Performance Modelling 2

Budapest University of Technology and Economics
Fault Tolerant Systems Research Group

Reminder

- **Stable state:**
  - Calculating with average values
  - $\lambda = X$ (arrival rate = throughput)
- **Maximum throughput (X^{max}):**
  - Upper bound of the reachable throughput
  - $X^{max} = \frac{K}{T}$ (in case of K resource instances)
- **Utilization (U):**
  - Ratio of the actual and maximum throughputs
  - $U = \frac{X}{K} \times T$ (in case of K resource instances)

Visitation number | Little's law | Zip's law | Changes in Workload

CONTENT

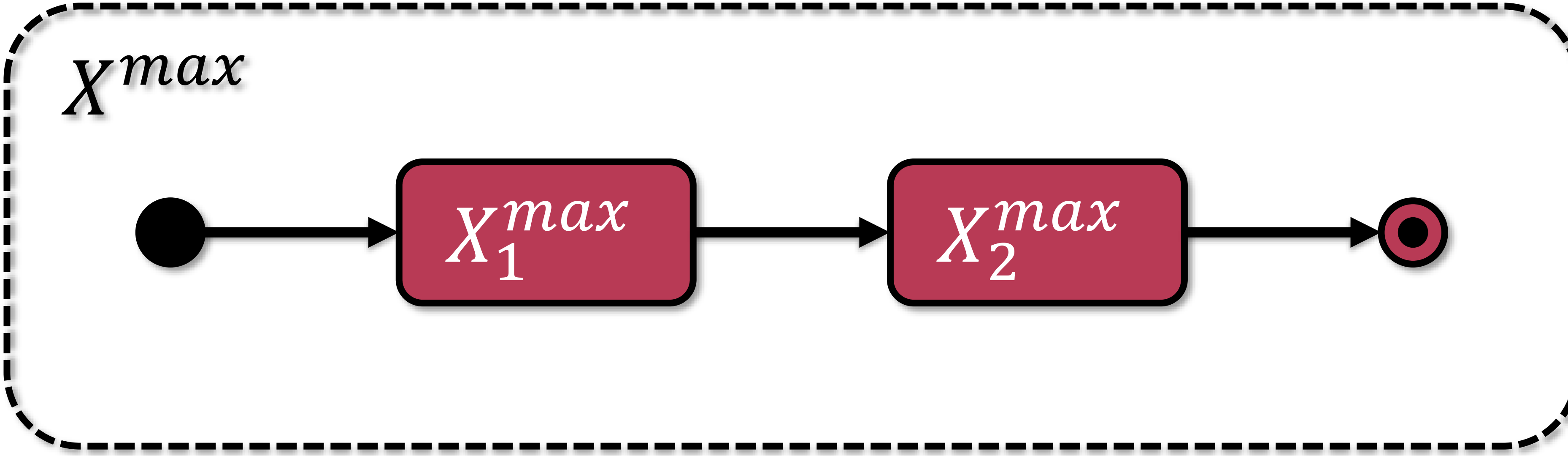
VISITATION NUMBER

Definition: Visitation Number

The **visitation number** indicates the average number of times a given activity/subprocess runs in a single execution of the whole process.

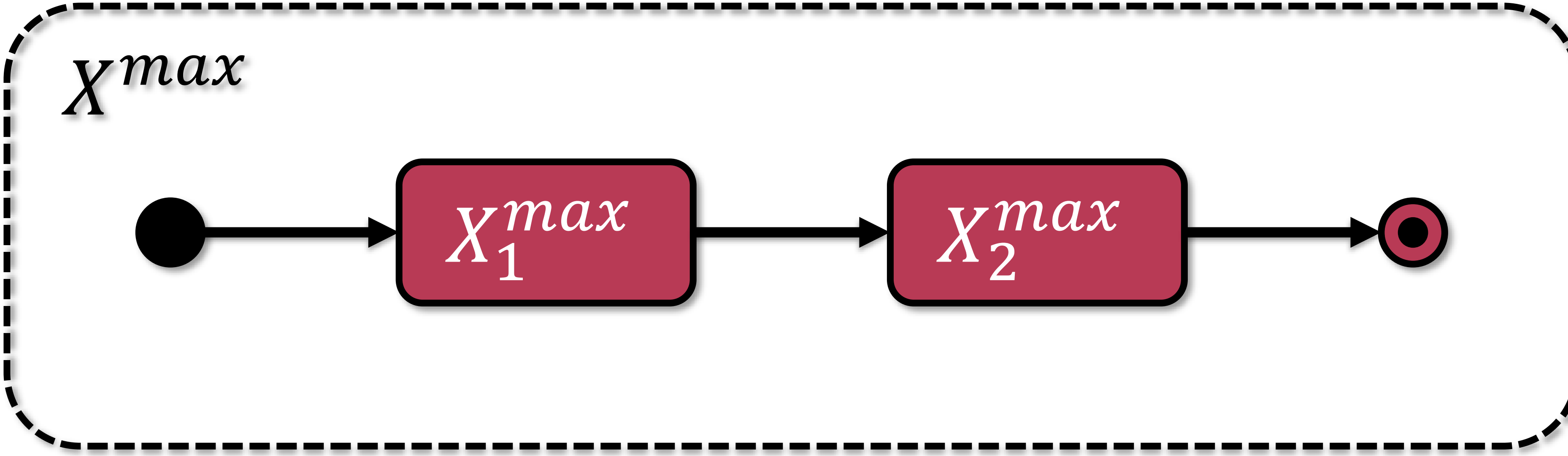
- How many times the process visits/repeats the given activity during a single execution?
- During a single execution of a process one of its activities can run not at all, or once, or several times. (Decisions, loops!)
  - If a choice between different outputs is described with probabilities, then these probabilities also play a role in determining the visitation numbers.

Sequential Composition



Each activity will be visited once.

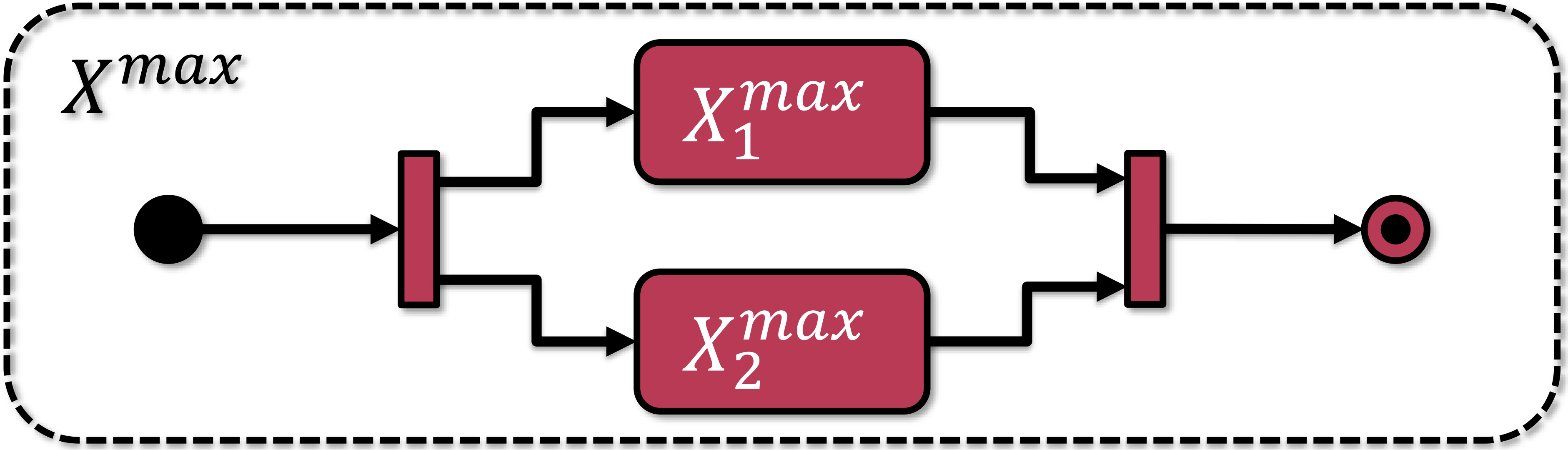
Sequential Composition



$$X^{max} = min(X_1^{max}, X_2^{max})$$

Bottleneck:
The component with the minimum throughput (or the corresponding resource).

Parallel Composition



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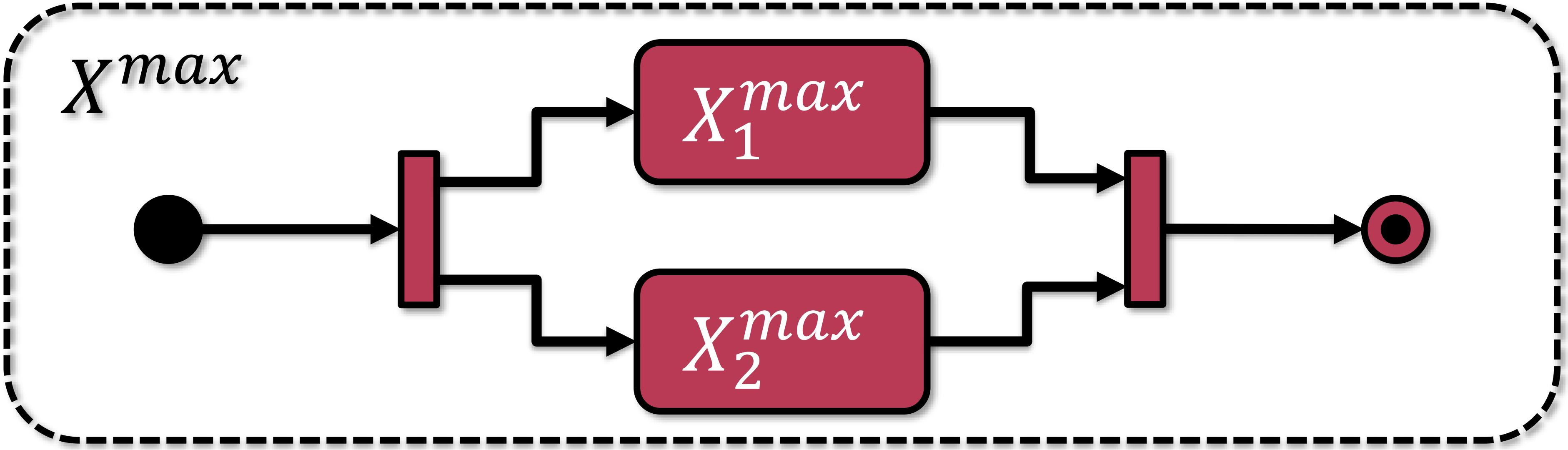

$$X^{max} = min(X_1^{max}, X_2^{max})$$

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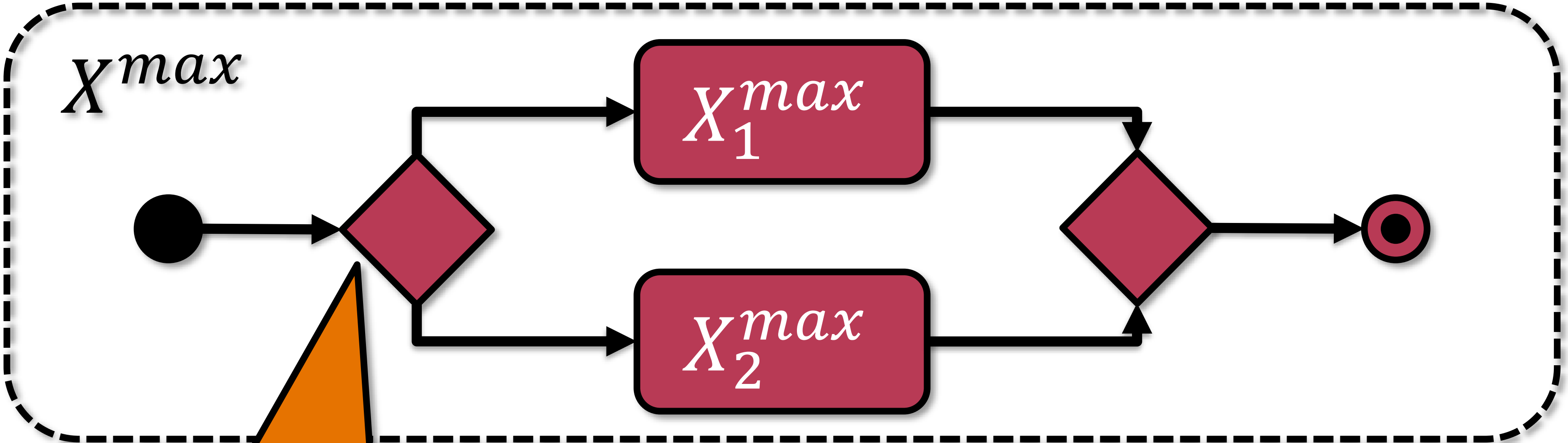
Composition of Free Choice

