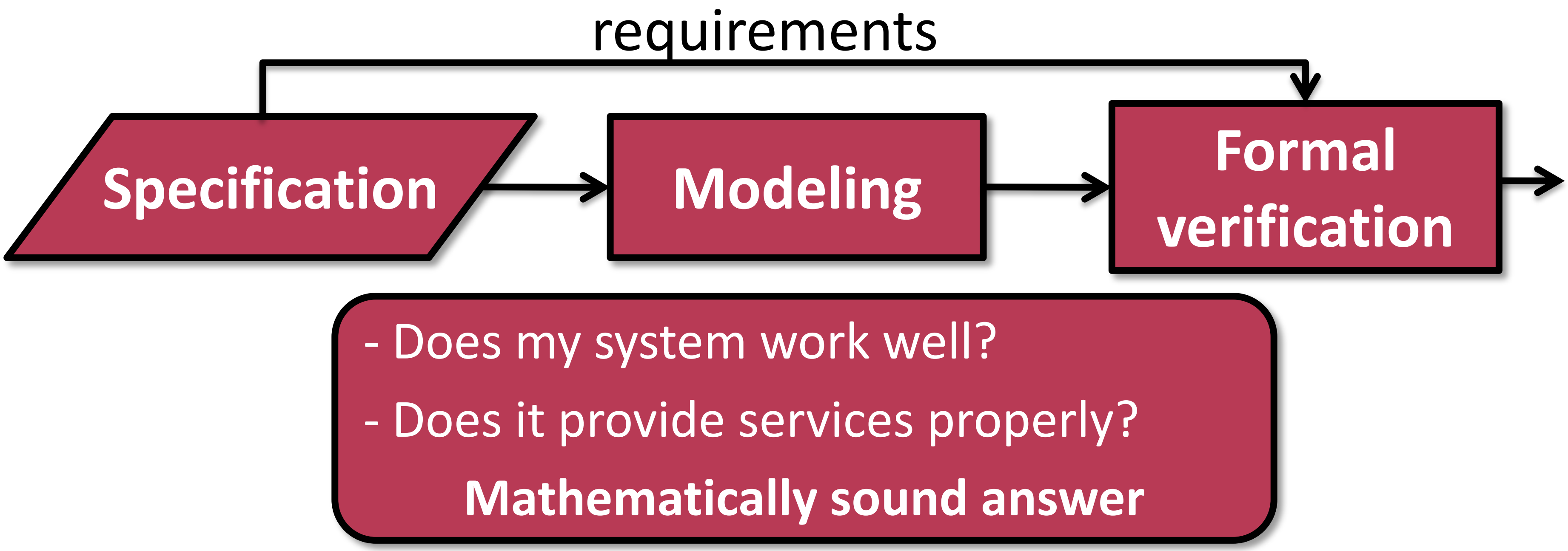
Saturation

András Vörös

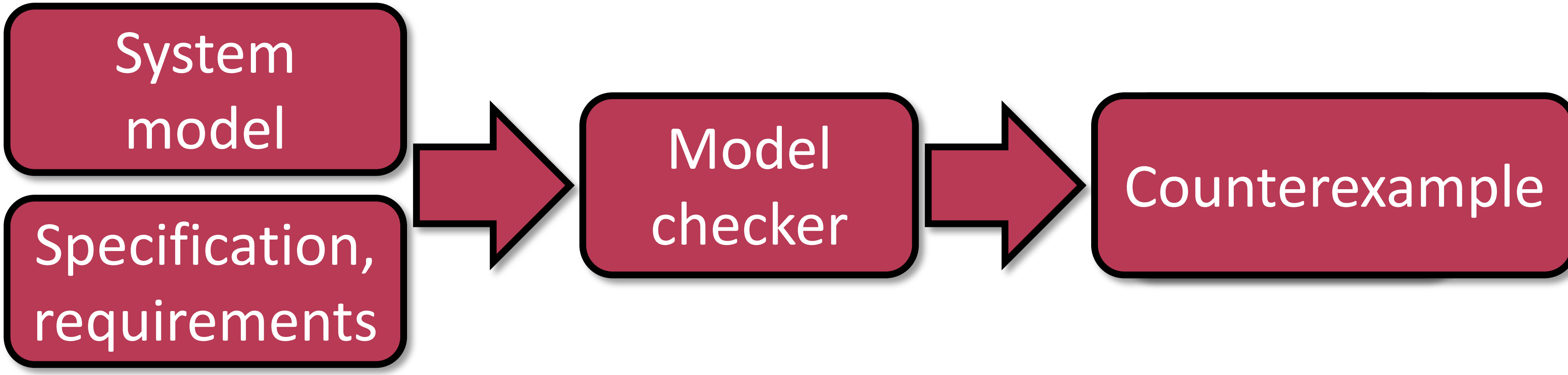
Formal methods

- Safety critical and embedded systems
  - Railway, automotive industry, air transportation
  - Reliability is an important issue
- Design time analysis
  - These models can be used for implementation



Model checking

- Automatic verification method



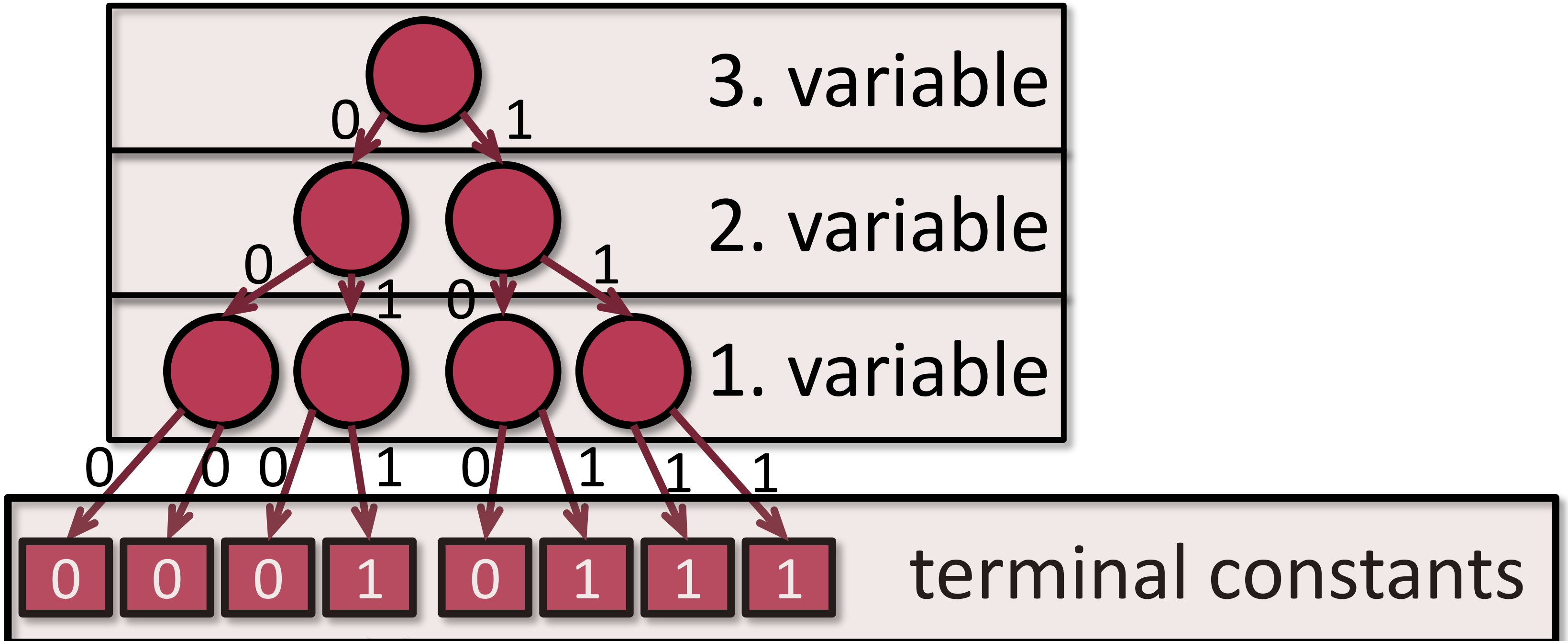
- Prerequisite:
  - Exploring and representing the reachable states
- Problem:
  - State space explosion
  - Time and space requirements

Saturation algorithm

- Efficient solution for:
  - State space generation
  - Model checking
- Symbolic algorithm
  - Encoding of states
  - Special underlying data structures
    - Multi Valued Decision Diagrams (MDD-s)
- Special iteration strategy
  - Efficient for asynchronous models

