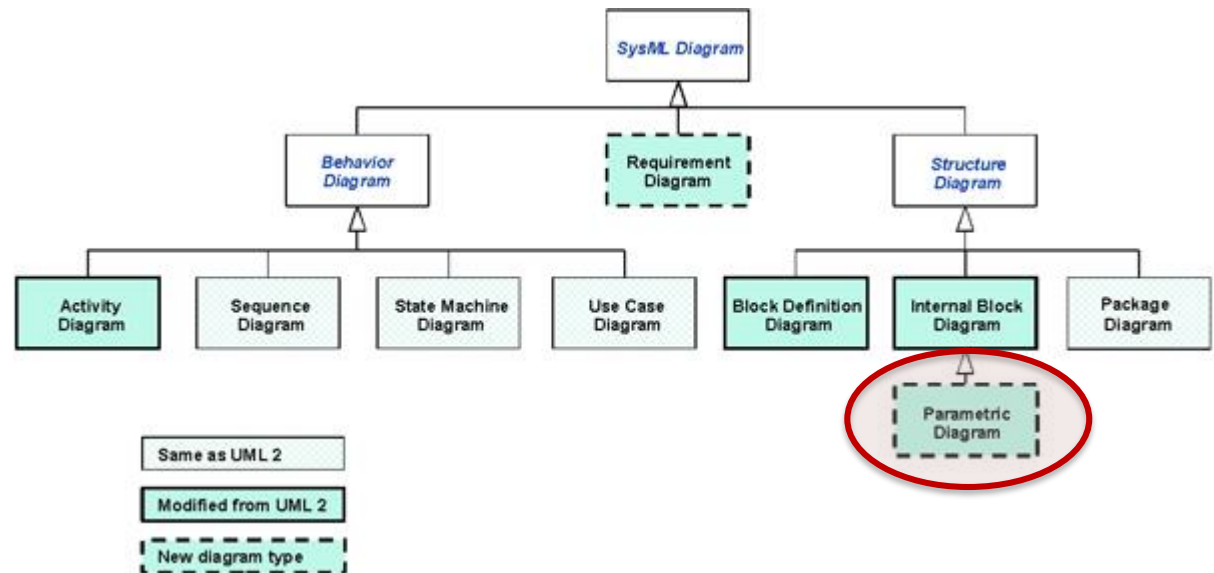
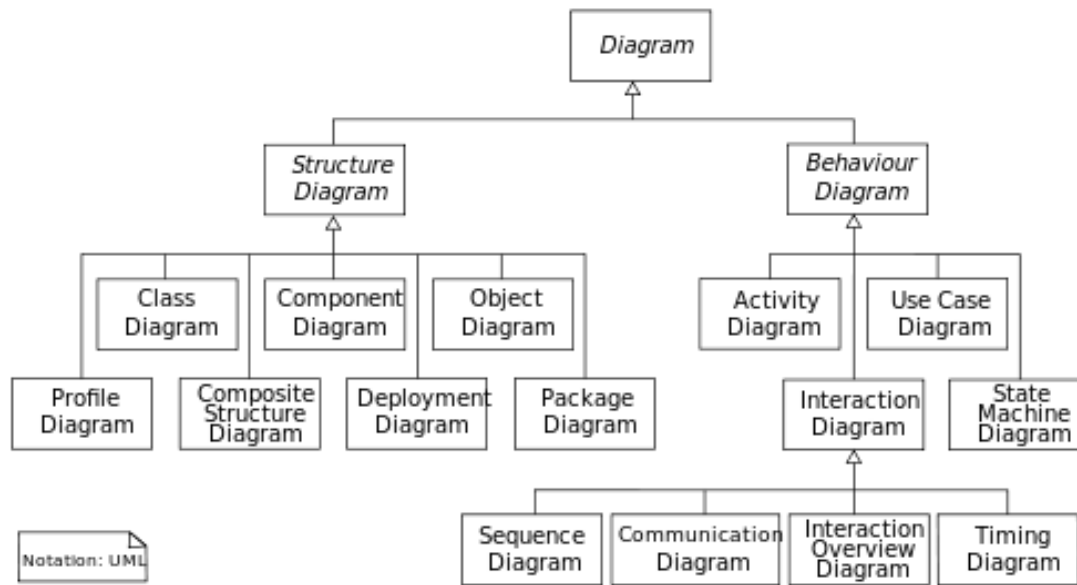
# Constraint definition