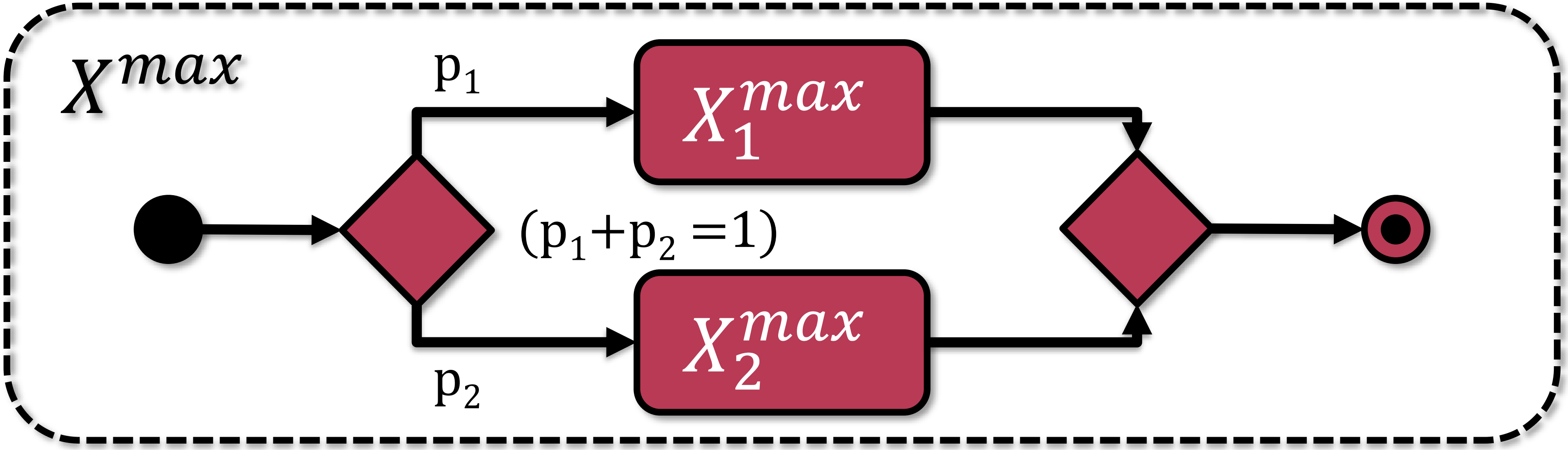


$$X^{max} = X_1^{max} + X_2^{max}$$

The tokens may choose a branch freely. If a branch is saturated, the other one may still be stable.

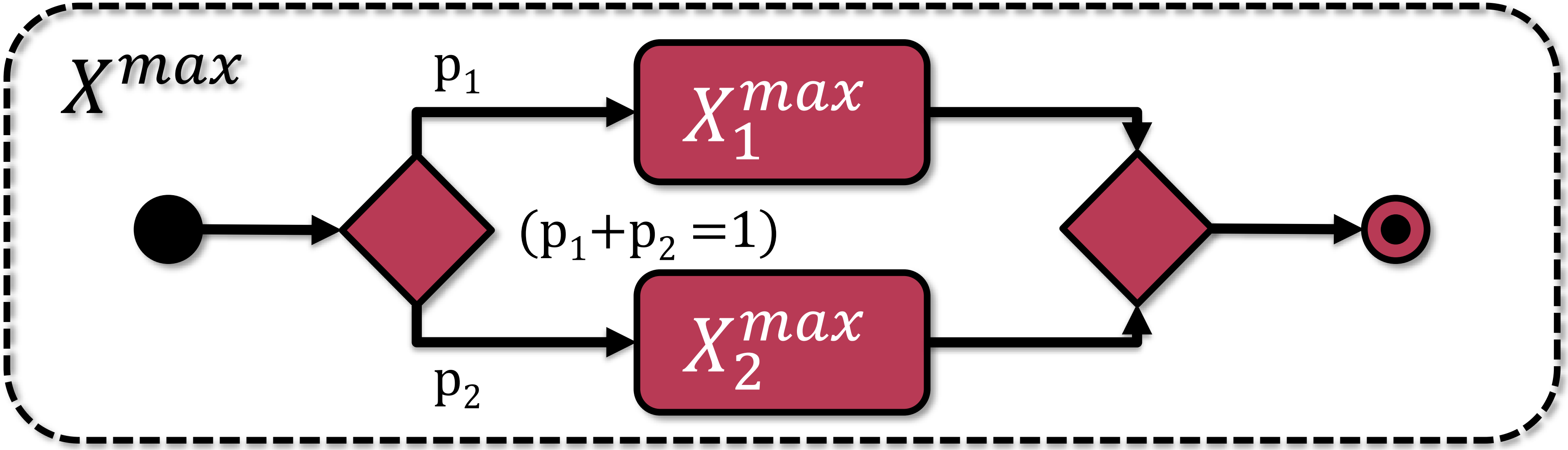
E.g. in a supermarkt: K cash-desks, 10 customer/h each

Composition of Stochastic Choice



Activity X_1 will be visited p_1 times in average,
activity X_2 will be visited p_2 times in average.

Composition of Stochastic Choice

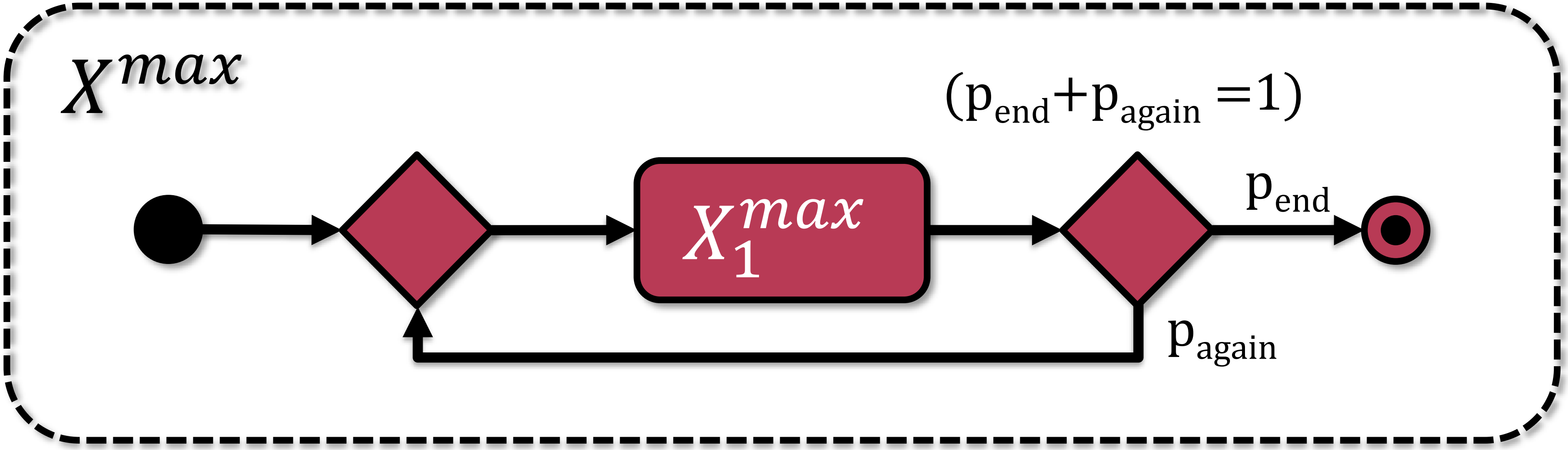


$$X^{max} = min(\frac{1}{p_1} \times X_1^{max}, \frac{1}{p_2} \times X_2^{max})$$

Bottleneck:

The component with the minimum throughput (or the corresponding resource).

Composition of Loop



Activity X_1 will be visited $\frac{1}{p_{end}}$ times in average.

Visitation Number

Choice: $$X^{max} = min(\frac{1}{p_1} \times X_1^{max}, \frac{1}{p_2} \times X_2^{max})$$

Loop: $$X^{max} = \frac{1}{\frac{1}{p_{end}}} \times X_1^{max} = p_{vége} \times X_1^{max}$$

- **Visitation number:** indicates the average number of times a given activity/subprocess runs in a single execution of the whole process.
  - In case of choice it is the decision possibility itself
  - In case of cycle it is the expected number of iterations

Visitation Number

Maximum throughput depending on the visitation number:

$$X^{max} = \frac{1}{v} \times X_1^{max}$$

- **Visitation number:** indicates the average number of times a given activity/subprocess runs in a single execution of the whole process.
  - In case of choice it is the decision possibility itself
  - In case of cycle it is the expected number of iterations

Visitation Number

Maximum throughput depending on the visitation number:

$$\frac{1}{X^{max}} = v \times \frac{1}{X_1^{max}}$$

- **Visitation number:** indicates the average number of times a given activity/subprocess runs in a single execution of the whole process.
  - In case of choice it is the decision possibility itself
  - In case of cycle it is the expected number of iterations

Visitation Number

Execution time depending on visitation number:

$$T_{process} = v \times T_{task}$$

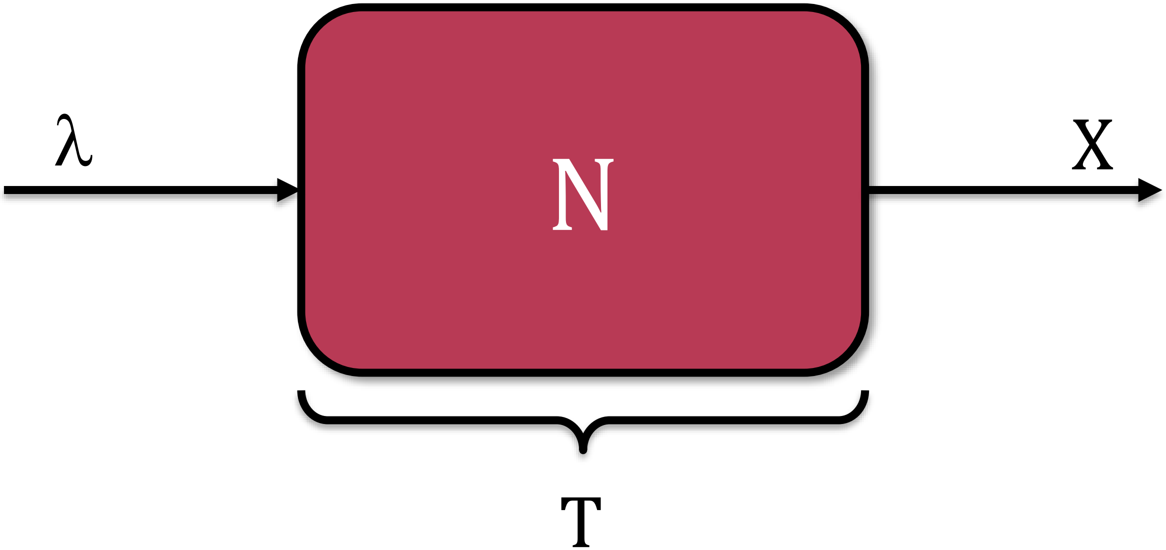
- **Visitation number:** indicates the average number of times a given activity/subprocess runs in a single execution of the whole process.
  - In case of choice it is the decision possibility itself
  - In case of cycle it is the expected number of iterations