Multi Valued Decision Diagrams

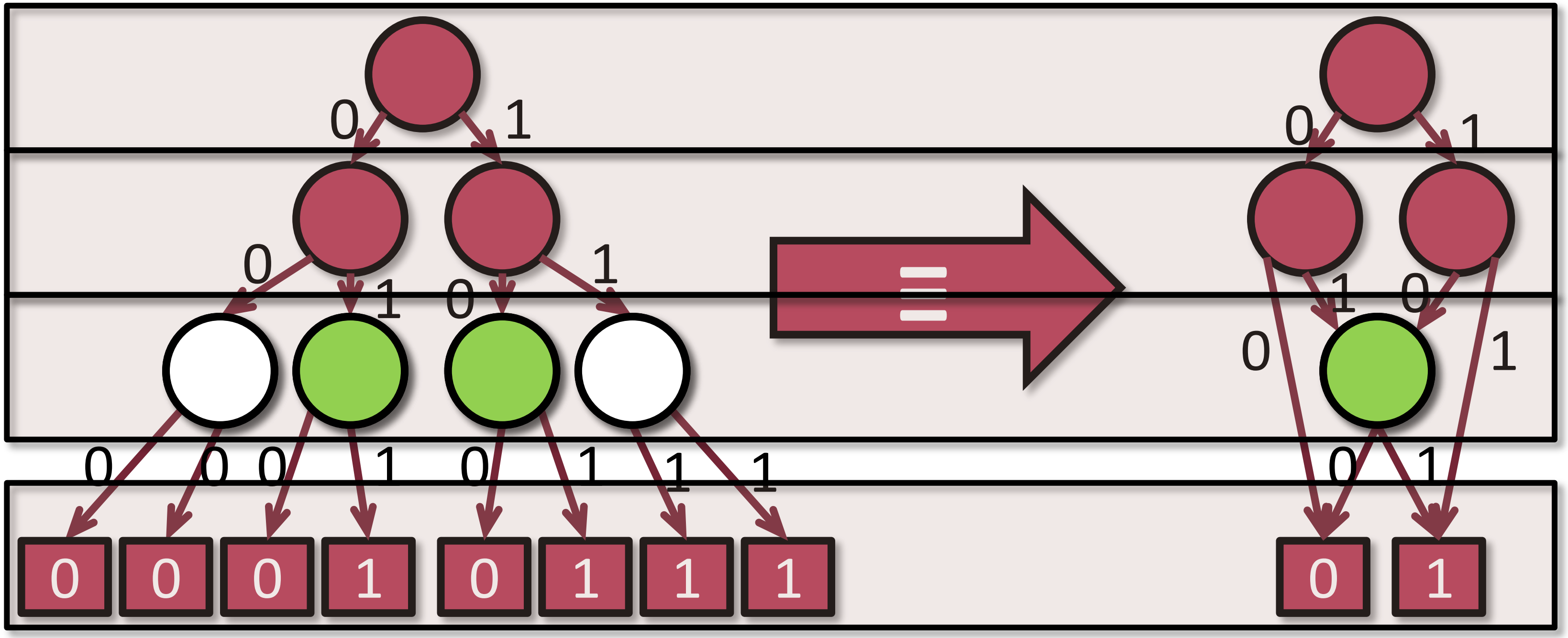
- Derived from decision trees
  - variables are ordered into levels
- Example:
  - only binary variables



Decision tree

Multi Valued Decision Diagrams

- Derived from decision trees
  - variables are ordered into levels
- Special reduction rules
  - in a bottom-up fashion, applying reduction from level-to-levels
- Compact representation of multi valued functions

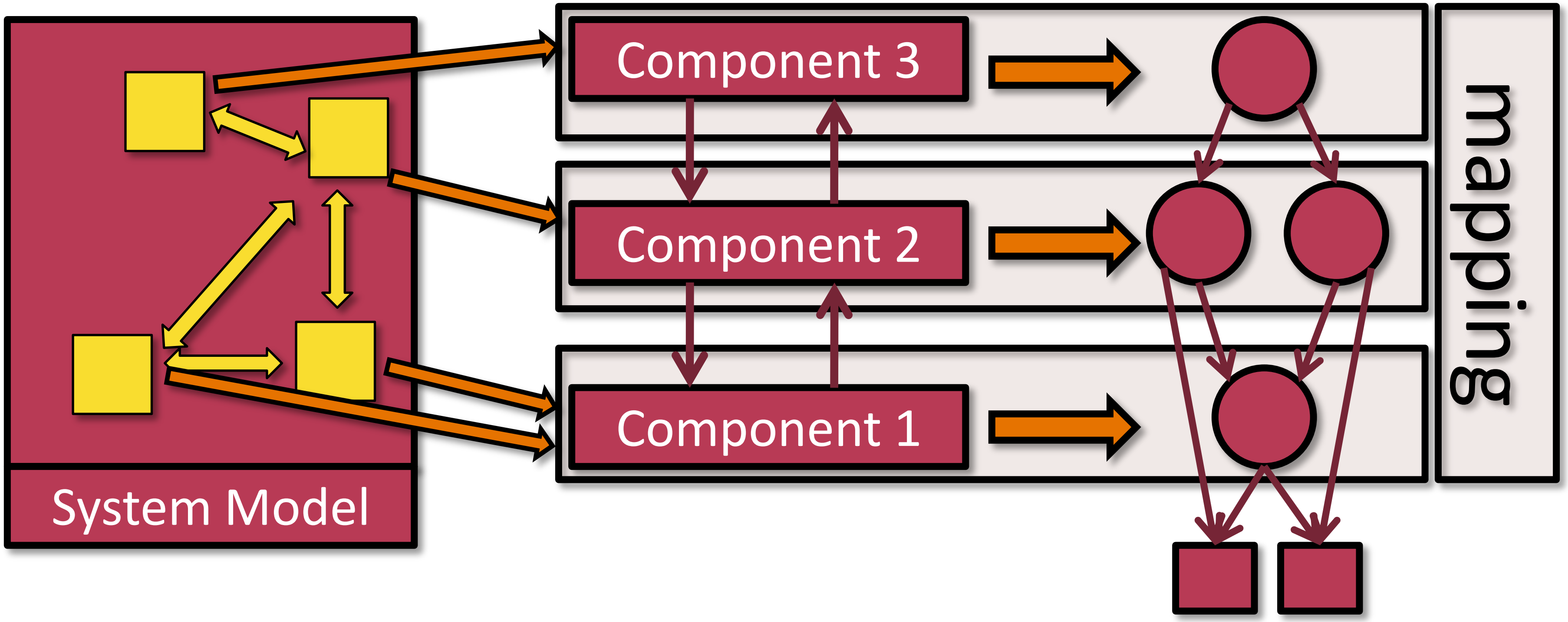


Decision tree

MDD

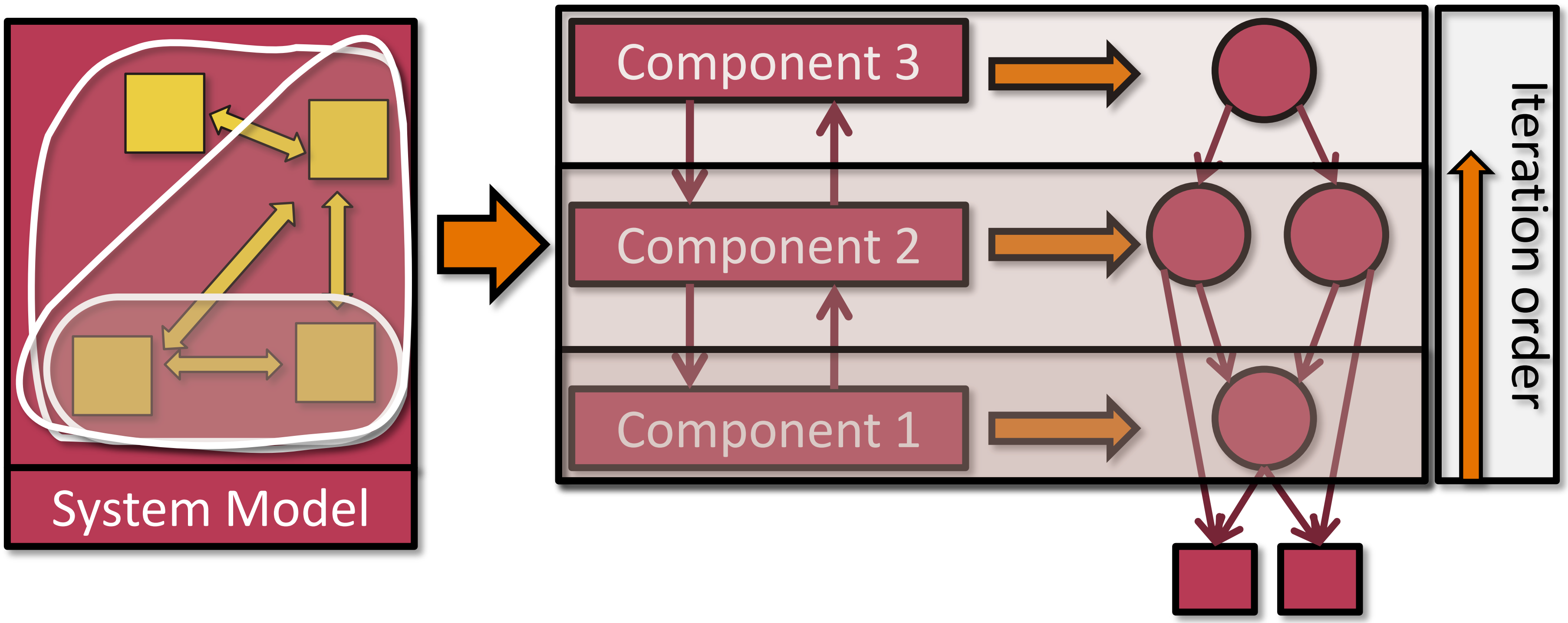
Symbolic algorithm

- Symbolic encoding instead of explicit state representation
  - Decomposition is needed
- Saturation uses component wise encoding