- **Composition** is used to define complex constraints from simple equations



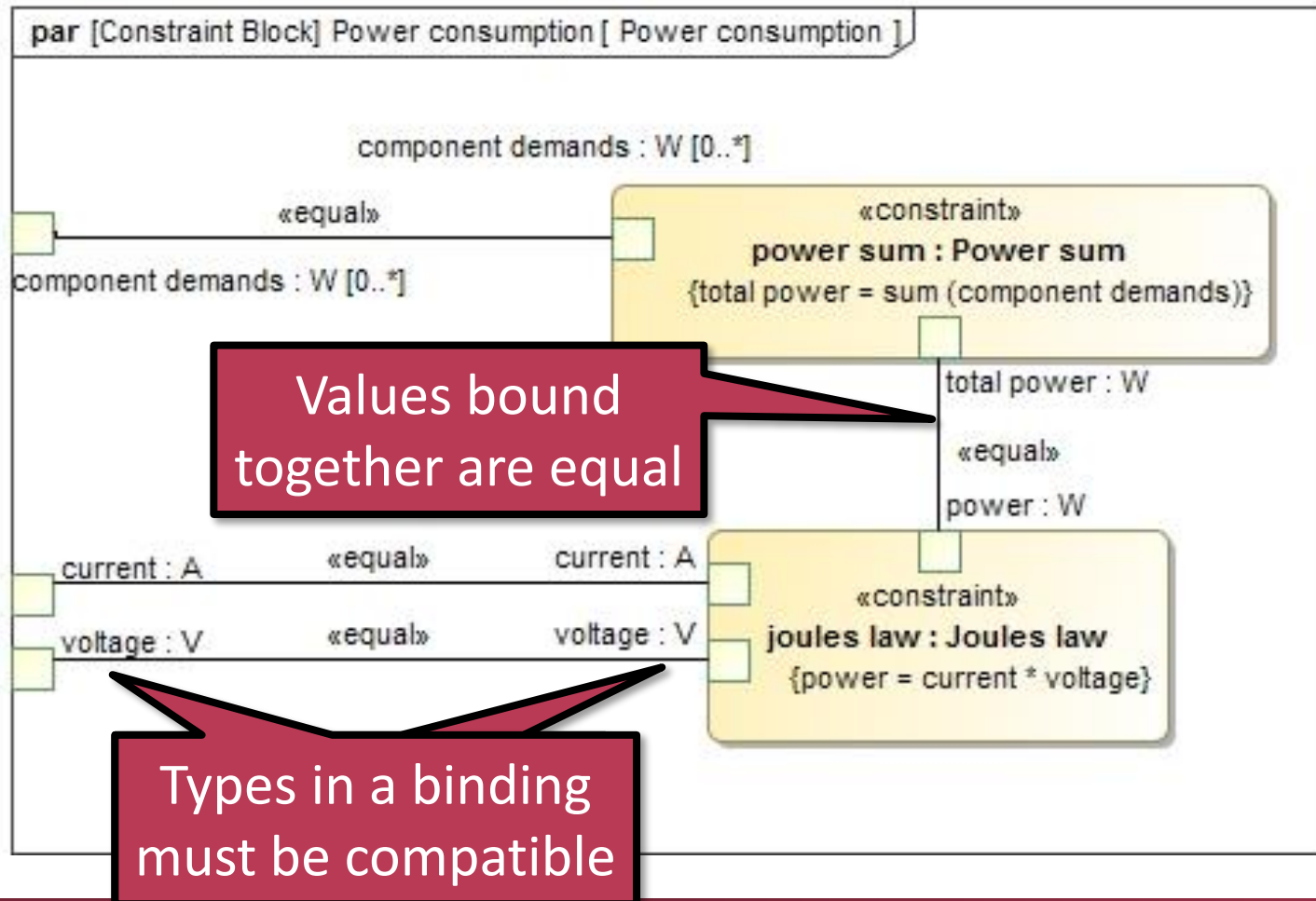
Hierarchy  
depicted in  
a BDD

# Parametric Diagram (PAR)



# Parameter bindings

- Goal: describe the application of constraints in a particular context

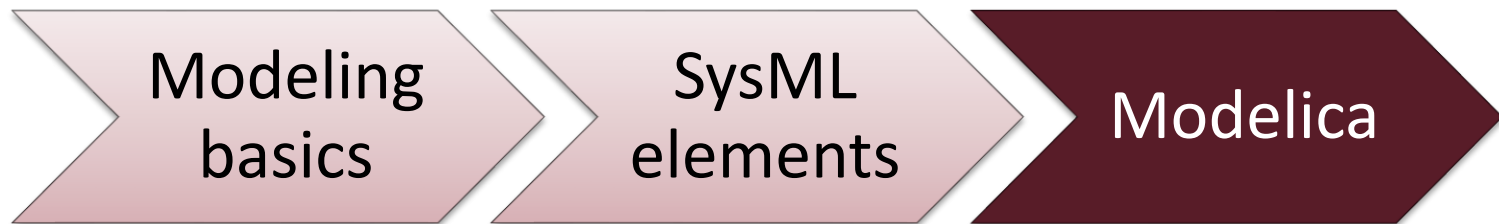


# Applications of parametrics

- Parametric specification
  - Define parametric relationships in the system structure
- Parametric analysis
  - Evaluating constraints on the system parameters to calculate values and margins for a given context
  - Checking design alternatives
  - Tool support: ParaMagic plug-in for MagicDraw
- Exact values may come from other sources
  - There are modeling standards with better support for this modeling aspect...
  - ...such as Modelica



# Modeling physical systems



# Modeling Tools

- Modelica

- OMEdit

- Dymola



- Matlab/Simulink



- Domain specific tools

- Ansys Simplorer (electrical systems)

- AUTOSAR → Course: BMEVIMIAV15

- CANoe (Engine control unit)



# Overview of Modelica

- **Modelica** is an object-oriented, equation-based language designed to model complex physical systems containing process-oriented subcomponents of different nature
  - Describing both continuous-time and discrete-time behaviour
- The **Modelica Standard Library** provides more than 1000 ready-to-use components from several domains
  - Full high-school + 1st year university physics (and much more)
- Implementations
  - Commercial e.g. by Dymola, Maplesoft, Wolfram MathCore
  - Open-source: JModelica
- Modeling and simulation IDE: OpenModelica - OMEdit

# Example: Modelica code for simple pendulum

Model name

```
model SimplePendulum
  parameter Real L=2.0;
  constant Real g=9.81;
  Real theta (each start = 1.0);
  Real omega;
equation
  der(theta) = omega;
  der(omega) = -(g/L)*theta;
end SimplePendulum;
```

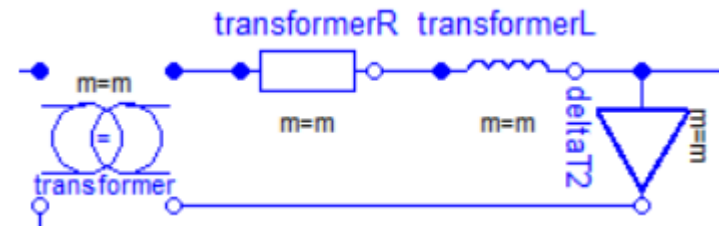
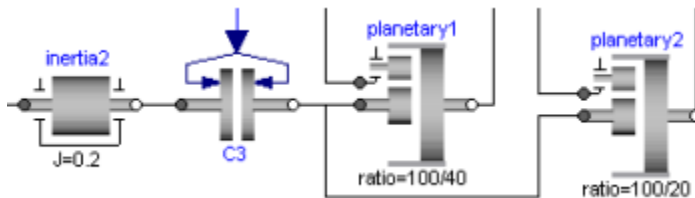
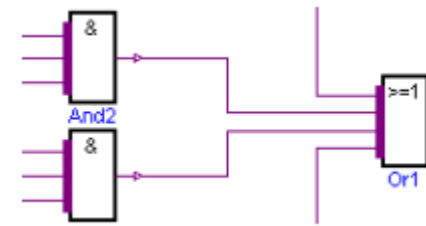
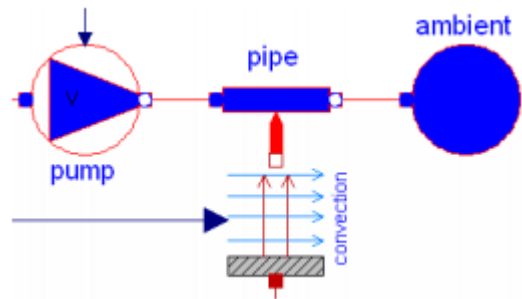
Continuous time  
variables, constants

Initial value

(Differential) equations

# Modelica Standard Library

- Provides reusable building blocks (called classes) for Modelica models
- Version 3.2.1. has more than 1340 classes and models
- Various domains



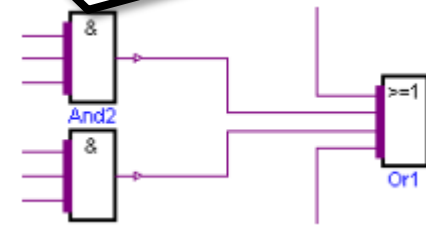
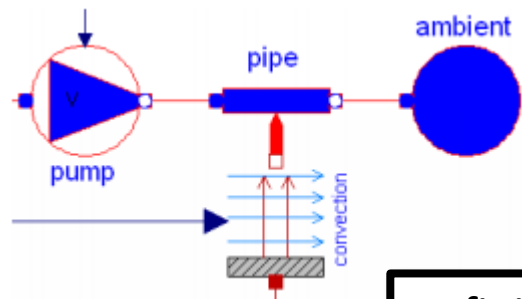


# Modelica Standard Library

- P Definition in Modelica textual syntax:

```

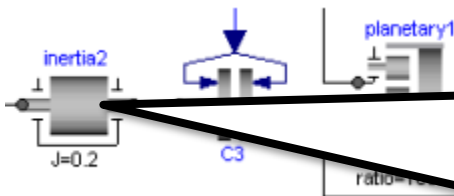
equation
  auxiliary[1] = x[1];
  for i in 1:n - 1 loop
    auxiliary[i + 1] = D.Tables.AndTable[auxiliary[i], x[i + 1]];
  end for;
  y = pre(auxiliary[n]);
  
```



- V Definition in Modelica textual syntax:

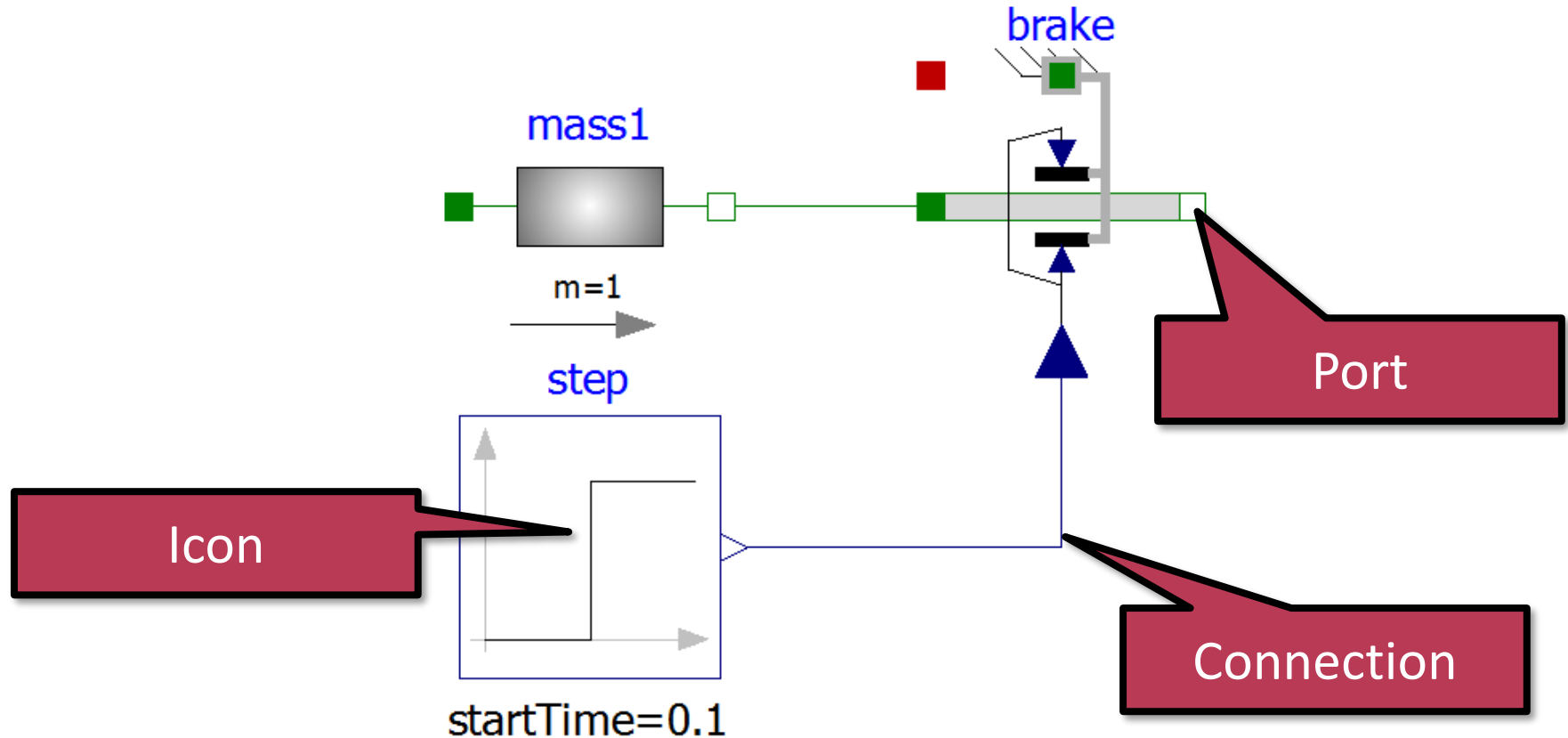
```

equation
  phi = flange_a.phi;
  phi = flange_b.phi;
  w = der(phi);
  a = der(w);
  J*a = flange_a.tau + flange_b.tau;
  