LITTLE'S LAW

The basic formula

Little's Law

- λ: arrival rate $\left[\frac{1}{s}\right]$ X: throughput $\left[\frac{1}{s}\right]$
- T: time spent in system $[s]$
- N: number of tokens in system $[1]$



Little's Law

- **In stable state** ($\lambda = X$) Little's law holds:

$$N = X \times T$$

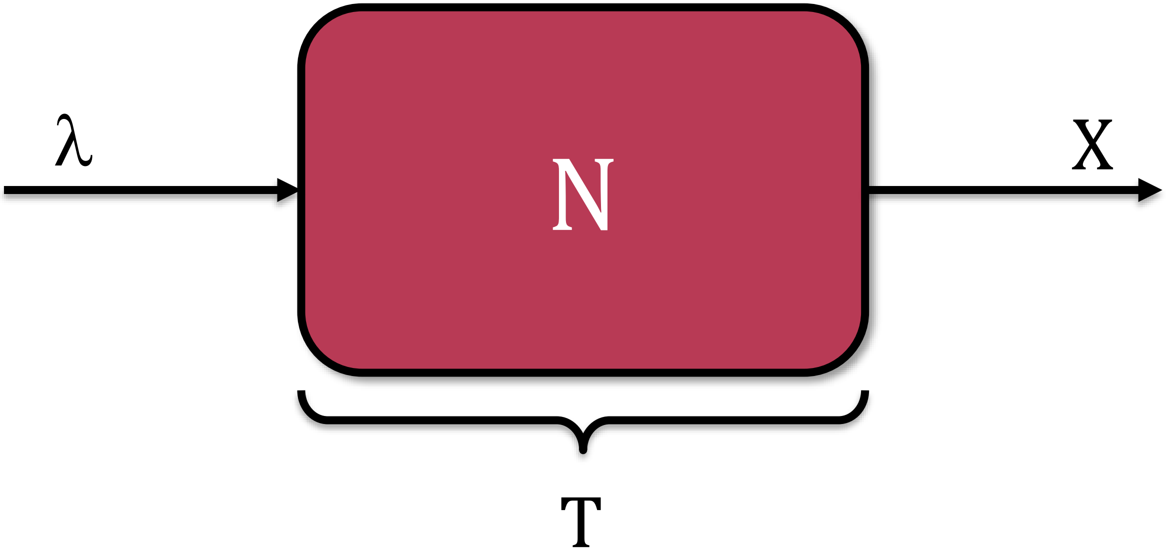


Illustration of Little's Law

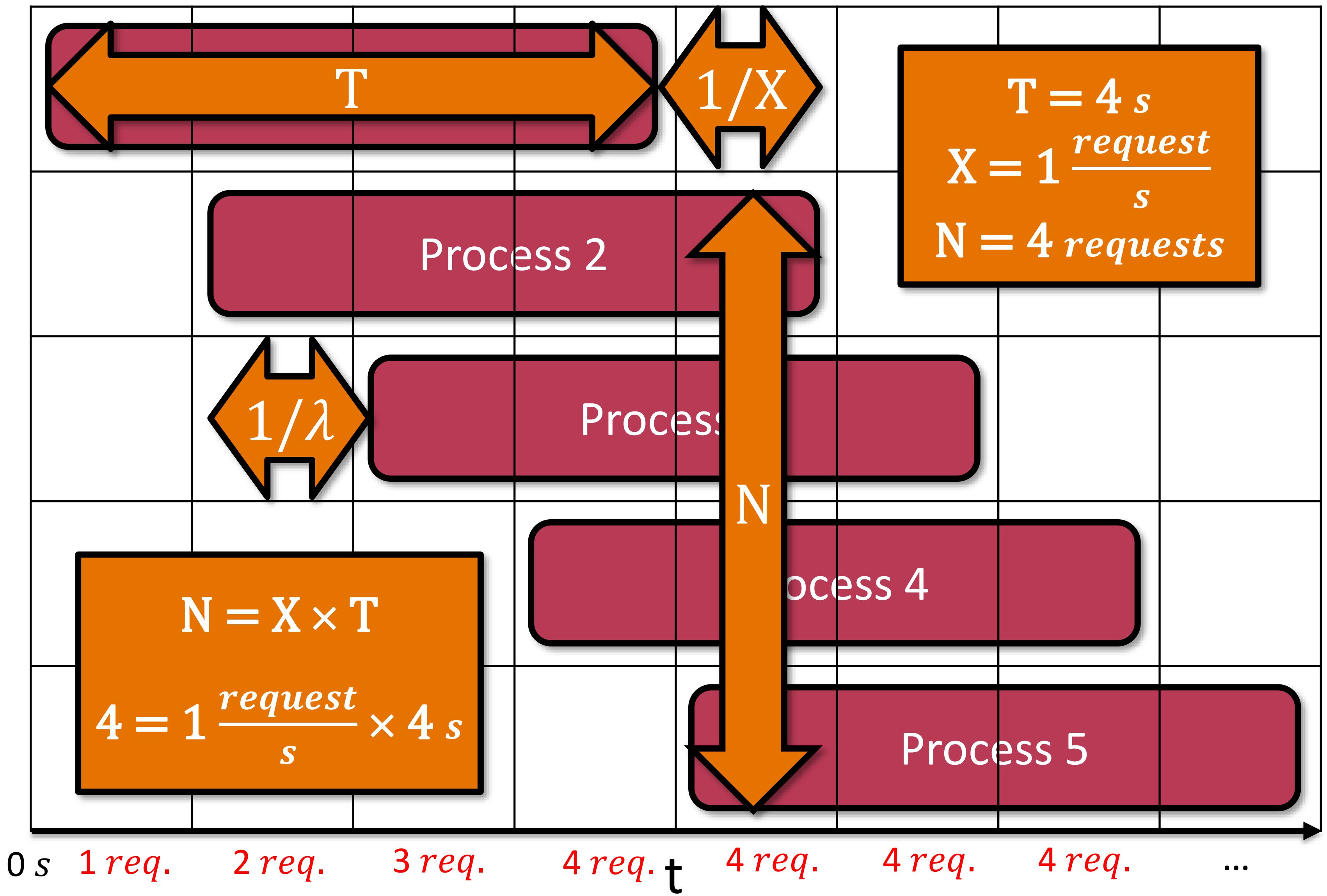
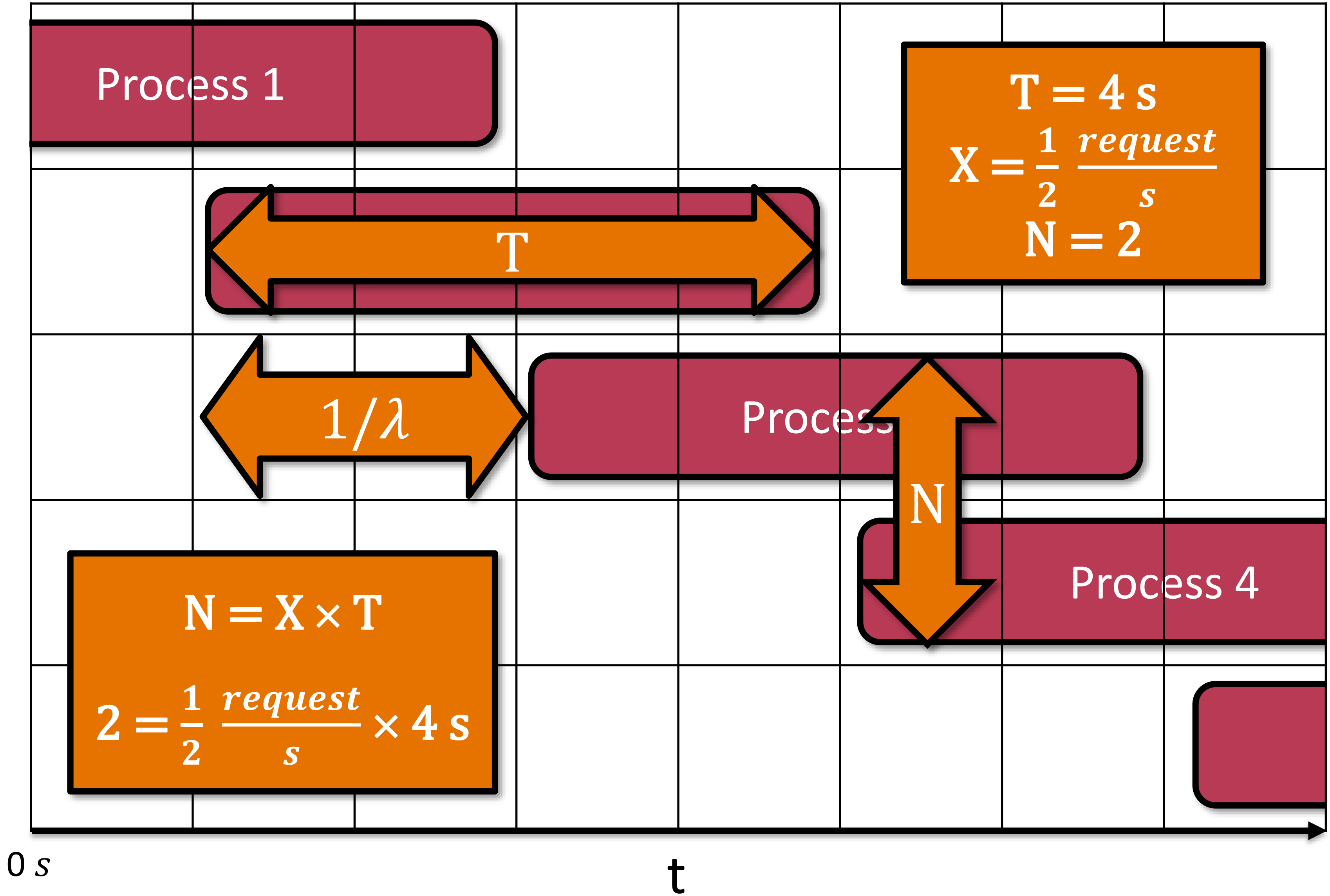


Illustration of Little’s Law



Utilization and Little's Law

- K resource instances: maximum K process instances under execution at the same time
- Little's law:
  number of process instances under execution (N)
- Average utilization can be derived as follows:

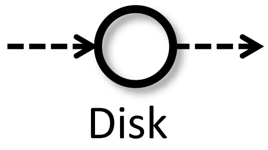
$$U = \frac{X}{K} \times T = \frac{X \times T}{K} = \frac{N}{K}$$