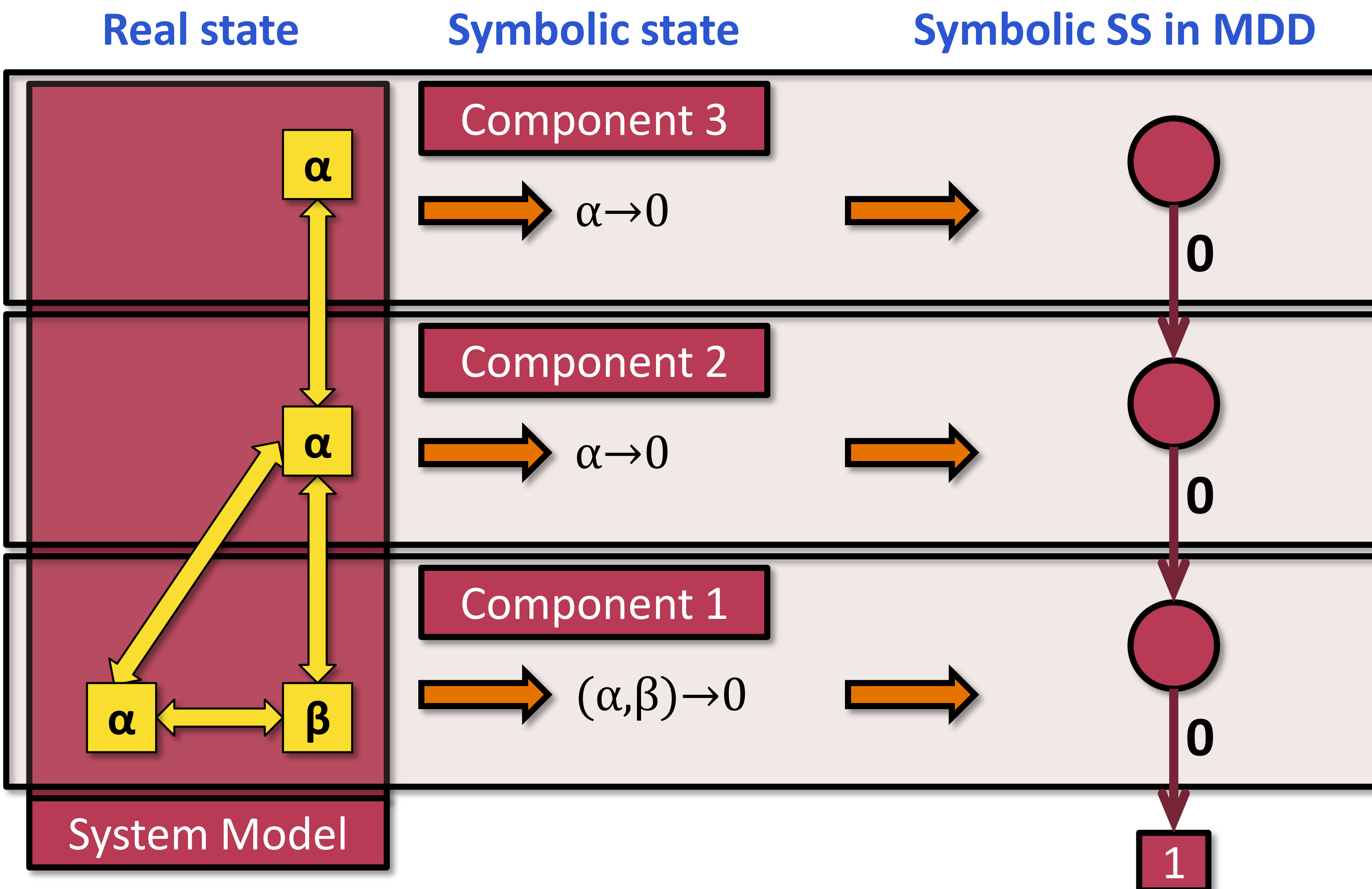


Special iteration

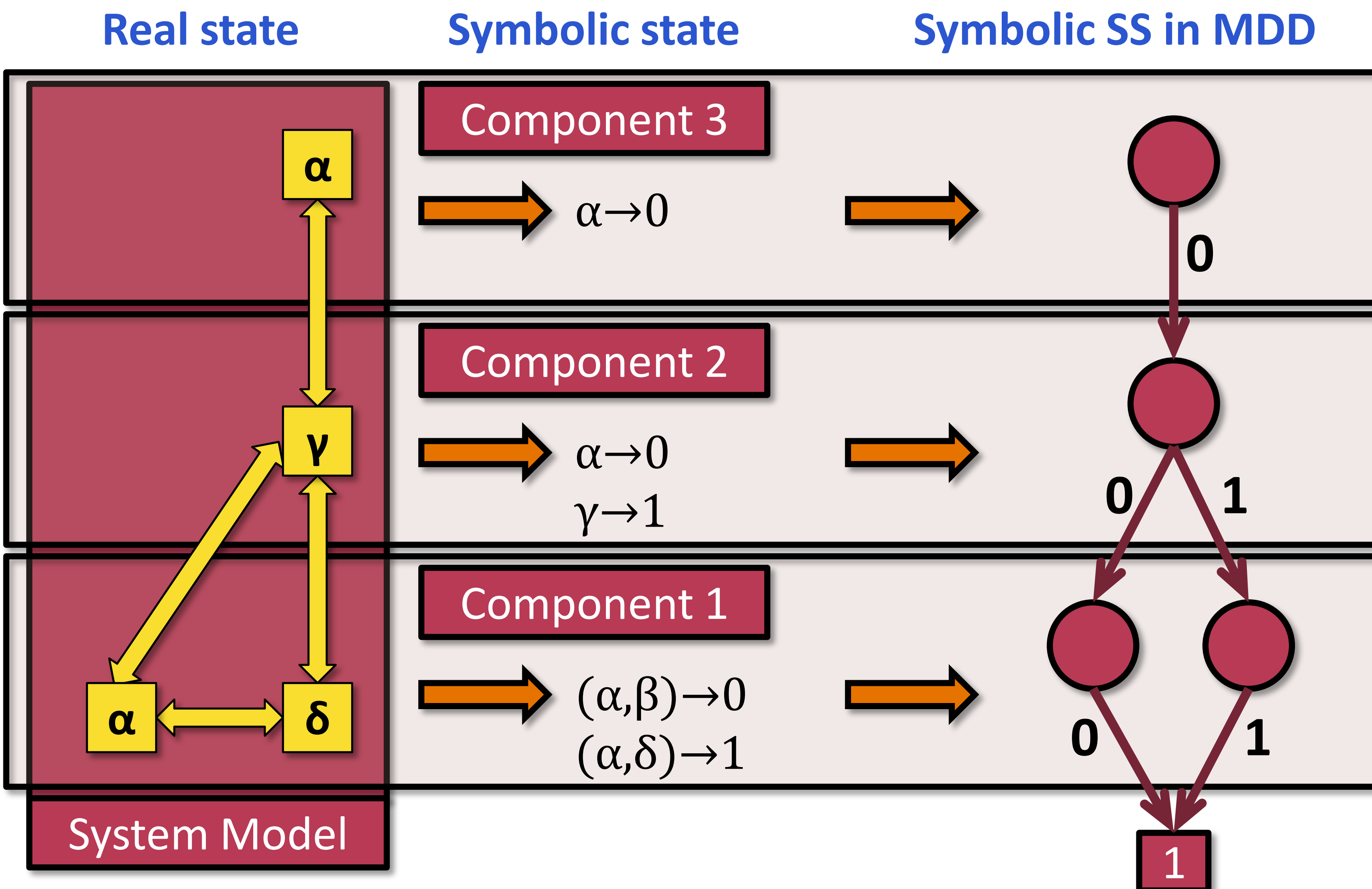
- Local exploration in a greedy manner
- Exploring global synchronization events if needed
- Uses the primarily defined order of the decision diagram variable encoding
- Efficient for Globally Asynchronous, Locally Synchronous models (GALS)