```



# Example plant model – train brakes

- Physical model for braking system carrying a mass



- Graphical notation in OpenModelicaEditor (~ibd)

# Simulation

Simulation  
basics

Discrete  
systems

Timed  
systems

Continuous  
systems

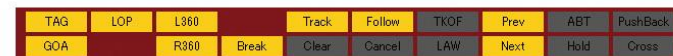
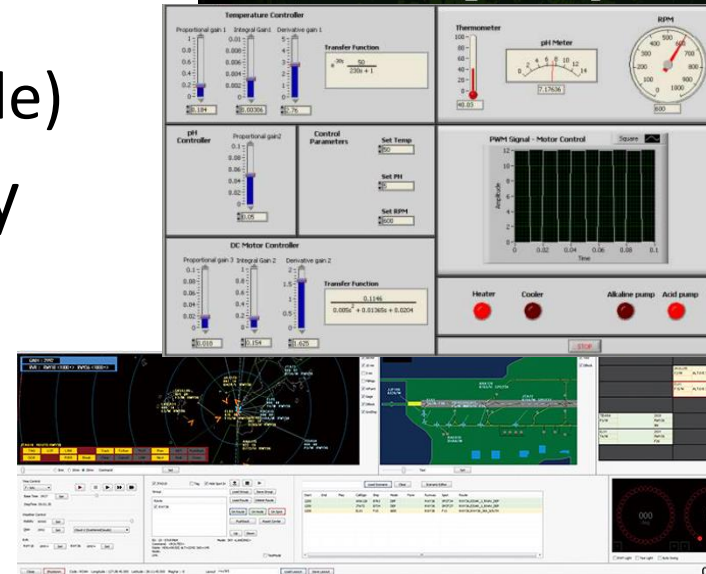
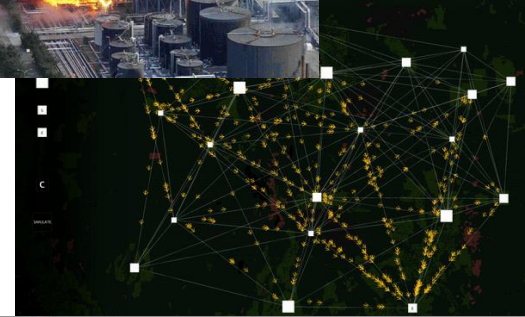
Practical  
aspects

# Model-based verification

- Modelling
  - Builds an abstract mathematical representation
- *Simulation*
  - Executes (some parts of) the **behaviour model**
  - Virtual experiment
- Testing
  - Executes (some parts of) the real system
- Other types of verification
  - Formal verification
  - Monitoring (can be used on simulated models)

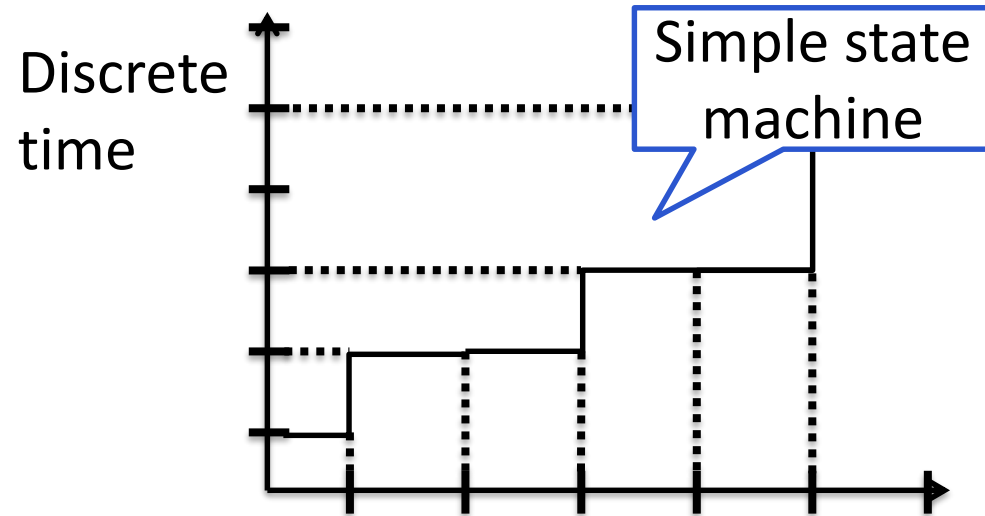
# Advantages over real experiments

- Real system doesn't change
  - Error in simulation doesn't cause real problem
  - Model can simply be reset
- Simulation is much faster
  - Hours can be simulated in seconds
  - (Real-time simulation is also possible)
- Parameters can be adjusted easily
- Easier to analyze
  - Can be controlled, replayed
  - No need for complex monitoring system