Utilization for K resource instances

Little's law
$(N = X \times T)$

LITTLE’S LAW: PRACTICAL EXAMPLES

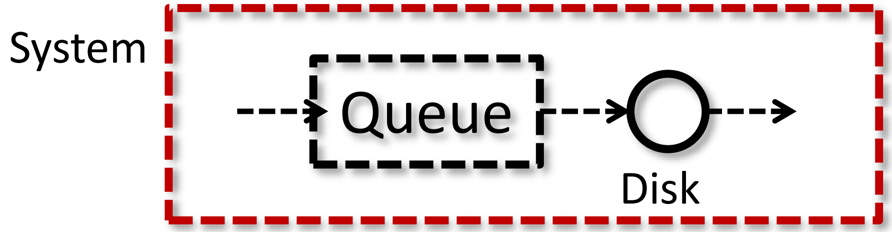
Little’s Law – Example



- Resource: disk
- Serves 40 requests per second (no overlap)
- Serving 1 request takes up 0,0225 seconds on average
- How much is the utilization?

$$U = X \times T_{disk} = 40\,\frac{request}{s} \times 0{,}0225\ s = 0{,}9 = 90\%$$

Little's Law – Example

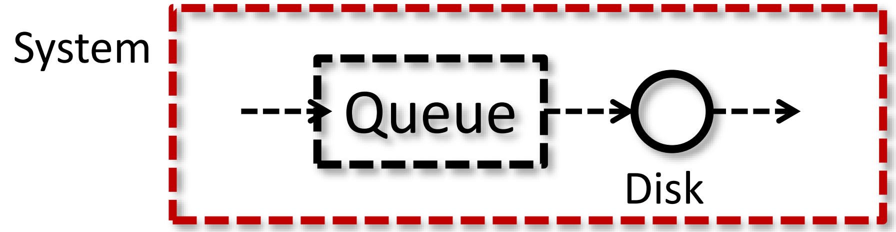


- Queuing before disk
- Disk: 40 $request/s$
- Average requests in system: 4

Average time a request spends in the system? (T_{system})

Average queuing time? ($T_{waiting}$)

Little's Law – Example



- Queuing before disk
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Queuing plus disk serving time

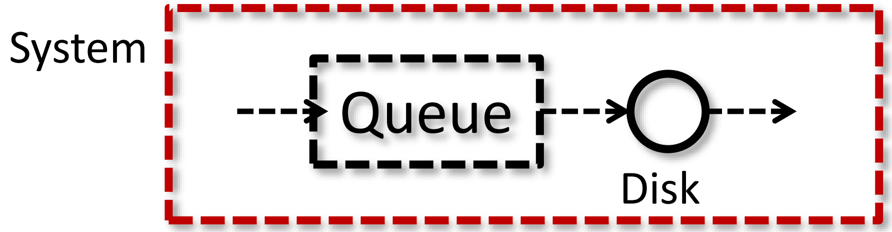
System

$N = X \times T \rightarrow T_{system} = 4\ requests / 40\ \frac{request}{s} = 0,1\ s$

Average queuing time

$(T_{system} - T_{disk}) = (0,1\ s - 0,0225\ s) = 0,0775\ s$

Little's Law – Example



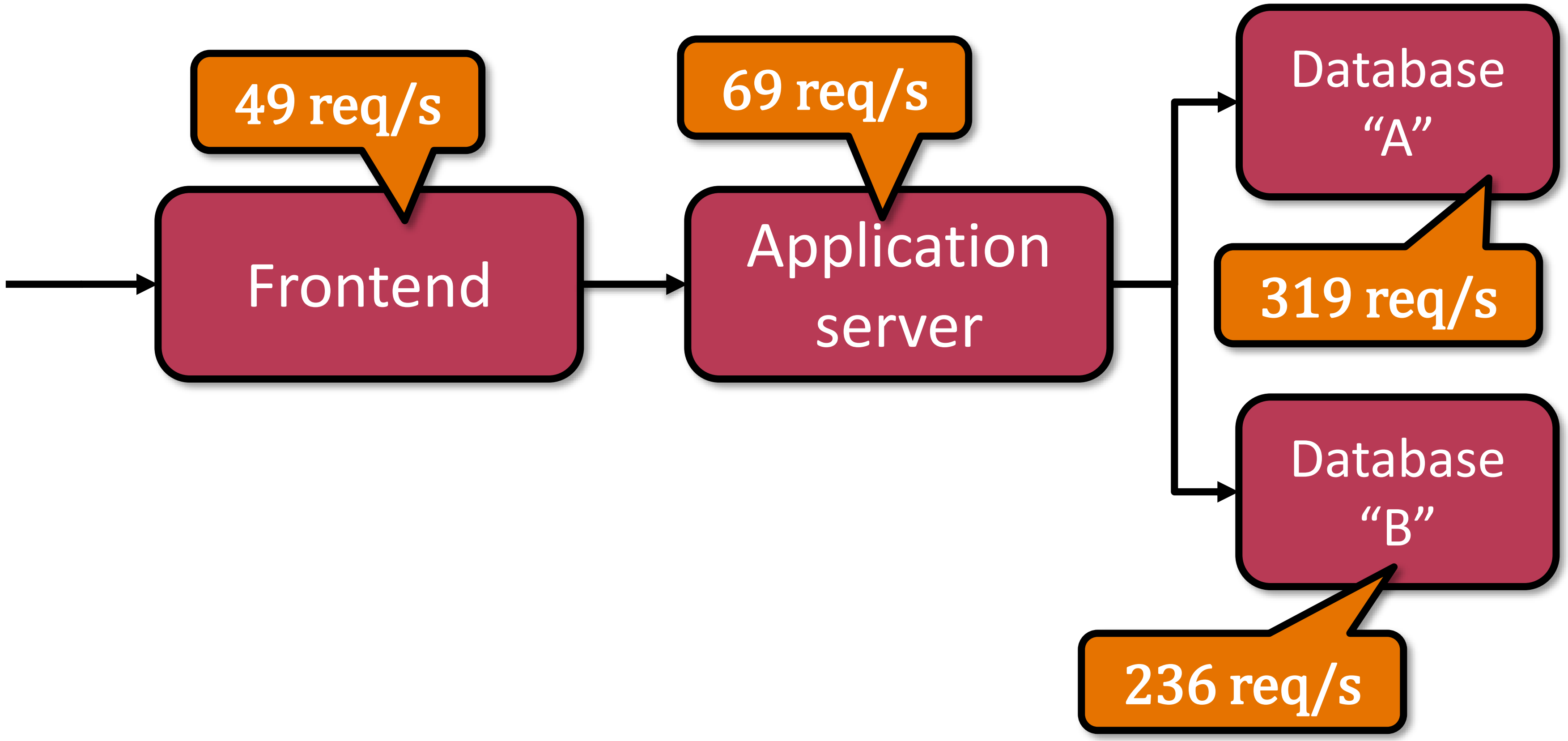
- Queuing before disk
- Disk: 40 $request/s$. In average 0,9 request
- Average requests in system: 4

Average number of requests in queue?
(N_{system} – N_{disk})
4 requests – 0,9 request = 3,1 requests

Little’s Law in Practice

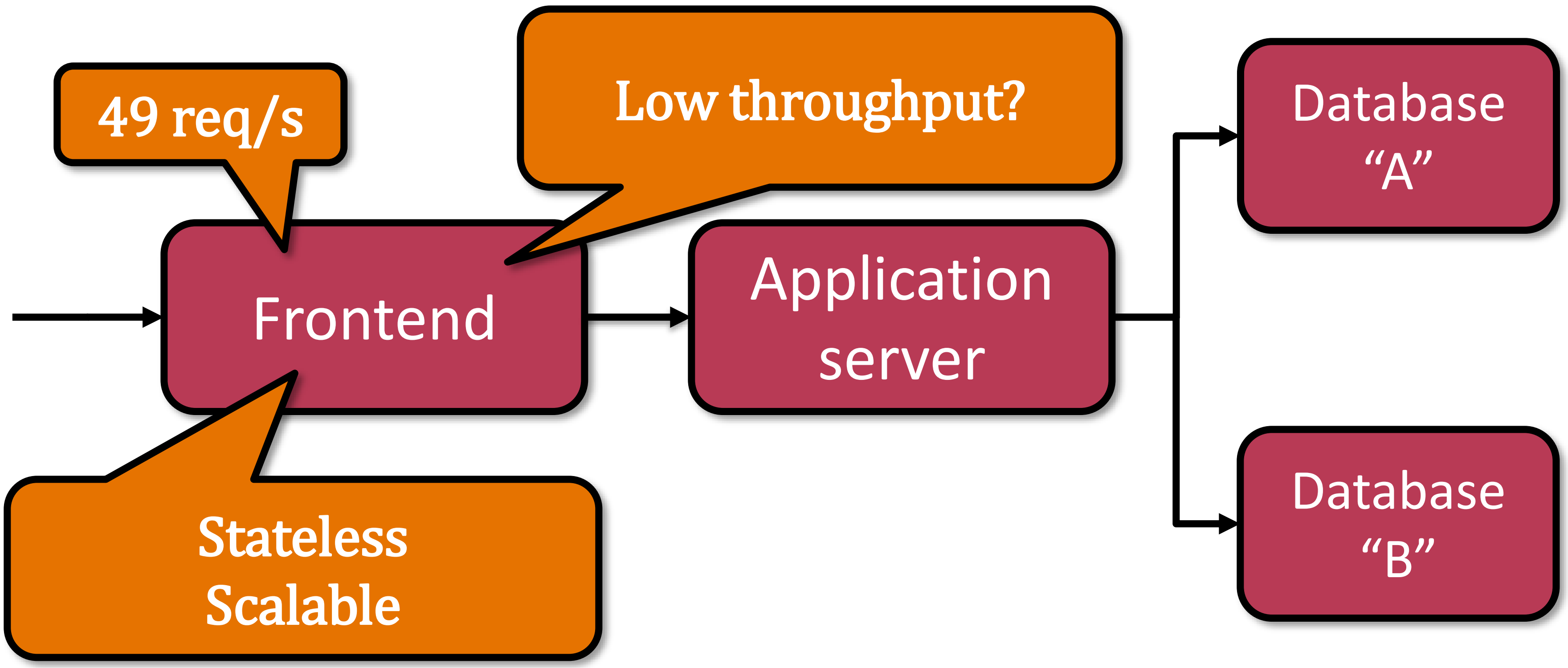
- Simulation
  - Dobson&Shumsky
  - https://youtu.be/UjzXQPGBaNA
- Why it is taught
  - http://pubsonline.informs.org/doi/pdf/10.1287/ited.7.1.106
- Examples
  - http://web.mit.edu/sgraves/www/papers/Little's%20Law-Published.pdf
    - E.g.: How long do the wine bottles stay in the cellar?
      - The cellar is filled up to ⅔ in average. (~160 bottles)
      - We bought 8 bottles per month in the last one year.
      - According to Little’s law, the bottles stay in average $T=N/X$, that is $160/8=20$ months in the cellar.

Performance of 3-tier Architecture



These metrics indicate the incoming load of the complete system! For instance „Database A" becomes the bottleneck if 319 requests arrive to the system each second.