State space representation with MDDs

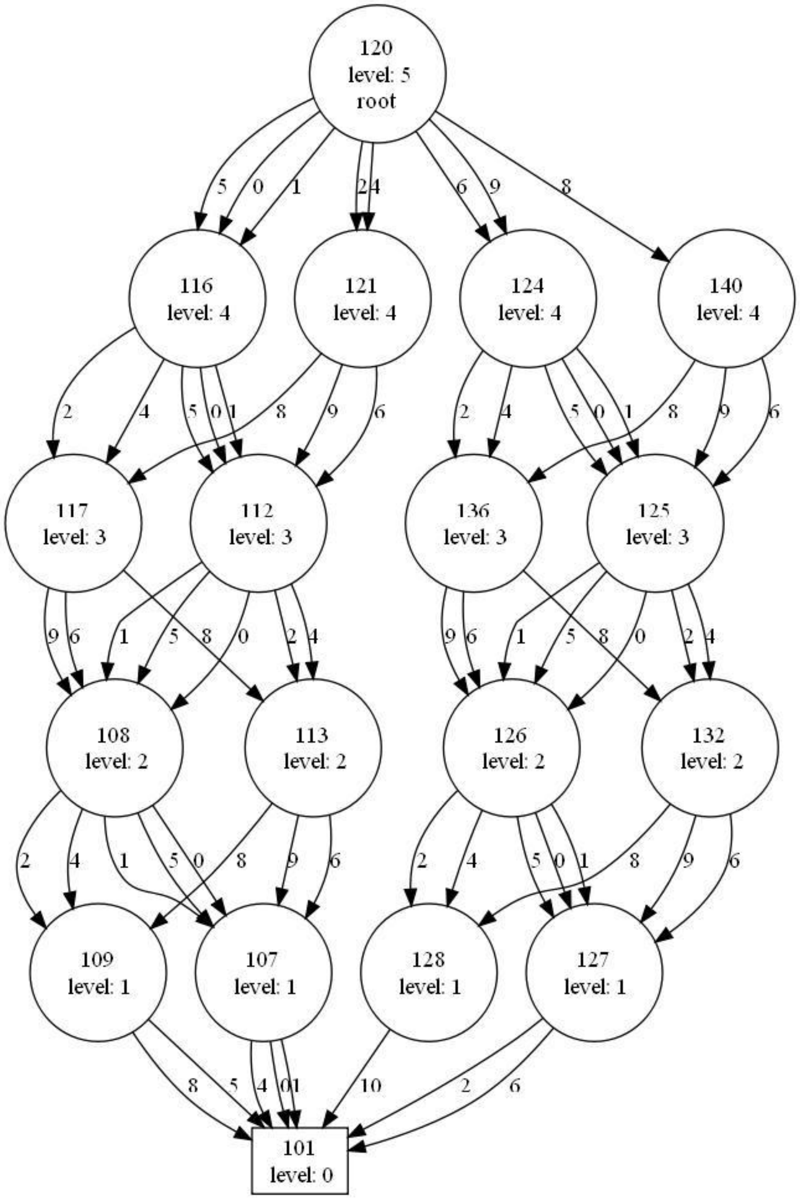


State space representation with MDDs



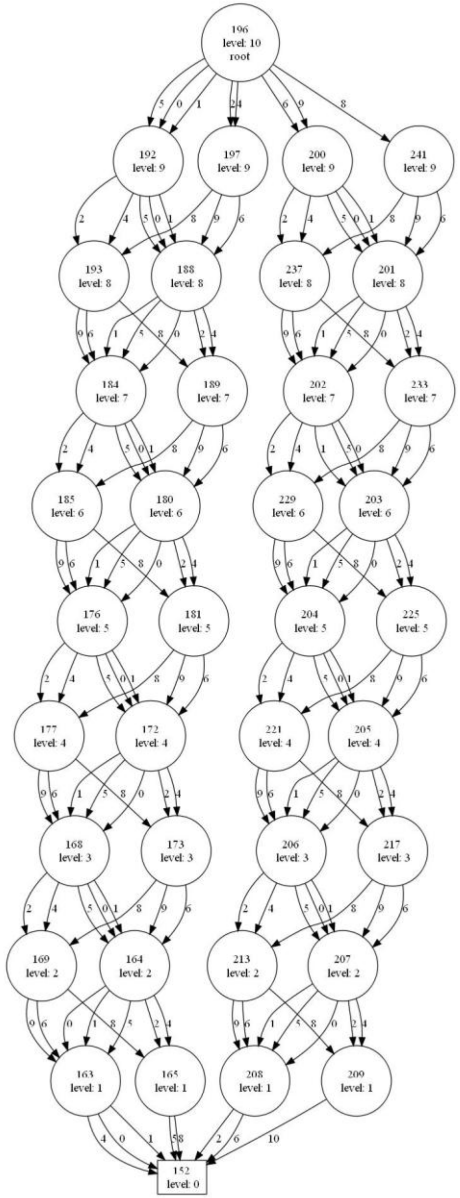
Experiments

- Dining philosophers
  - 5 philosophers
- State space representation
  - 1364 states
  - 19 nodes



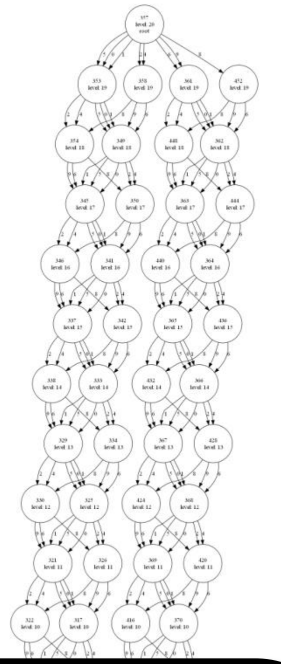
Experiments

- Dining philosophers
  - 10 philosophers
- State space representation
  - 1,860,498 states
  - 40 nodes



Experiments

- Dining philosophers
  - 20 philosophers
- State space representation
  - 3,461,452,808,002 states
  - 80 nodes



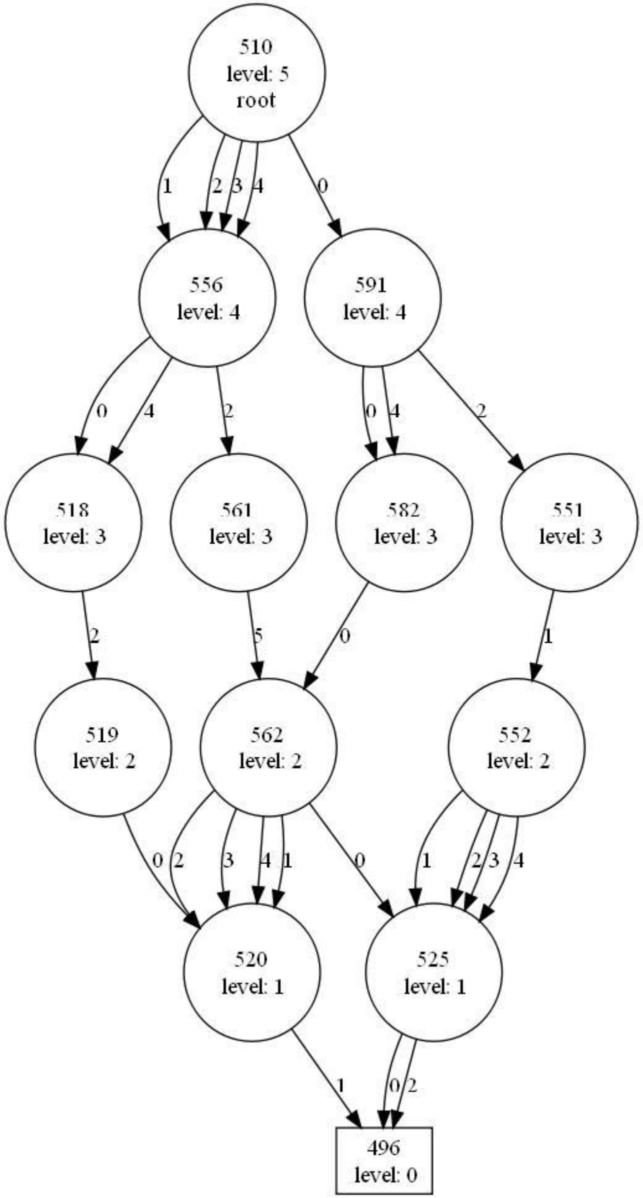
Exponential growth in the state space

⇕

Linear growth in the state space representation

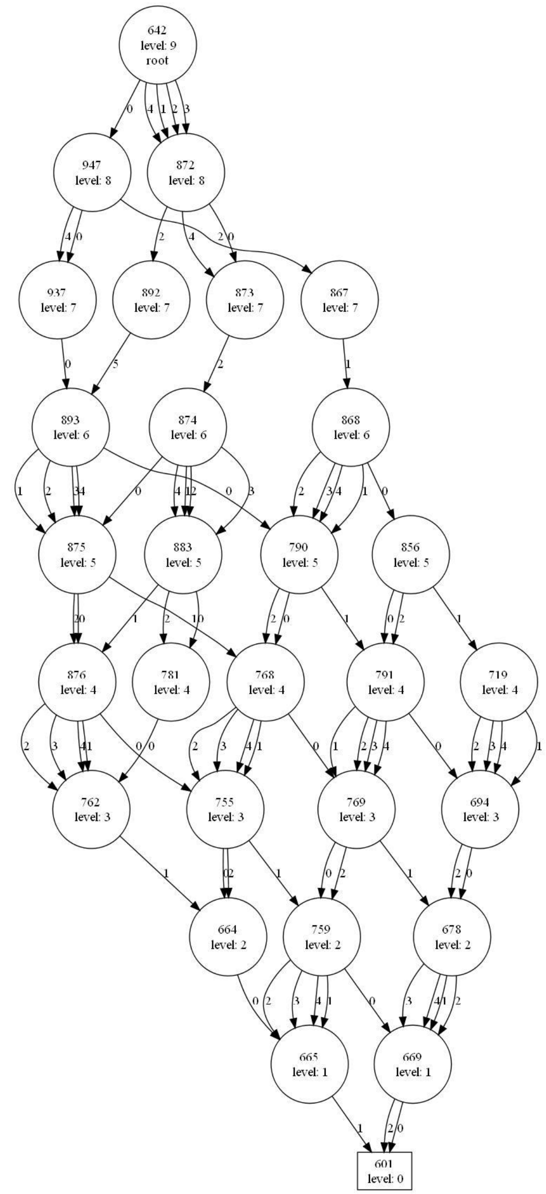
Experiments

- Slotted Ring communication protocol
  - 2 slots
- State space representation
  - 52 states
  - 14 nodes