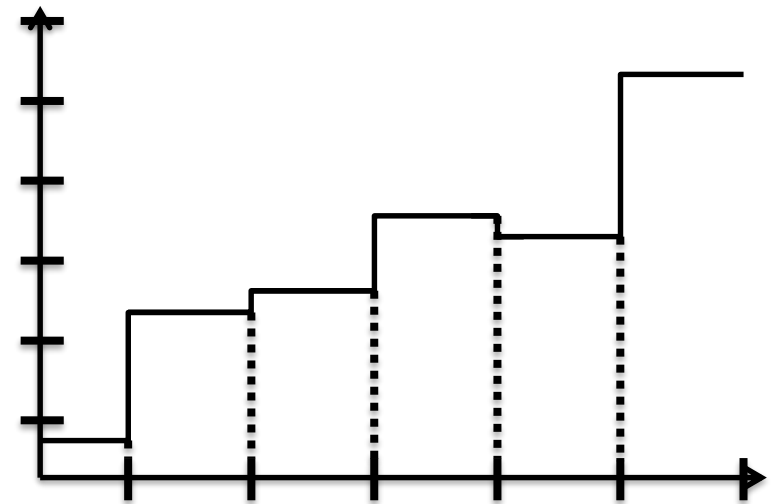


# Types of behavior models

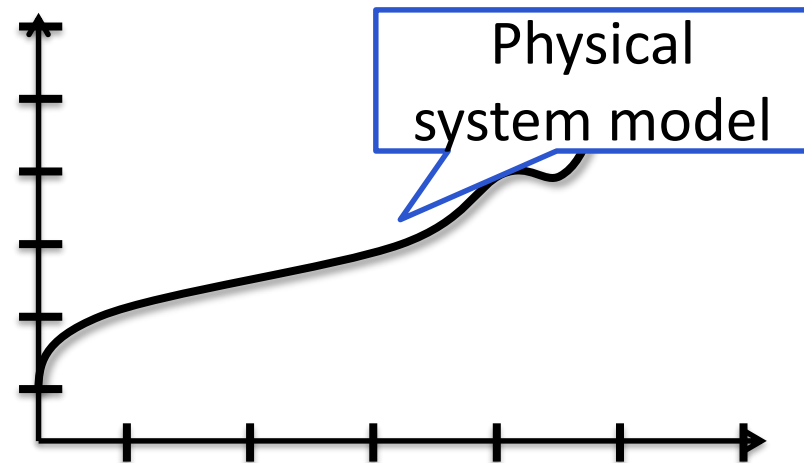
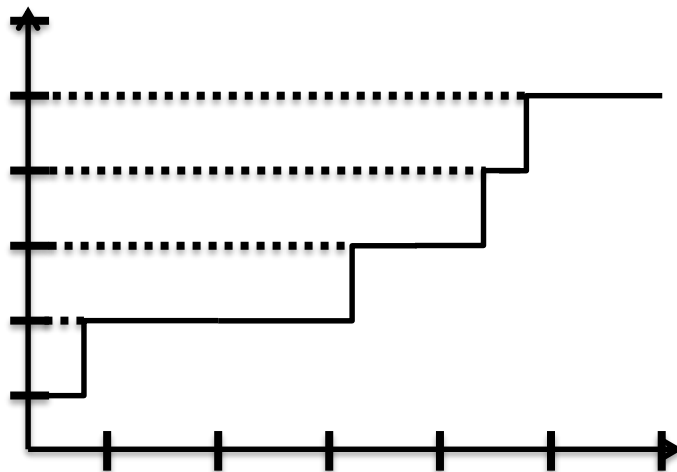
Discrete system state



Continuous system state



Cont.  
time

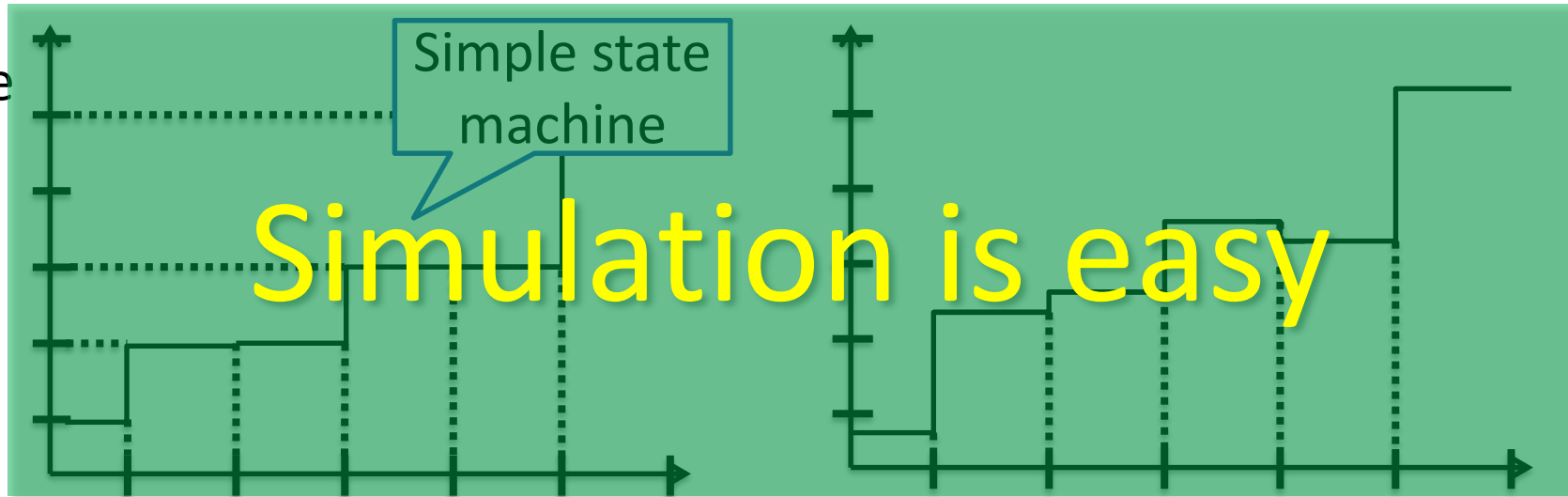


# Types of behavior models

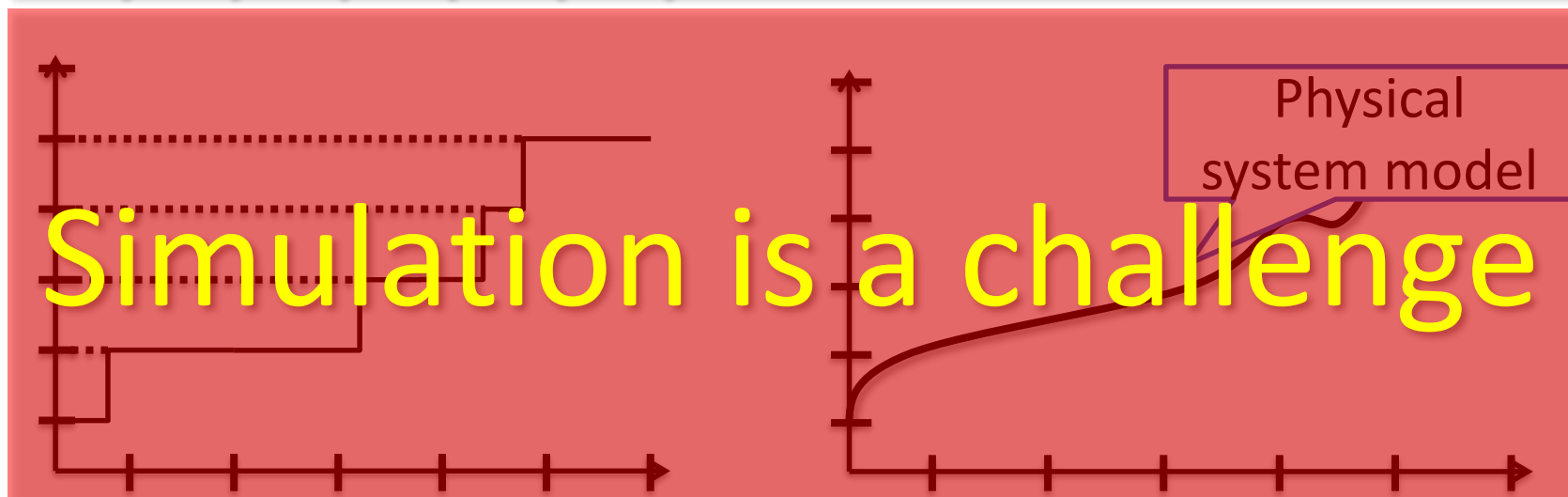
Discrete system state

Continuous system state

Discrete  
time



Cont.  
time





# Types of simulation

- DES: Discrete event simulation
  - Event-based model (e.g. timed state machine)
  - Simulation step by step
    - Considering events (timestamps), guards and actions
    - Event queue – order is important
  - Challenges: synchronization
- Time stepped dynamic model
  - Continuous model  
(performance model, physical model)
  - Problem: discrete time simulator
    - Discretization of time