Performance of 3-tier Architecture



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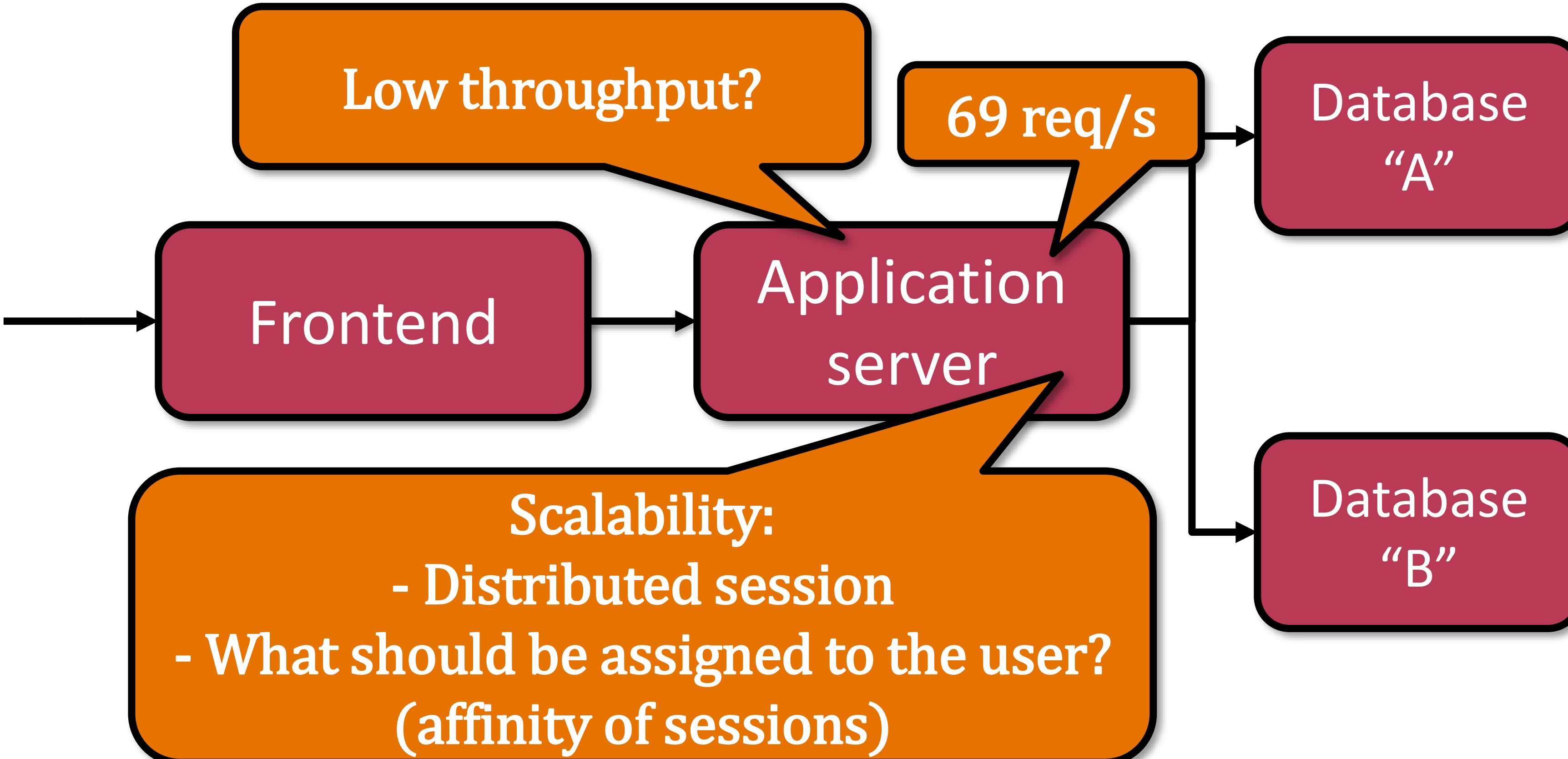

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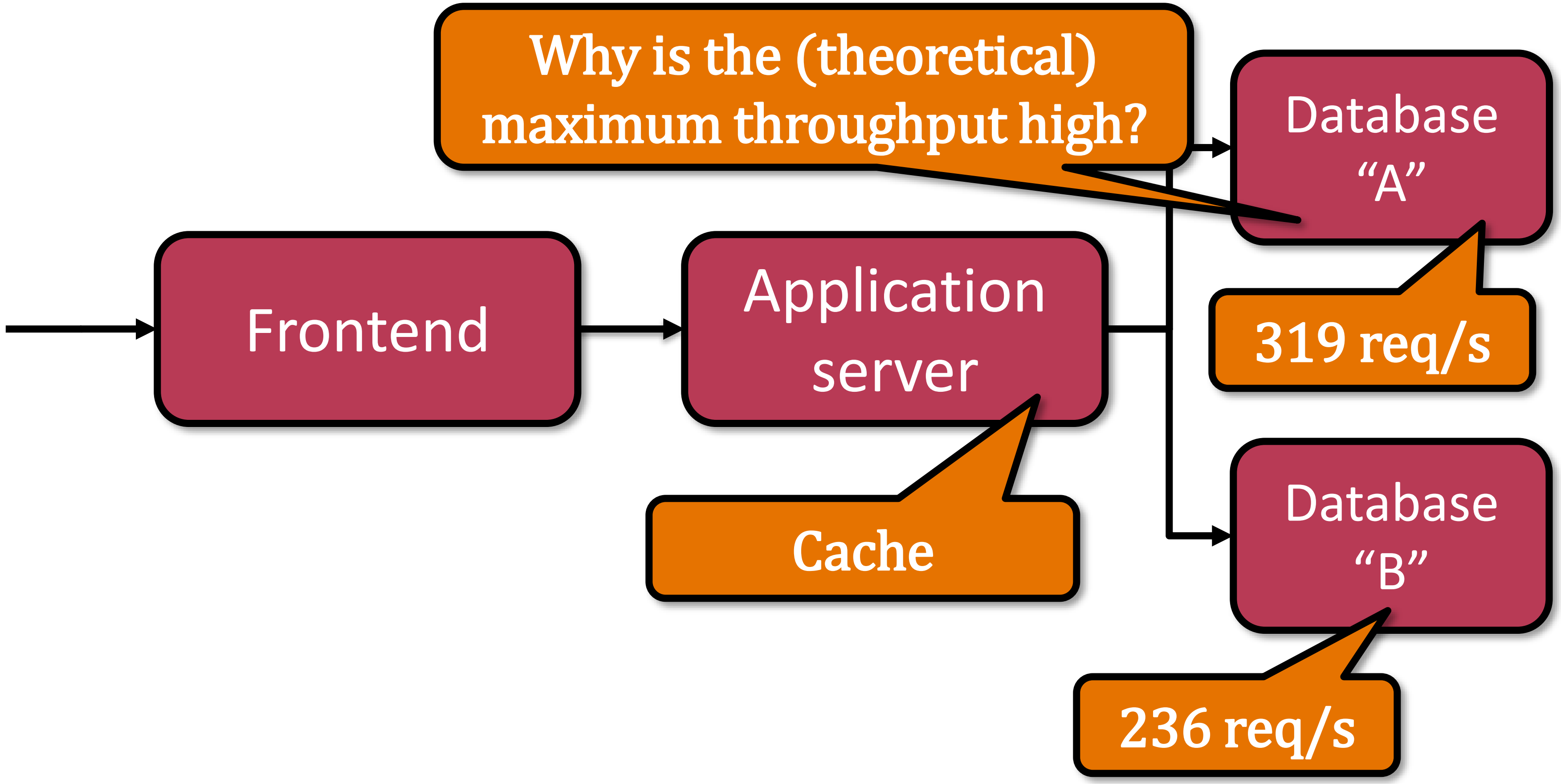

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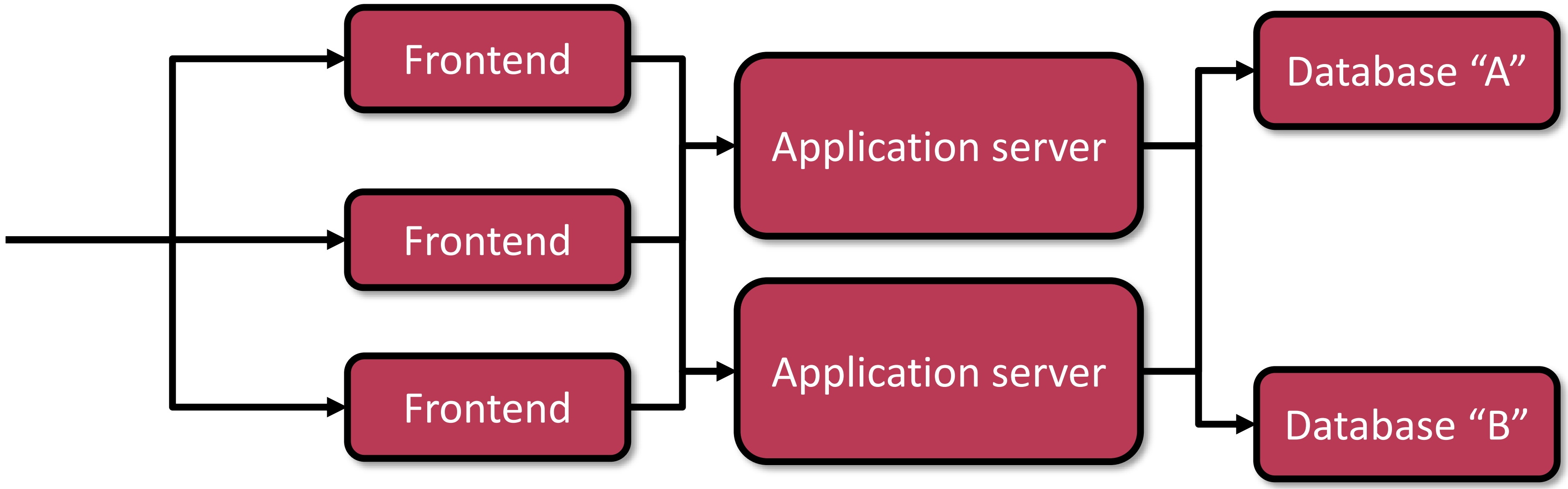
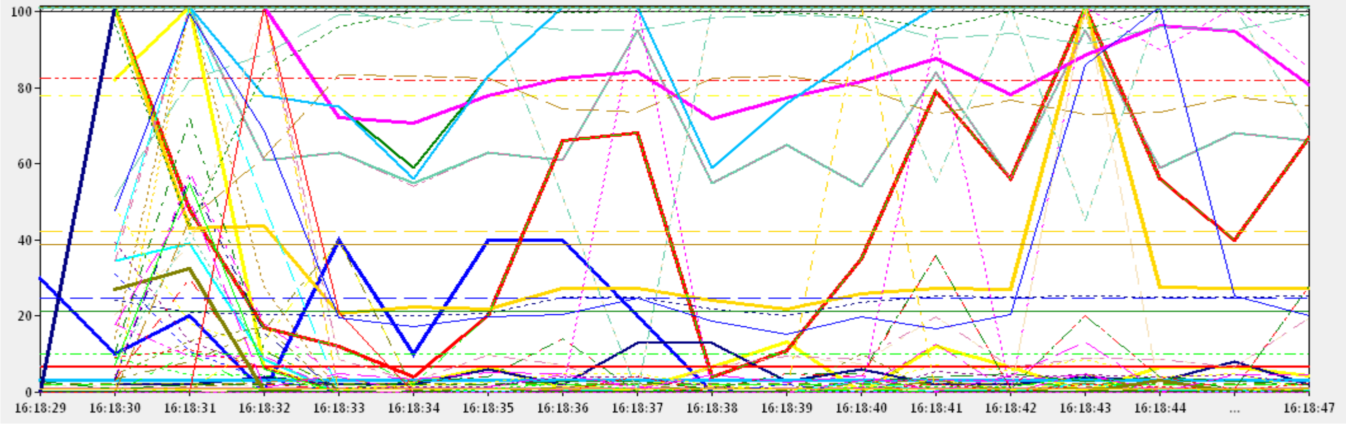
3-tier Architecture in Reality