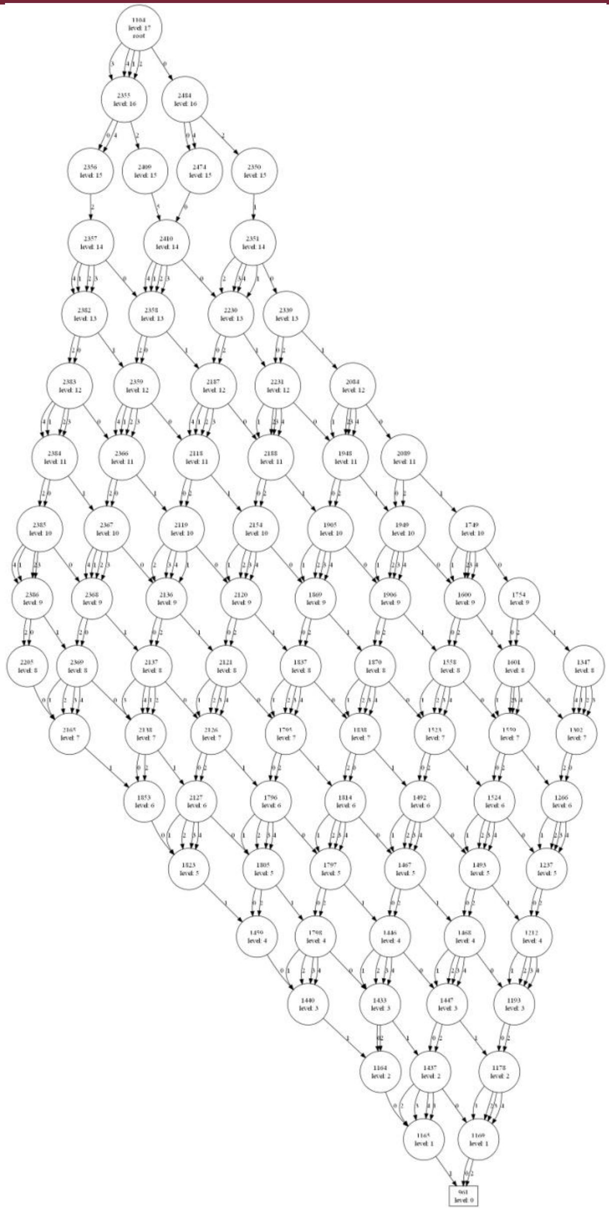
Experiments

- Slotted Ring communication protocol
  - 4 slots
- State space representation
  - 5136 states
  - 30 nodes



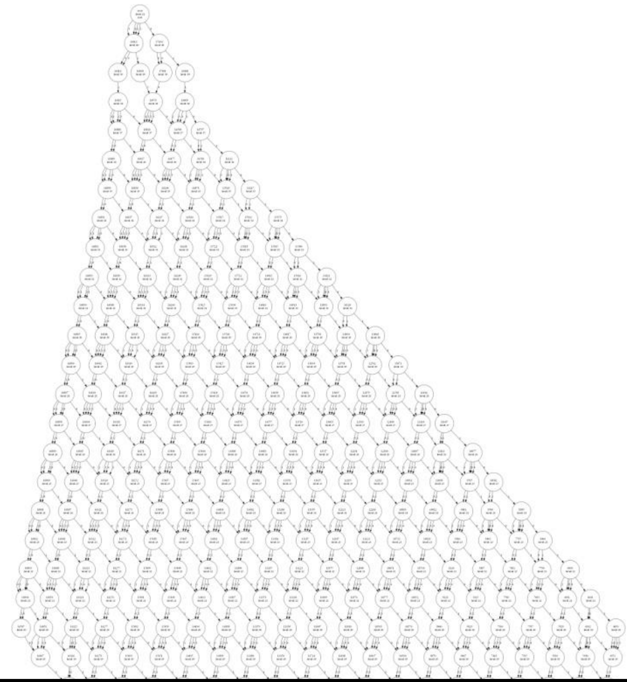
Experiments

- Slotted Ring communication protocol
  - 8 slots
- State space representation
  - 68,026,624 states
  - 103 nodes



Experiments

- Slotted Ring communication protocol
  - 20 slots
- State space representation
  - 10^{20} states
  - 487 nod

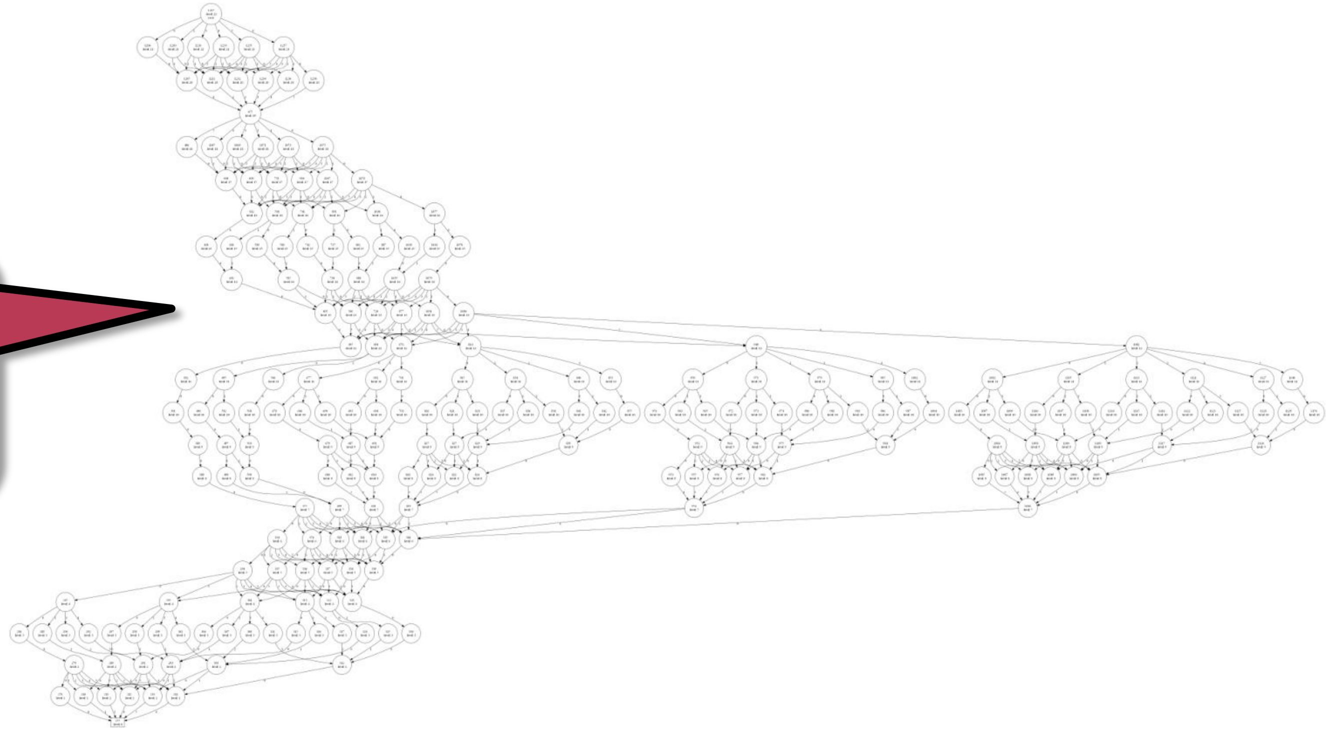


Scales up to about 200 slots in the ring and about 10^{200} states

(60902 nodes in the state space MDD, the full state space generation lasted 222 seconds long)

Experiments

- Flexible manufacturing system
  - 5 item
- State space representation
  - about 2,900,000 states
  - 248 nodes



Not so nice, but still efficient ☺

Experiments

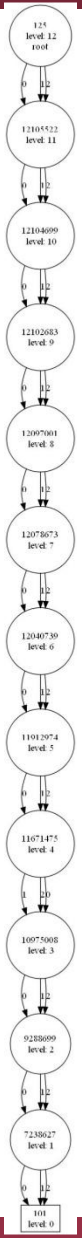
- Tower of Hanoi game
  - It consists of three rods, and a number of disks of different sizes which can slide onto any rod.
  - Rules:
    - Only one disk may be moved at a time
    - Each move consists of taking the upper disk from one of the rods and sliding it onto another rod, on top of the other
    - No disk may be placed on top of a smaller disk

Synchronous model:

- at most 4 transitions are enabled from each state

Experiments

- Tower of Hanoi game
  - 12 disks
- State space representation
  - 531 441 (3^{12}) states
  - 12 nodes
- **Unfortunately**:
  - during the exploration we construct more nodes
  - the state space generation took 58 seconds
  - huge number of transitions in the model



Experiments

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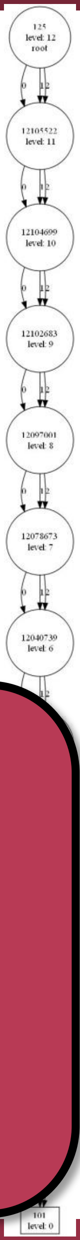


Conclusion:

Efficient state space representation

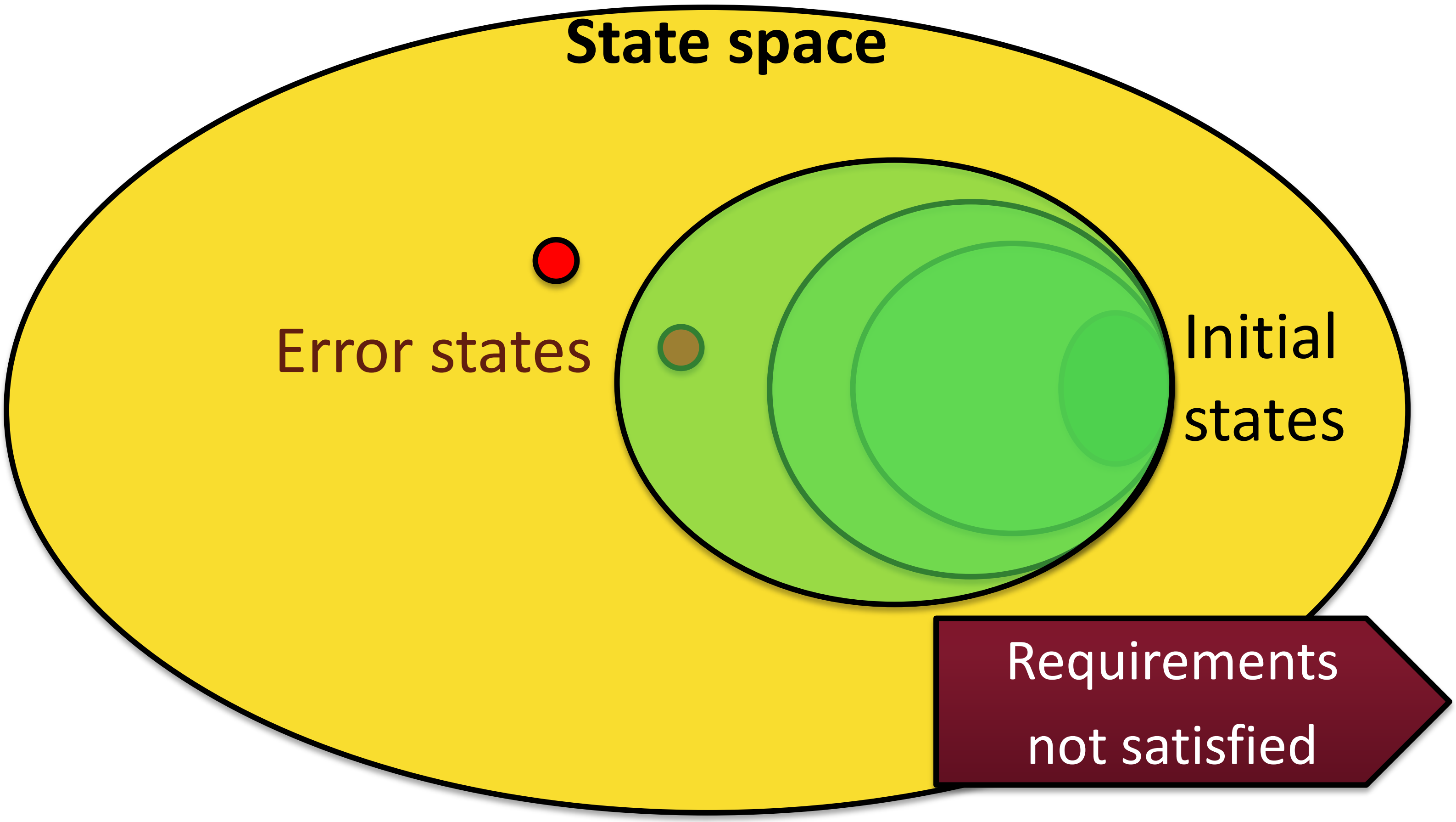
Efficient iteration

- For asynchronous models -

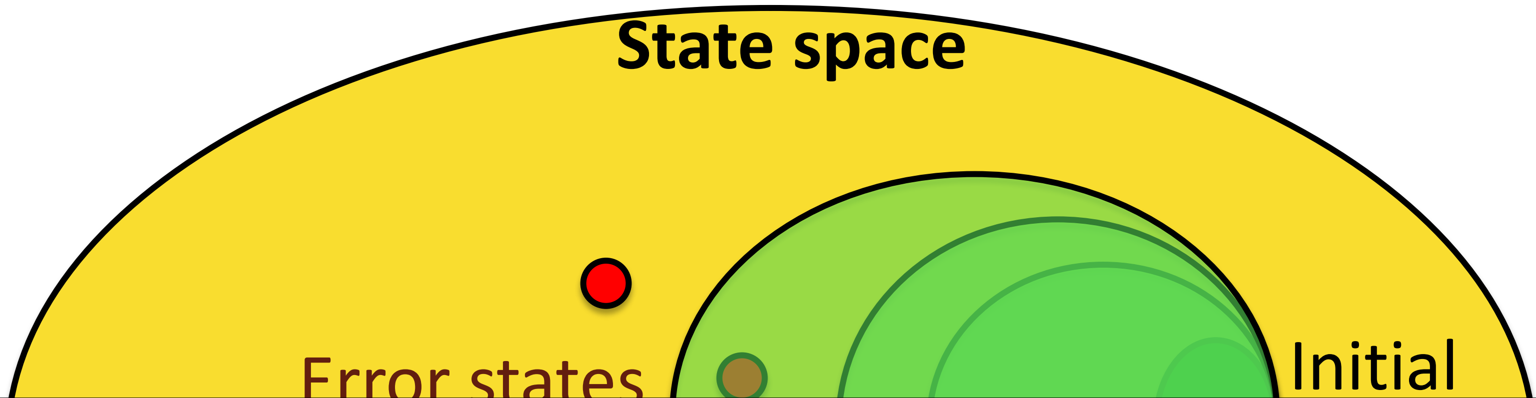
Problems

- Efficiency of the algorithm highly depends:
  - Decomposition
  - Variable ordering
    - Bottleneck of symbolic methods
- Best performance if this information is provided manually

Motivation for bounded model checking



Motivation for bounded model checking



Bounded model checking (BMC)

- explores a *k*-bounded part of the state space (usually in a breadth first manner)
- examines the specification on this smaller part

Bounded Saturation

- Bounded model checking
  - explores a k-bounded part of the state space
    - usually in a breadth first manner
  - examines the specification on this smaller part
- Saturation
  - Explores the state space in an irregular recursive order
  - Difficult to bound the exploration
  - There is no distance information in the MDD-s

Bounded Saturation

- Bounded model checking
  - explores a k-bounded part of the state space
    - usually in a breadth first manner
  - examines the specification on this smaller part
- Saturation
  - Explores the state space in an irregular recursive order
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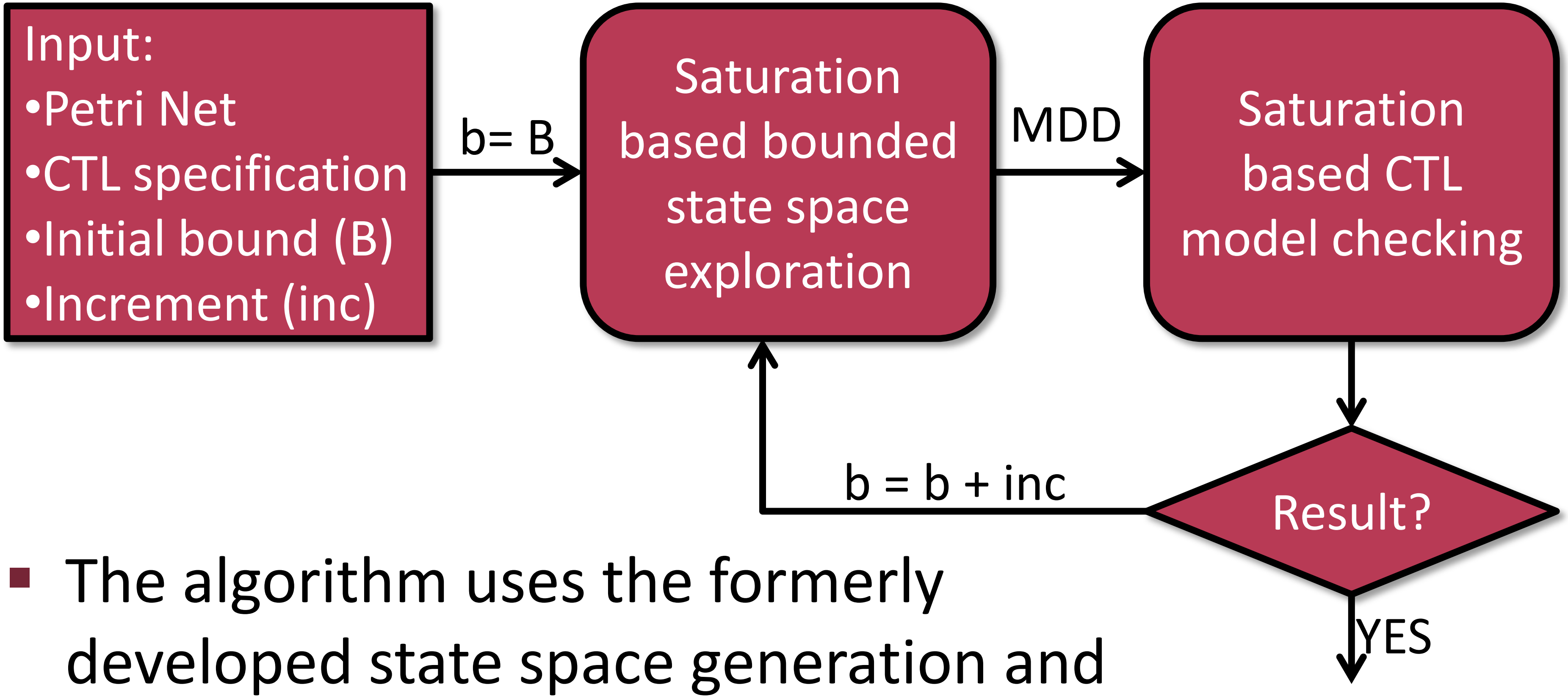
  - T

New data structure:

Edge Valued Decision Diagrams (EDDs)

- MDD based data structure enriched with distance information

Saturation Based bounded model checking



- The algorithm uses the formerly developed state space generation and CTL model checking algorithms
- Efficient for some models and specifications