# Simulation

Simulation  
basics

Discrete  
systems

Timed  
systems

Continuous  
systems

Practical  
aspects

# Goals of System Simulation

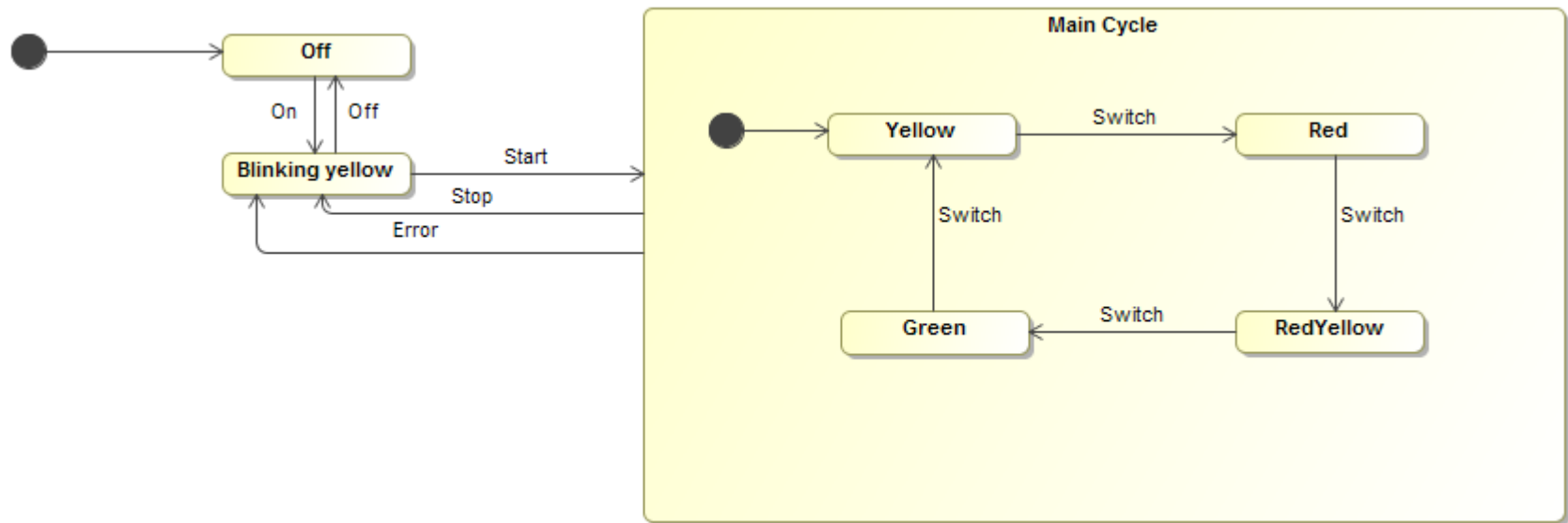
- Check the design of the system
  - Material flows -are there bottlenecks?
  - Queue locations and sizes -do they get blocked or starved?
  - Resources -are they sufficient, do they starve important operations?
  - Failure modes -what are they and what causes them?
- Check if it has the required capacity
- See what different types of downtime do to performance
- Improve the design

# Components of a D. E. Simulation

- Simulations contain
  - **Events** causing changes in the system state
    - **Event space** – set of possible events: input events, timed events, etc.
  - **Queues** where entities wait their turn
    - Significant in case of *asynchronous* communication
    - *Synchronous* systems – **logical clock** model
- Only one event at once
  - Two events can have the same *time stamp* but they have to have an *order* (see: state charts with *exit*, *entry* and *do* actions)

# DEMO CAMEO simulation

- Example: Traffic Light
  - Events: On, Off, Start, Stop, Switch, Error



# Simulation

Simulation  
basics

Discrete  
systems

Timed  
systems

Continuous  
systems

Practical  
aspects

# Variables and time

- System state:
  - Represented by variables (explicit or implicit)
  - Continuous/discrete
- Representation of time
  - Continuous/discrete (logical clock - tick)
- Simulation uses virtual time
  - Virtual time  $\neq$  runtime
  - (Except for real time simulation)
- Time is a variable!
  - *Although a special one*

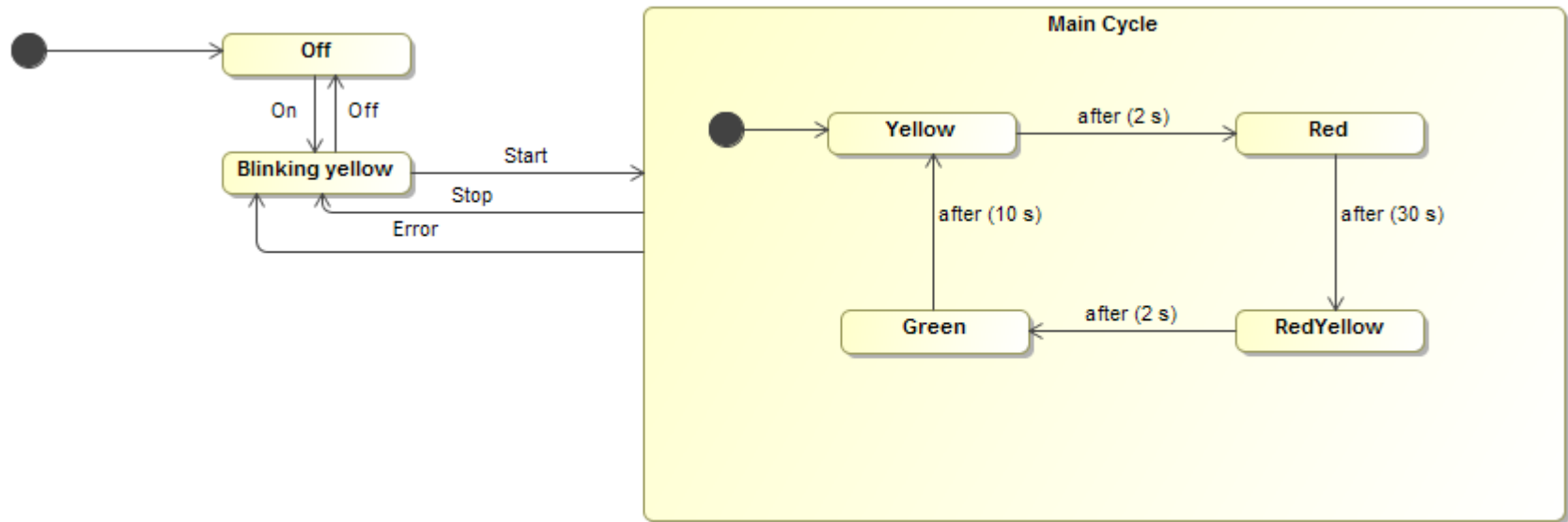
If the only continuous component is time, discrete simulation is still possible.

# DEMO Timed simulation with Cameo

## ■ Example: Timed traffic Light

Timed event

- Events: On, Off, Start, Stop, **Switch**, Error





# Simulation

Simulation  
basics

Discrete  
systems

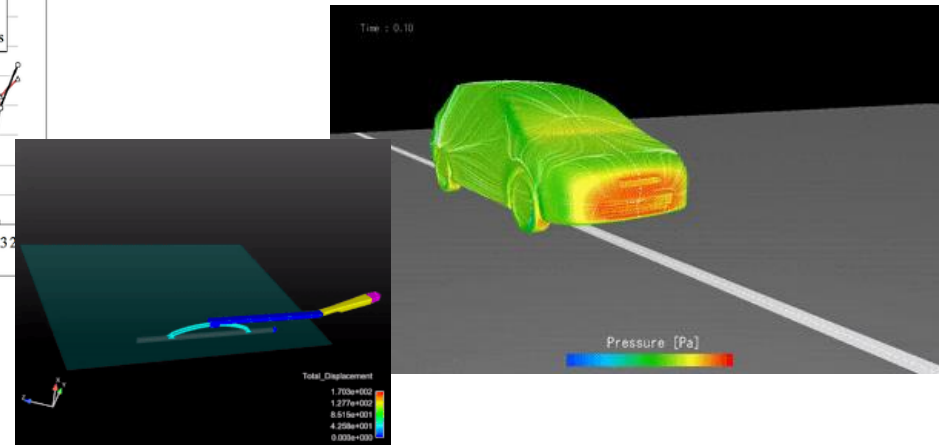
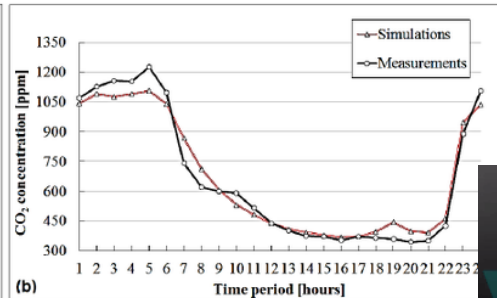
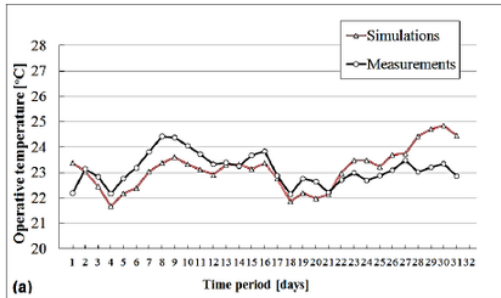
Timed  
systems