(Example: technological background for the interested)
http://www.projectclearwater.org/wp-content/uploads/2013/05/Clearwater-Deployment-Sizing-10-Apr-13.xlsx
http://www.projectclearwater.org/technical/clearwater-performance/

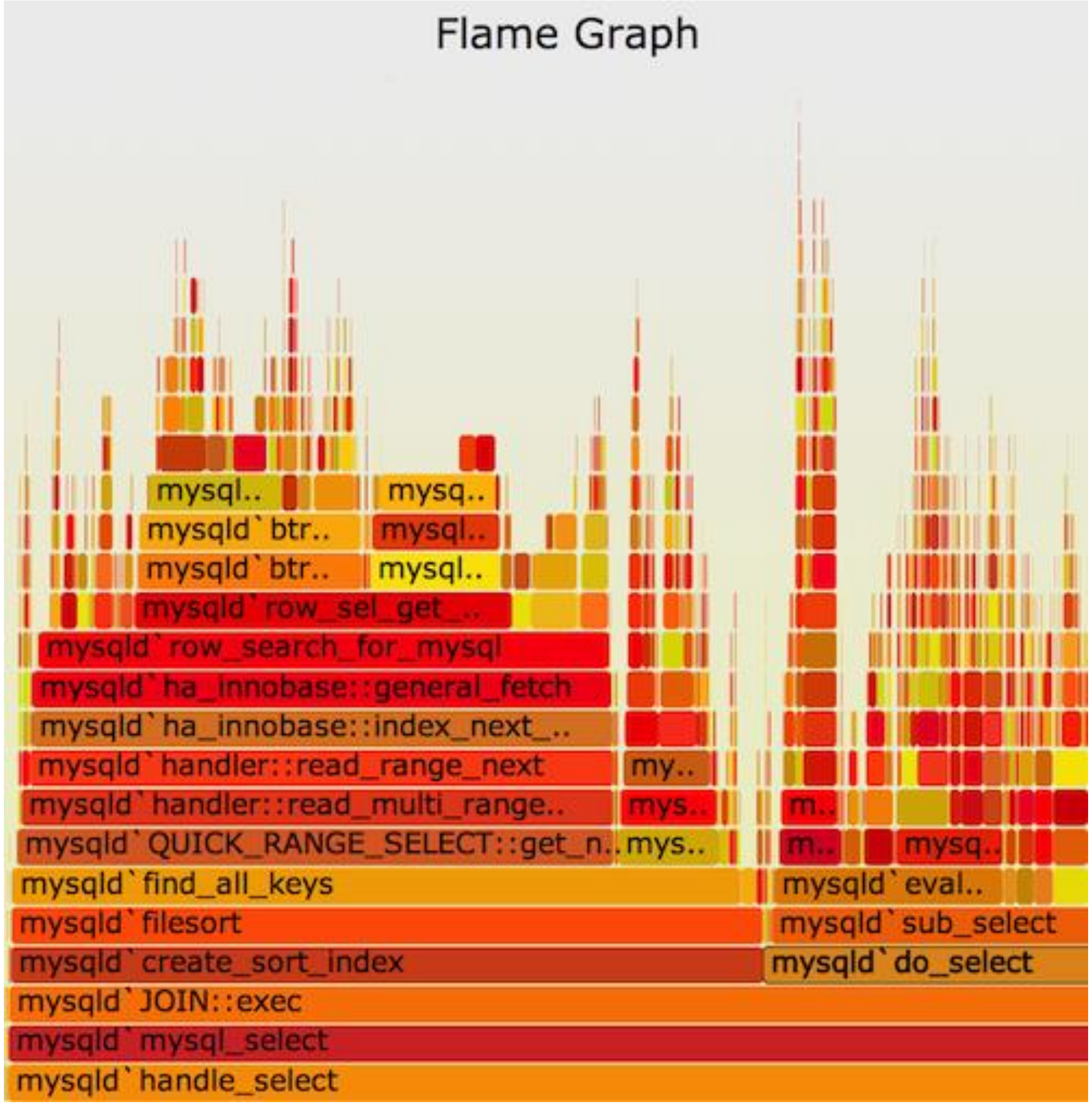
What to Measure? / What is Important?

- Metrics "in small"
  - E.g. Task manager, Resource monitor, the same on server-side…



- Metrics "in big"
  - E.g. virtualized systems
- Which metrics are interesting?

E.g.: What Takes so Much to Compute?



http://www.brendangregg.com/flamegraphs.html

Where Do We Approximate?

- In practice the values are difficult to measure
  - (e.g. Response time fluctuation, …)
- Applications compete
  - $(2 * \lambda \neq \lambda + \lambda)$
- Decision between resources
  - → Load balancer is also critical
  - → E.g. Requests of the same user to the same server
- We ignored the actual order/pattern of arrival
  - Advantage of Little's law
- Execution of a task may be data-dependent
- Structure/parameters of the system may change

"Watch my hands, because …"
(picture: wikipedia)

Where Do We Approximate?

- In practice the values are difficult to mea
  - (e ... n, …)

Everything is „lies and deception” only.

- Applications compete
  - $(2 * \lambda \neq \lambda + \lambda)$
- Decision between ... rces
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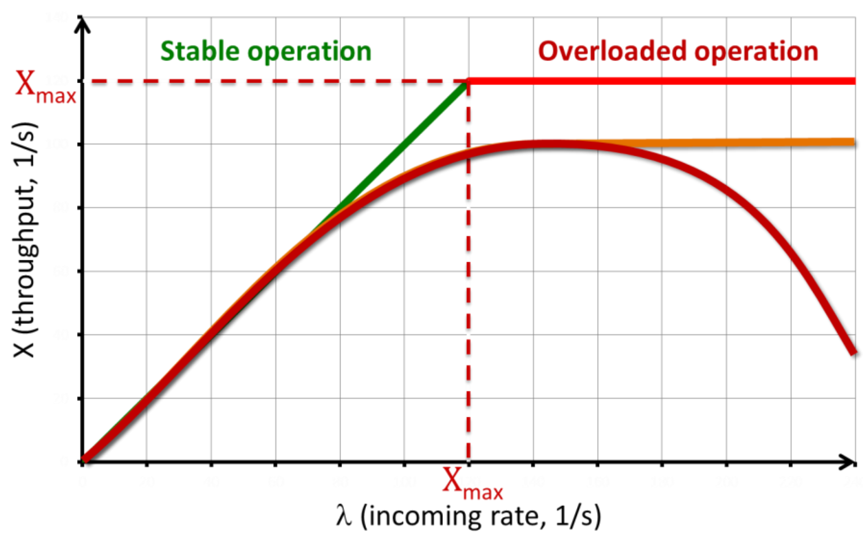
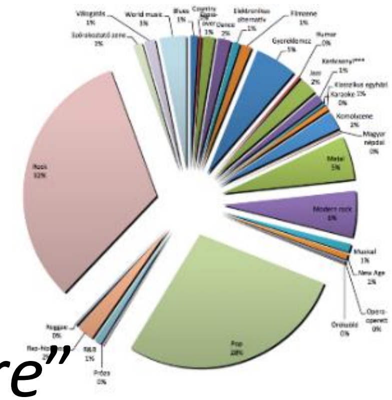
edia)



LOAD MODELS: ZIPF'S LAW

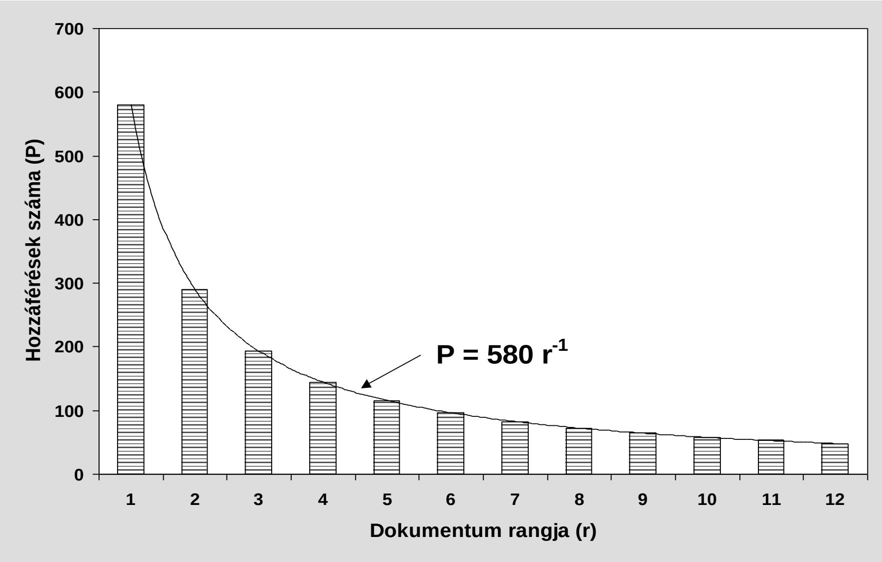
What is the Content of the Requests?



- Up to now: each requests are alike
  - “I need the details of a book”
- Actually: requests have content
  - “I need the details of *Foundation and Empire*”
  - See Pareto principle (80% – 20%)
  - Majority of the requests concerns minority of data
- Essential, because…
  - Has technical effects
    - Cache, pool size, static storage, …
  - Concerns the system model
    - Special handling of frequent requests

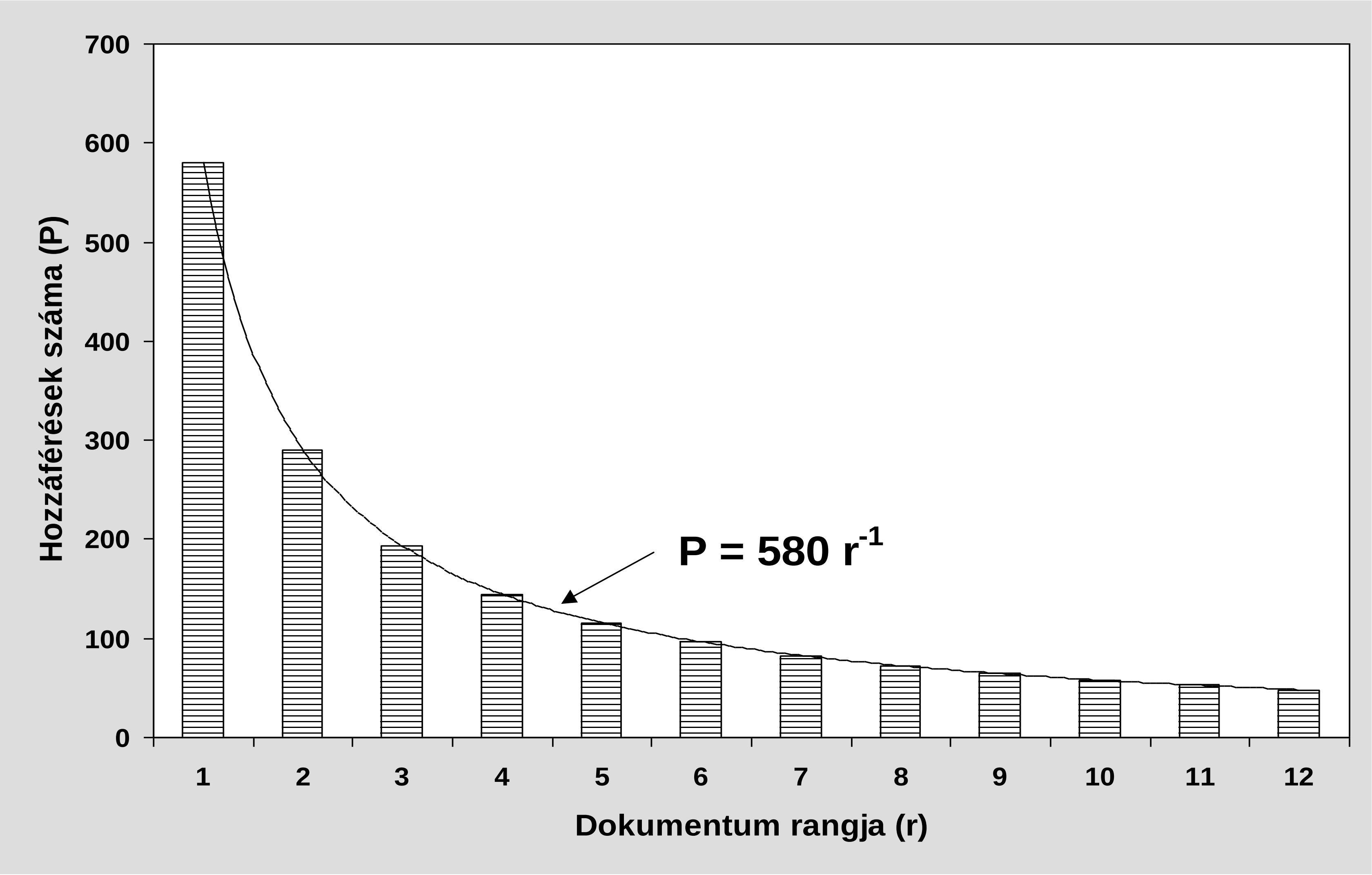
Zipf's Law

- Originally: number and frequency of words in *corpora* shows a characteristic distribution
  - True for not only language texts