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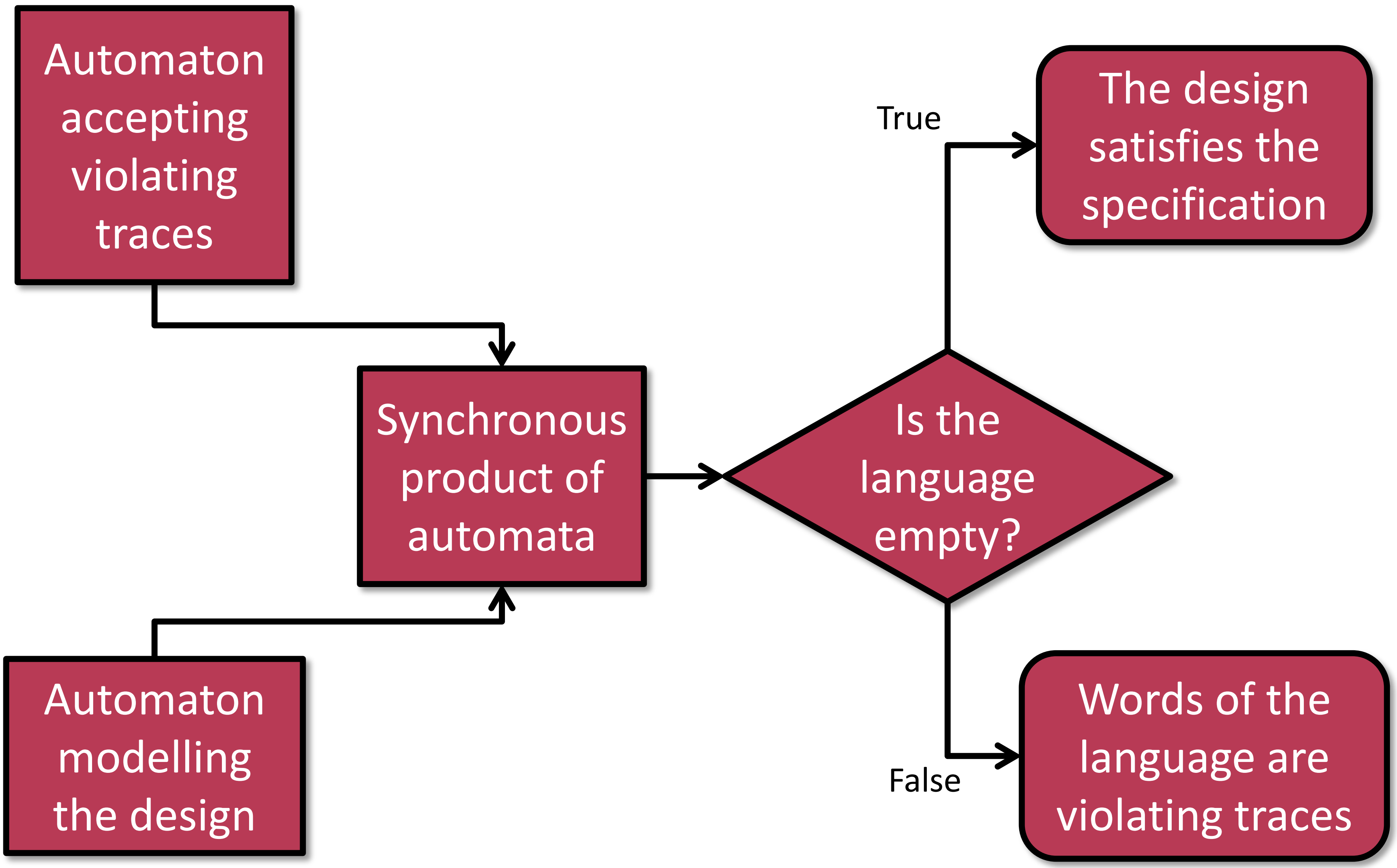
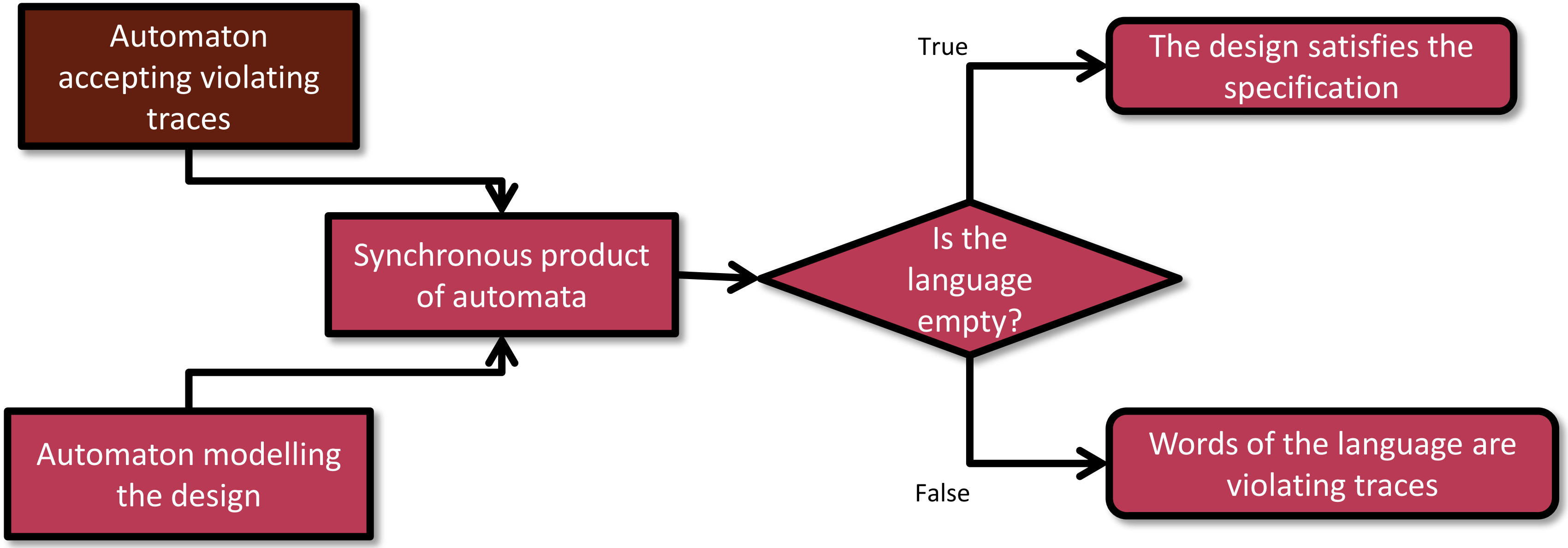
Automata theoretic model checking