Continuous  
systems

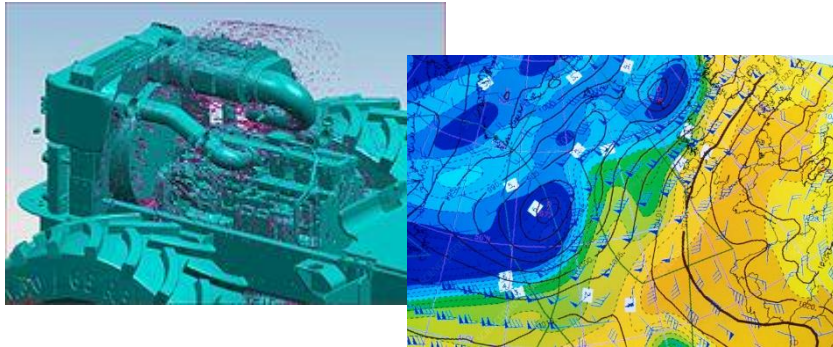
Practical  
aspects

# Goals of system simulation

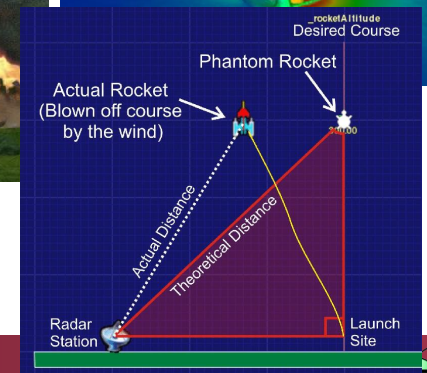
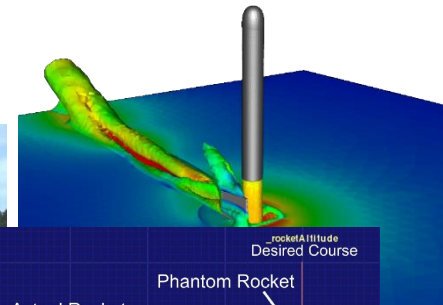
- Ensure model correctness
- Ensure correct interactions with designed system



- Analyse/predict system behaviour



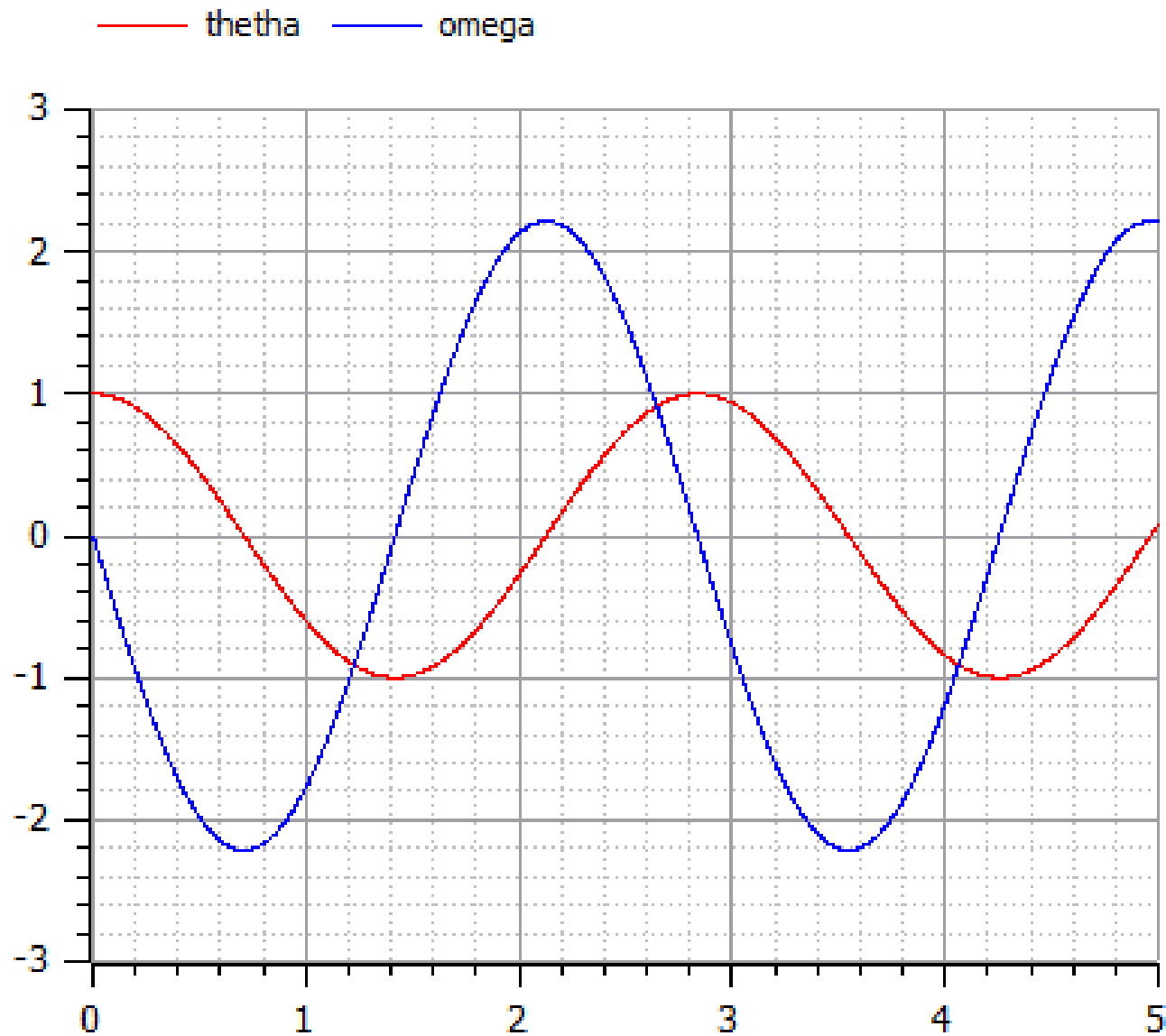
- Perform tests



# Simulation of continuous systems

- Simulating a model means to calculate the values of its variables at certain time instants
- Different algorithms and strategies for simulation
  - The task is to solve Ordinary Differential Equations (ODEs) generated from the model
  - Numerical techniques
  - ODE specific solvers exist for this purpose

# Example: Pendulum simulation results



# Challenge

- Good model can be simulated incorrectly

aggajjar



OFFLINE

1 Posts



Thank you

## Bouncing Ball Simulation Problem

Apr-24-17 06:40:35



Dear all,

I am not able to understand how to use when loop for bouncing ball problem. The standard code given in Michael Tiller's book is as follows:

```
model BouncingBall "The 'classic' bouncing ball model"
type Height=Real(unit="m");
type Velocity=Real(unit="m/s");
parameter Real e=0.8 "Coefficient of restitution";
parameter Height h0=1.0 "Initial height";
Height h;
Velocity v;
initial equation
h = h0;
equation
v = der(h);
der(v) = -9.81;
when h<0 then
reinit(v, -e*pre(v));
end when;
end BouncingBall;
```

*Good model of  
a bouncing ball*

*Condition of  
bouncing:  $h < 0$*

This code works extremely fine but let's say when I make  $h == 0$  (instead of  $h < 0$  i.e. ball hits the ground) in when statement then I get weird output in simulation and I am not able to understand it. So, any help would be appreciated.