George Kingsley Zipf
(1902–1950)
US American linguist
and philologist

Zipf’s Law – Example



Zipf’s Law - Examples

- Hit lists
- Population of cities by their ranks
- Characteristics of internet traffic
- Popularity of websites’ subpages
- Evolution of open source systems

Zipf's Law - Formula

$$R_i \sim \frac{1}{i^\alpha} \qquad f \sim \frac{1}{p}$$

- R_i – is the incidence of the i^{th} word
- α – a value charasteristic of the corpus
  - close to 1
- Simplified (α = 1):
  - f - frequency
  - p – popularity: rank of the text (decreasing order)

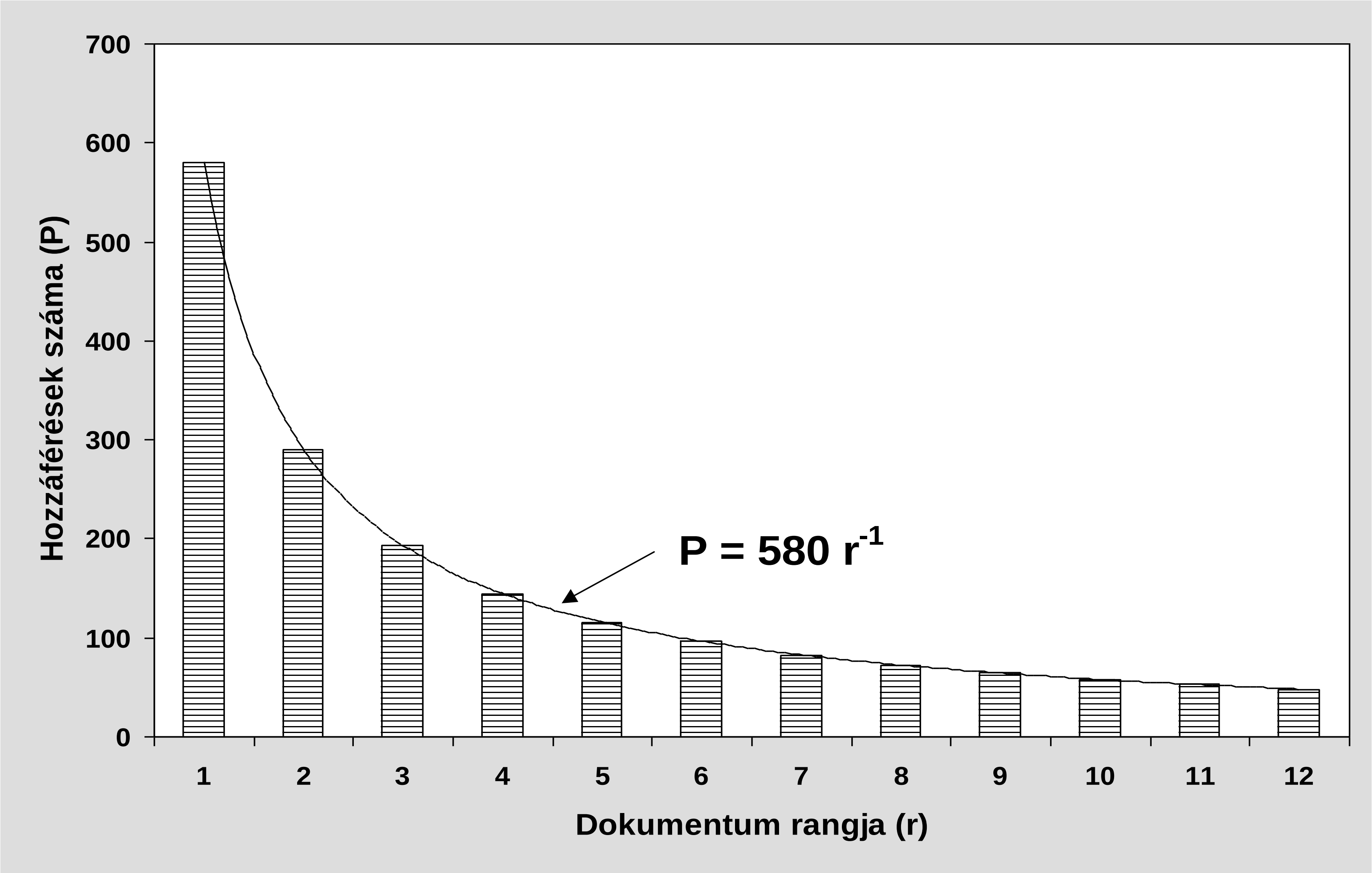
Zipf’s Law – Example: Web Documents

$$P = \frac{k}{r}$$

- P – references (hits)
- r – rank (1 = most frequent)
- k – positive constant

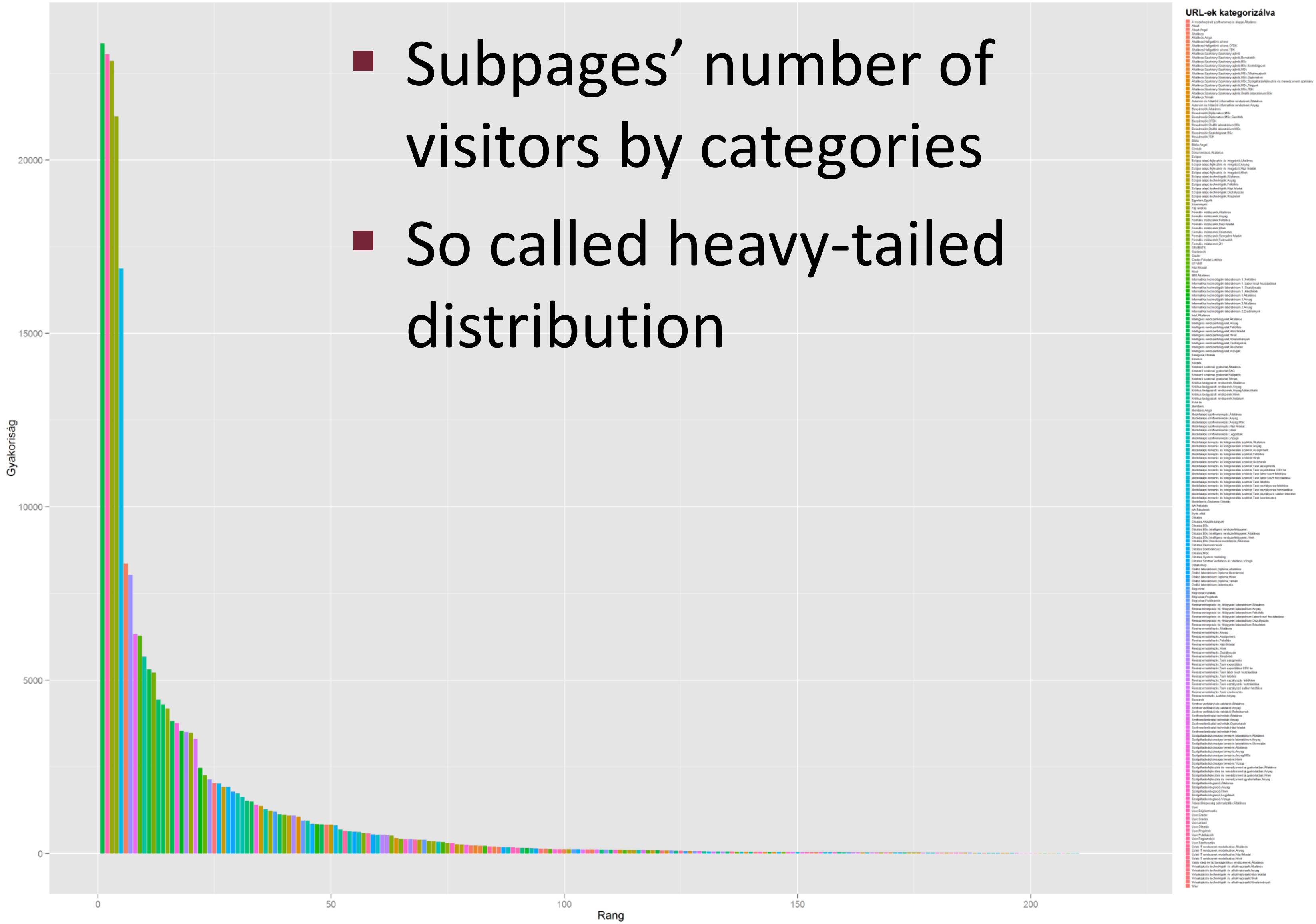
For more information see:
http://www.hpl.hp.com/research/idl/papers/ranking/adamicglottometrics.pdf