Verifying ω-regular properties

Traditional approach

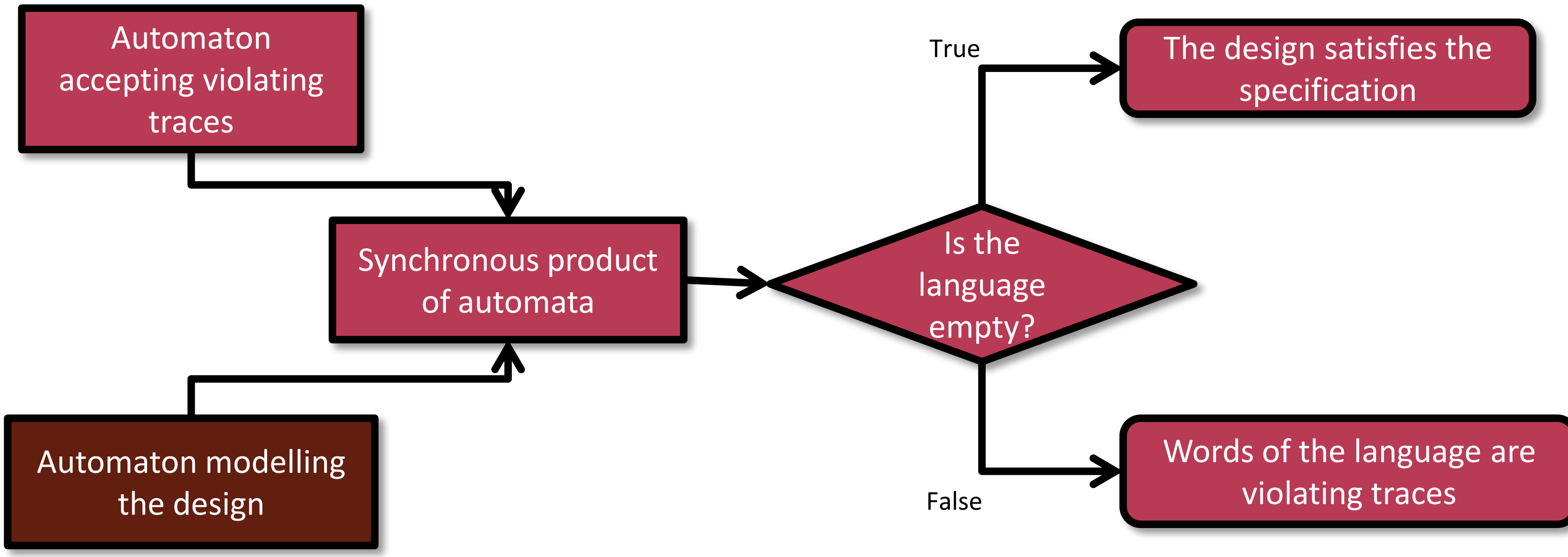


Detailed steps



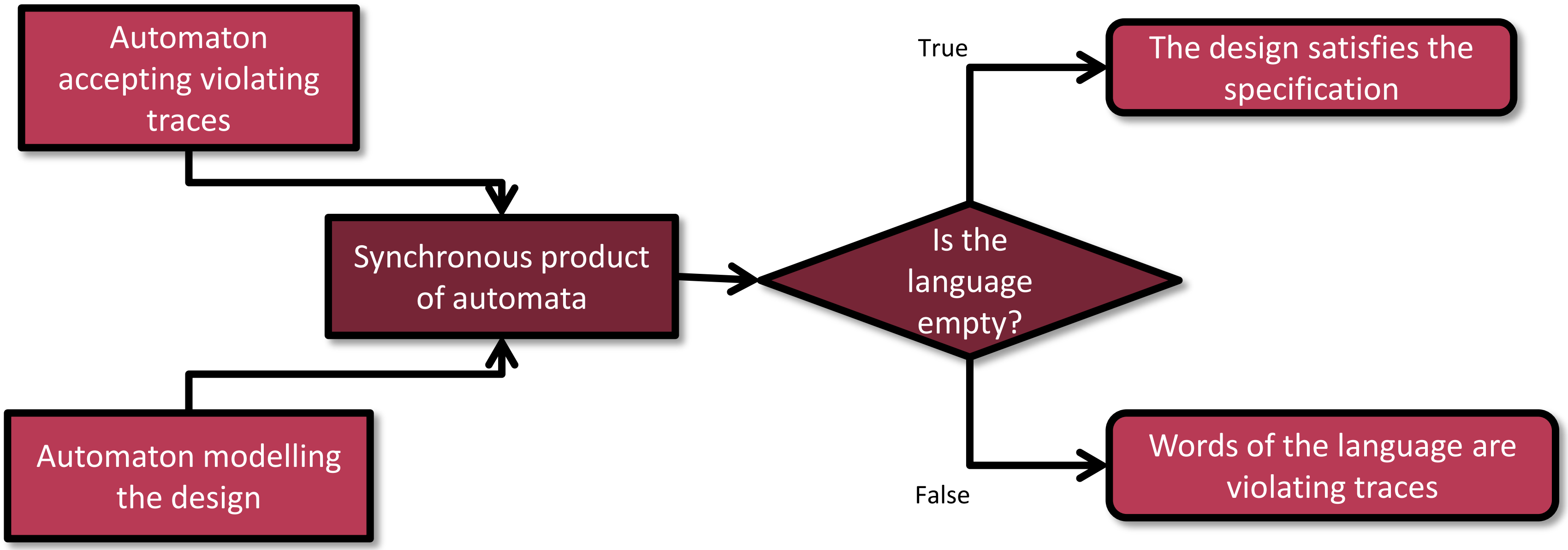
- Can be obtained by:
  - Complementing the Büchi-automaton describing the specification (not always possible)
  - **Compiling the negation of the specification described in linear temporal logic**
- Compilation algorithm and optimization is crucial
  - It is the „exponential part" of the model checking process
  - Product computation complexity is linear in the size of the automaton

Detailed steps



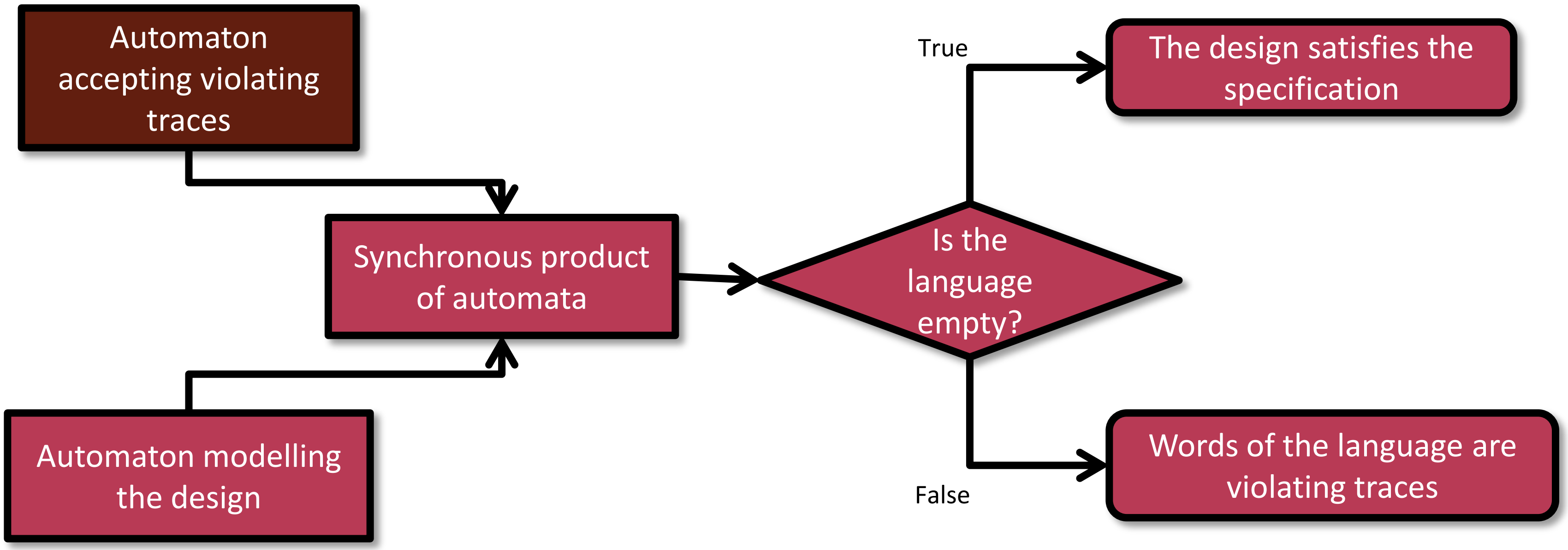
- Can be obtained from higher level models by the means of state-space generation
  - State space can be huge, especially in asynchronous systems
- Symbolic representation can solve storage problems
  - But it complicates the generation process
- Saturation algorithm provides efficient symbolic state space exploration
  - Especially in asynchronous systems…

Detailed steps



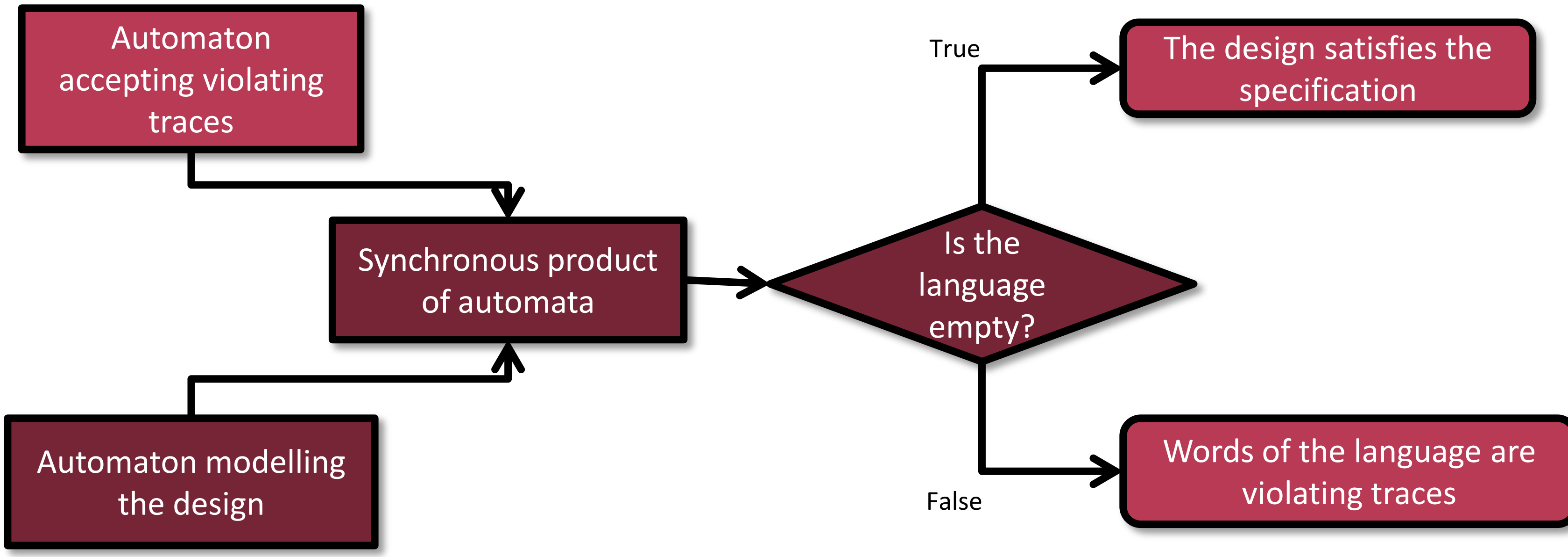
- Product state space is the Cartesian product of the sets of states of the automata
  - But only a part is reachable from the initial state
- A word of the product language is an accepting trace of the product automaton
  - An accepting trace is a reachable loop that contains accepting states
- Task: find such a loop or prove that there does not exist one
  - Efficient on-the-fly algorithms on explicit graph representations
  - Symbolic approaches are cumbersome

Detailed steps



- Compilation algorithm handling linear temporal logics (LTL)
  - Past and future time linear temporal expressions
- Heavy optimization and minimization of the specification automaton
  - Reduction of the temporal expressions
  - Removing redundancy from the automaton
- **A compiled and optimized automaton can be used multiple times!**

Detailed steps



- Product state space is generated together with the system state space
  - Using a modified version of the saturation algorithm
- Emptiness checking is performed simultaneously during the generation process
  - Unlike traditional symbolic approaches, accepting loops are being detected incrementally
  - State space generation halts immediately when a loop is found
- **This gives us an incremental, on-the-fly LTL model checking process**
  - Supporting LTL, PLTL and any ω-regular specification properties