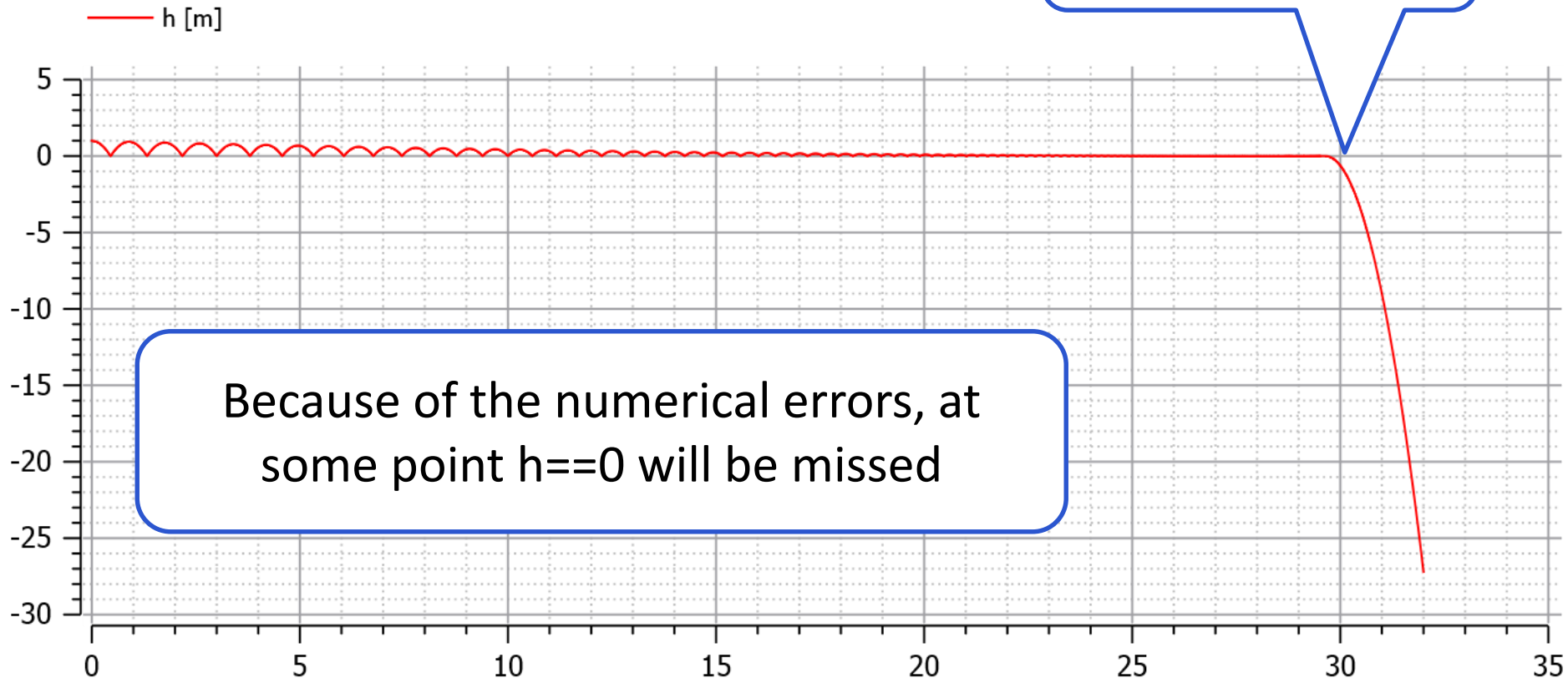
Thank you.

Edited by: aggajjar - Apr-24-17 06:43:15

*Problem when  
changed to  $h == 0$*

# Incorrect simulation example

## ■ Bouncing ball



# Continuous simulation challenges

*„The problem with simulation is that no matter what results you need, you are probably going to be able to get them.”*

[ApPLIED 2019]

- People tend to forget the limitations of simulation
  - Model limitations
    - Complex physics
    - Modeling connections is hard
    - Some environmental impacts will always be neglected
  - Limitations caused by time-stepped simulation
    - Communication delays
    - Induced reactions
  - Differential equation solver limitations
  - Propagation of numerical errors
- Correct configuration requires both domain and simulation knowledge

# Simulation

Simulation  
basics

Discrete  
systems

Timed  
systems

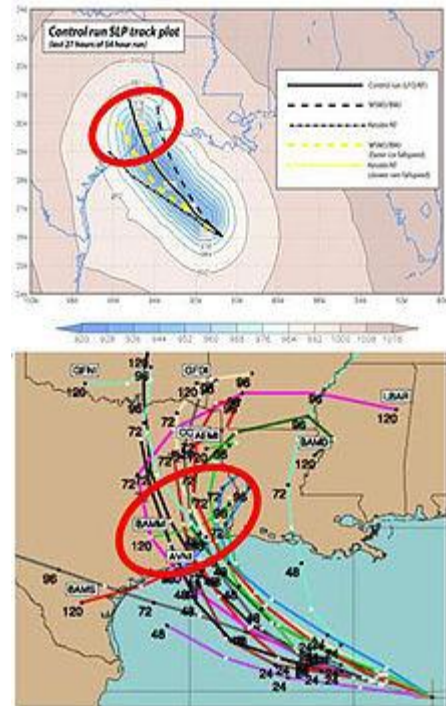
Continuous  
systems

Practical  
aspects



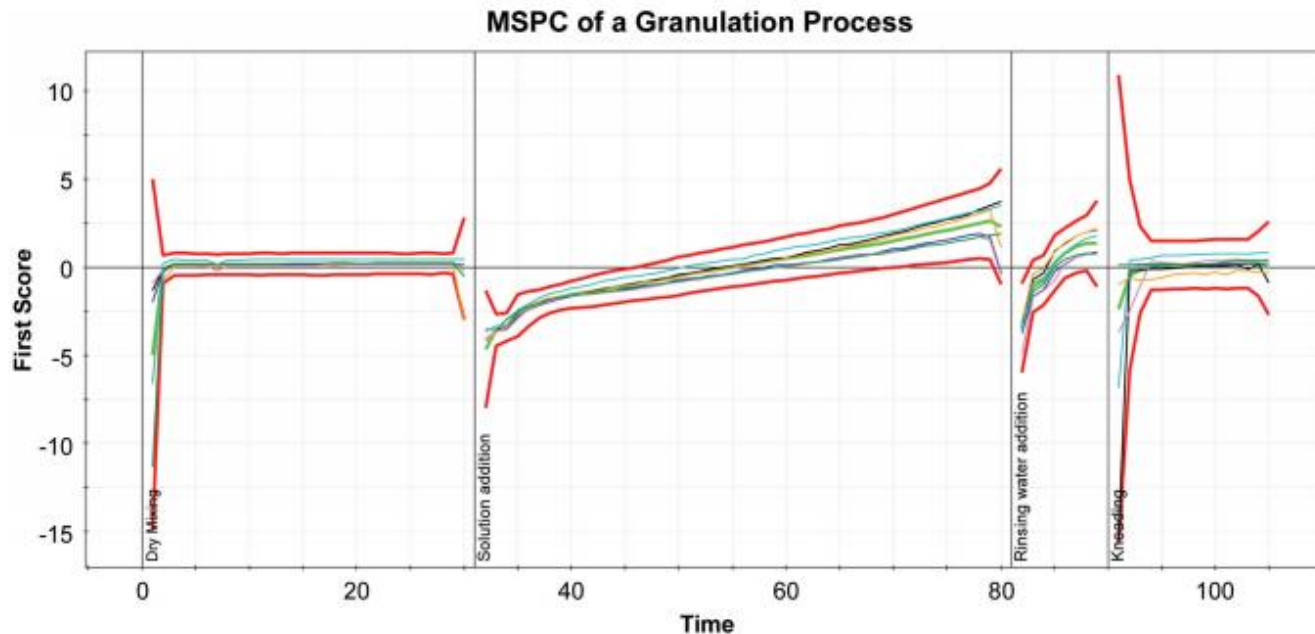
# Ensemble simulation

- Problem: uncertainty in initial physical conditions
  - ➔ No exact initial state
- Solution: Ensemble simulation
  - Repeat simulation multiple times from different states
  - Approximate probabilities of outcomes
- Example: weather
  - Hurricane Rita, 2005 September 07
  - ~ a month after Katarina destroyed New Orleans
  - ➔ Most probably heading for Houston



# Hybrid simulation

- Most industrial models are hybrid
  - Contains both discrete and continuous components
  - Discrete changes effect the dynamic behaviour



# Co-simulation

- Models of system components may differ
  - Model domain characteristics
    - E.g. discrete/continuous
    - Desired simulation techniques
  - Modeling environment and capabilities
  - Creator (protection of intellectual property)

*Co-simulation: The parallel simulation of different models in a controlled environment, allowing them to communicate, synchronize, etc. without raising IP protection concerns*

- Solution: FMI standard



# TLM simulation

- Most simulation limitations exist to avoid loops
  - Cyclic dependencies between variables
  - There will be a delay
    - Easy solution: always calculate with previous values
    - Better solutions: at least one delay per cycle
- Transmission Line Modeling
  - Every transmission/propagation (energy, force, etc.) in the model has a velocity
  - Delay exists in the real system!
  - Control simulation so that the simulation delay is the same
  - More precise, (much) less efficient simulation