Zipf’s Law – Example



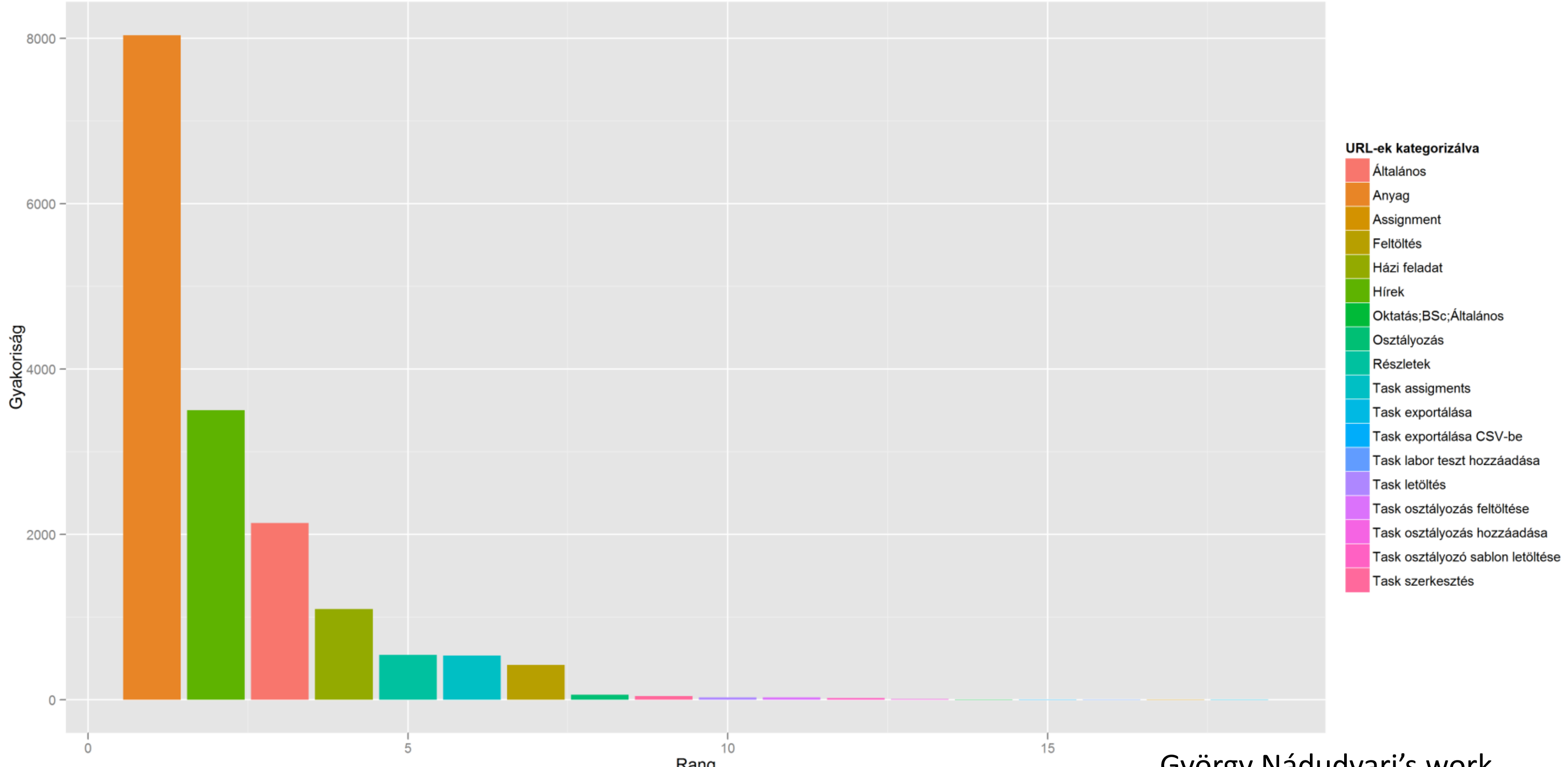
Zipf – Example: Website of our Group

- Subpages' number of visitors by categories
- So called heavy-tailed distribution



Zipf – Example: Website of our Group

- Visitors of the webpages of the System Modelling course



György Nádudvari's work

Visitation number | Little's law | Zip's law | **Changes in Workload**

CHANGES IN THE WORKLOAD

What kind of workload?

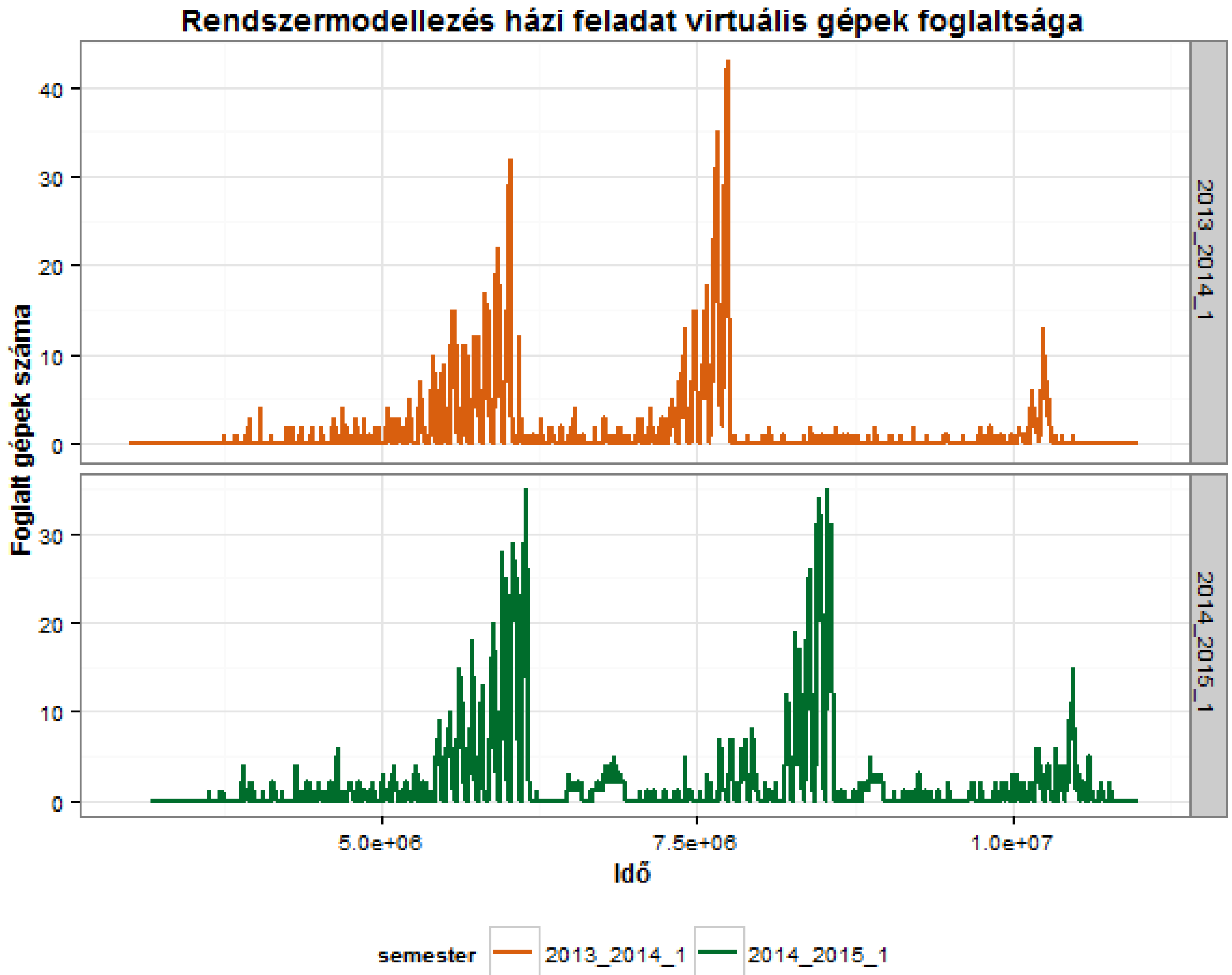
- Up to now:
  - We calculated with average values
  - Regarded the system’s behaviour depending on the *load (intensity)*
  - But: In reality the increase of the load is not necessarily predictable
- In reality
  - The behaviour of the system *changes over time*
  - This has technological effects
    - Switching between tasks, resource reservation, etc. (see: Operating systems)

Changes in the Workload – Example

- Dimensioning a systems for producing the (at that time) new identity cards
  - It is predictable how many new cards will be applied for in a year. (expirations, next age group)
  - It is predictable how many hours there are in a year.
    → We have the avg. arrival rate of the applications $[card/h]$
    Can it be used for dimensioning the system?

- Consider two different hours
  1. the 24th December 10-11PM
  2. the 15th June 4-5PM
     (End of working day shortly before the main summer holiday time)

System Modelling (7th semest.) – in the cloud



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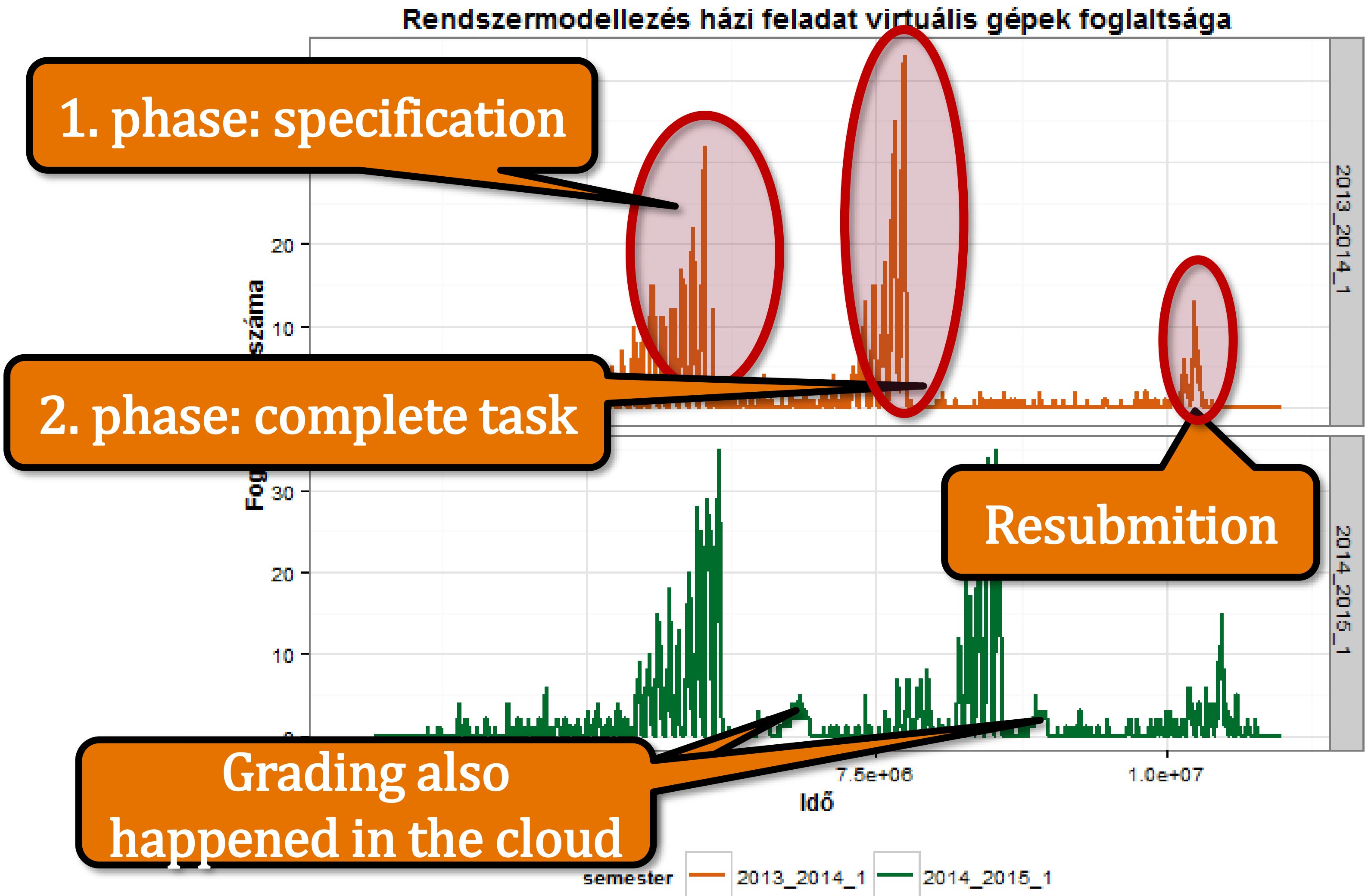

Real (historical) Load Example (iwiw)

Approximations can be applied to sections
- Linear, exponential, logarithmic
- Regression, detailed in Probability Theory

Source: http://www.sg.hu/cikkek/42924/