# Motivating case studies

Case studies from the OpenCPS Project

# OpenCPS project

- Open Cyber-Physical System Model-Driven Certified Development

- 4 countries    

- 18 industrial partners, including



SAAB



ERICSSON

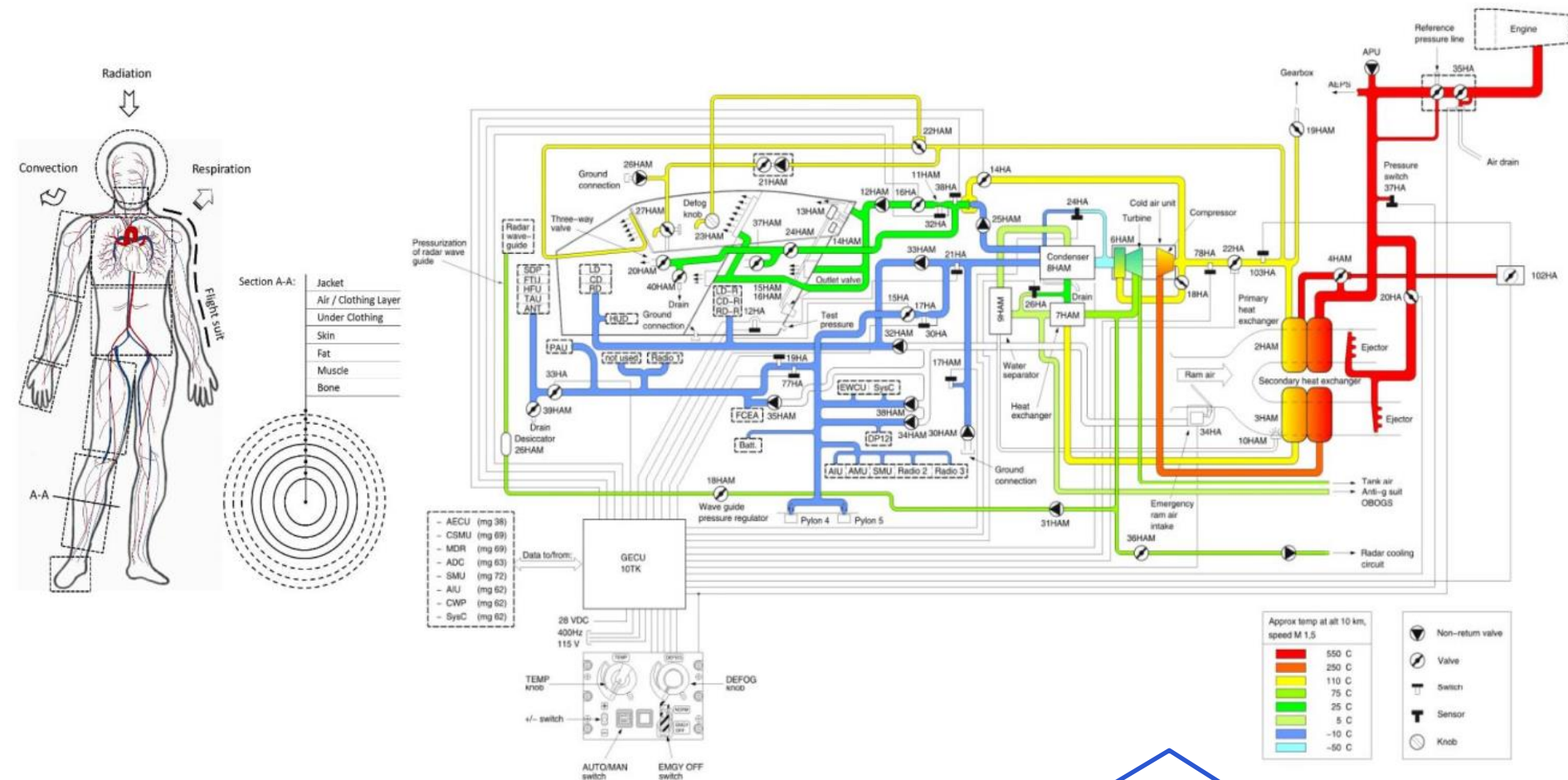


SIEMENS



- Industrial demonstrators in various fields
  - Building, **Aeronautics**, **Mechanics**, Naval, Power plant, Gaz turbines, **Automotive**

# Thermal model of an aircraft



Huge, complex system

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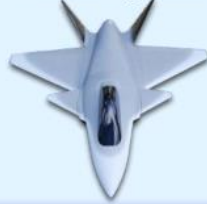
**SAAB**

# Thermal model of an aircraft

## Boundary Conditions

- Flight mission (Mach, altitude, ...)
- Pressure, Temp., Humidity with altitude
- Sun radiation, Sun position,
- Pressure, Temperature, Humidity change over horizontal distance
- Non standard atmospheres model?
- Time varying heat loads from e.g. sensors

## Geometry Data



Model description  
[language/tool origin]

Functional Mock-up  
Unit (FMU)

Physical connection

Information signal

## Engine [Modelica]

Simple model of an aircraft engine;  
bleed pressure, temperature.

## FMI master simulation environment

Multidomain, hybrid  
co-simulation required

## ECS Control System [Simulink]

Control system model, takes sensor  
signals as input, sends control signals  
to a number of electrically actuated  
valves in the ECS.

## Aircraft Environmental Control System (ECS) [Modelica]

Model of the aircraft environmental  
control system, including realistic  
provision of cockpit comfort air.

## Cockpit Model [Matlab]

Simulate the temperature, pressure  
and humidity of the cockpit air  
depending on the inputs from the ECS  
system and the pilots physiological  
performance.

## Pilot Physiological Model [Matlab]

Simulate human (pilots) body  
temperatures depending on personal or  
automatically set cockpit  
environmental aircraft data.

## Flight Management System [UML]

Model providing the pilot with alarms,  
warnings, and suggested actions  
depending on current aircraft state.

## Pilot Psychological Model [UML]

Model which simulate the  
psychological behavior and  
reactions of the pilot depending  
on the environmental comfort.



# Project experiences, lessons learnt

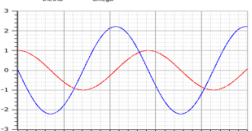

- Saab's aircraft model
  - Huge complexity
  - Continuous components required very small simulation step size → highly inefficient simulation
  - Solution: better tools, distributed algorithm
- SKF's bearing model
  - Required very precise simulation
  - Solution: TLM simulation
- Sherpa's hybrid electric vehicle model
  - Required fast and accurate simulation
  - (My) solution: New simulation algorithm



# Summary

## Discrete vs continuous models

Discrete	Continuous
Reactive, state-based models	Differential equation systems
Changes described by assignments	Behaviour described by mathematical equations
Simulation - logical clocks	Simulation - numerically solving equation systems



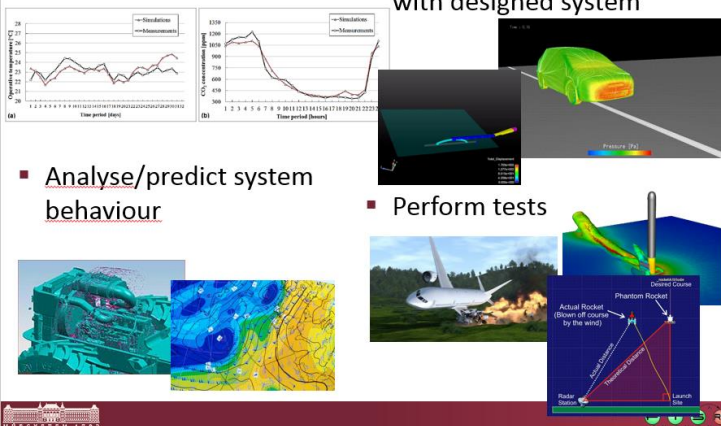
- Reality: Practically all real systems are hybrid
  - Reactive components mixed with continuous changes

## Requirements

- Model **can be simulated** → Modeling tools
  - Neither over- nor underdetermined equation system (theoretical requirement)
  - Modeling tools have additional constraints
- Representative** model
  - Obeys physical laws → formulate in model
  - Accurate representation of real world systems → compare simulation with real measurements
- General **usability**
  - Maintainable, reusable, etc. → block-based modeling

## Goals of system simulation

- Ensure model correctness
- Ensure correct interactions with designed system
- Analyse/predict system behaviour



## Simulation of continuous systems

- Simulating a model means to calculate the values of its variables at certain time instants
- Different algorithms and strategies for simulation
  - The task is to solve Ordinary Differential Equations (ODEs) generated from the model
  - Numerical techniques
  - ODE specific solvers exist for this purpose