| Formal Methods |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| First Mid-term Exam | 1. | 2. | 3. | 4. | 5. | 6. | $\Sigma$ |
| Name: |  |  |  |  |  |  |  |
| NEPTUN code: | 10 points | 6 points | 8 points | 6 points | 12 points | 8 points | 50 points |

1. Theoretical questions ( $\mathbf{1 0}$ points)
1.1. For each of the following statements indicate (with an X ) whether it is true, false or not decidable.

| Statement | True | False | Not decidable |
| :--- | :---: | :---: | :---: |
| Each transition in a Labeled Transition System (LTS) can <br> have only one action assigned. | X |  |  |
| If bounded model checking finds a counterexample for an <br> invariant property, it may be the case that it is not a <br> counterexample if we use unbounded (full) model checking. |  | X |  |
| Changing the order of variables in an ROBDD will always <br> yield an ROBDD with the same size. |  | X |  |

1.2. Give a sequence of labeled states for which the properties $\mathbf{G}(\mathbf{P} \vee \mathbf{Q})$ and $\mathbf{X X}(\mathbf{P} \mathbf{U} \mathbf{Q})$ hold, but the property $\mathbf{X X} \mathbf{Q}$ does not hold, using as few states as possible!

3 points

1.3. Give an example for a temporal logic expression that is syntactically valid in CTL* but invalid in CTL. Explain why it is invalid in CTL!
e.g.: $\mathbf{A}(\mathbf{X X} p)$ or $\mathbf{E}(\mathbf{X} p \vee \mathbf{F} q)$, because the path expressions may not be combined in CTL.
1.4. Describe the behavior of committed states in timed automata (of UPPAAL)!

The incoming and outgoing transitions are executed as an atomic step, i.e., no other automaton can fire its transition while the committed location is active.

The following figures show two timed automata (modeled in UPPAAL) that describe the states of the controller of a cooler (Idle, Cooling or Finished) and the states of the water in a water tank (Empty, Warm, Cold or Frozen). The automata use a single logical variable (bool frozen), and three channels (chan refill, freeze, finished). The logical variable is initially false. Note that guards use " $==$ " whereas assignments use " $=$ ".

2.1. Construct the Kripke structure corresponding to the whole system, i.e., reachable combinations of the states of the controller and states of the water tank, including the transitions! Label each combined state with the names of the states that it represents (you can use the initial letters of the states)!


## 3. Binary Decision Diagrams ( 8 points)

Please provide the solution on a new sheet!
A Kripke structure is given in the left side of the figure, where the states are encoded in three bits using the variables $x, y, z$ (for example 010 corresponds to $x=0, y=1, z=0$ ).

3.1. Give the characteristic function for the initial state of the Kripke structure!

Give the characteristic function for the transitions outgoing from the initial state!

$$
\begin{gathered}
C_{000}=\neg x \wedge \neg y \wedge \neg z \\
C_{000 \rightarrow 001}=(\neg x \wedge \neg y \wedge \neg z) \wedge\left(\neg x^{\prime} \wedge \neg y^{\prime} \wedge z^{\prime}\right) \\
C_{000 \rightarrow 010}=(\neg x \wedge \neg y \wedge \neg z) \wedge\left(\neg x^{\prime} \wedge y^{\prime} \wedge \neg z^{\prime}\right)
\end{gathered}
$$

3.2. Draw the ROBDD representing the states of the Kripke structure! Use the following order for the variables: $x, y, z$ !

3.3. Give the ROBDD corresponding to the intersection of the states of the Kripke structure (constructed in the previous task) and the states encoded by the ROBDD in the right-hand side of the figure, using ROBDD operations! The variable order should remain $x, y, z$.


## 4. CTL model checking ( 6 points)

Consider the following Kripke structure:

4.1. Check if the following CTL expression holds from the initial state using the iterative labeling algorithm presented in the lectures: $\mathbf{E}((\mathbf{p} \vee \mathbf{q}) \mathbf{U}(\mathbf{A X q}))$.
For each iteration give the expression that is currently used for labeling and enumerate the states that are labeled!

1. iteration: (labeling with $\mathbf{p} \vee \mathbf{q}$ )

S, A, C, D
2. iteration: (labeling with $\mathbf{A X} \mathbf{q}$ )

B, D
3. iteration: (labeling with $\mathbf{E}((\mathbf{p} \vee \mathbf{q}) \mathbf{U}(\mathbf{A X q})))$
$\mathrm{B}, \mathrm{D} \quad$ (because of $\mathbf{A X} \mathbf{q}$ )
A, C (first iteration backward, both predecessors of B or D and labeled with $\mathbf{p} \vee \mathbf{q}$ )
$\mathrm{S} \quad$ (second iteration backward, predecessor of A and labeled with $\mathbf{p} \vee \mathbf{q}$ )

## 5. LTL requirement formalization ( 12 points) Please provide the solution on a new sheet!

On each day we record the status of the weather and our equipment to protect ourselves. The weather can be sunny, windy or rainy. Our equipment can be an umbrella and/or a coat.
Use LTL expressions to formalize the following three requirements, which must apply to the behavior of the system in every moment!
5.1. If the weather is rainy we will eventually take an umbrella or a coat with us.

$$
\mathbf{G}(\text { rainy } \Rightarrow \mathbf{F}(\text { umbrella } \vee \text { coat }))
$$

5.2. If the weather is rainy or windy we are taking our coat with us until the weather becomes sunny.

$$
\mathbf{G}((\text { rainy } \vee \text { windy }) \Rightarrow(\text { coat } \mathbf{U} \text { sunny }))
$$

5.3. If the weather is sunny for three consecutive days, we are not bringing our umbrella on the second day and we will not take an umbrella, neither a coat on the third day.

$$
\mathbf{G}((\text { sunny } \wedge \mathbf{X} \text { sunny } \wedge \mathbf{X X} \text { sunny }) \Rightarrow(\mathbf{X}(\neg \text { umbrella }) \wedge \mathbf{X X}(\neg \text { umbrella } \wedge \neg \text { coat })))
$$

5.4. Use the tableau method to check if the requirement $\neg(\mathbf{x ~ U} \mathbf{y})$ holds for the Kripke structure below! Explain and document your solution! If the requirement does not hold, give a counterexample based on the tableau!

Negated requirement: $\neg(\neg(\mathbf{x} \mathbf{U} \mathbf{y}))=\mathbf{x} \mathbf{U} \mathbf{y}$
Negated normal form: $\mathbf{x} \mathbf{U} \mathbf{y}$


## 6. UML statecharts (8 points)

Consider the following statechart, in which for all states $s_{k}$ there is also an entry action $s_{k}$ entry and an exit action $s_{k}$ exit that is not displayed in the figure! The expressions on the arrows (transitions) have the following form: transition_name: trigger [guard] / action.


The statechart starts from the default initial state, the value of the logical variable " $a$ " is "false". The incoming event is " $y$ ".
6.1. Which transitions are enabled?

1 point
t1, t4, t6, t10, t17
6.2. Which enabled transitions are in conflict (cannot fire together)?

1 point
( $\mathrm{t} 17, \mathrm{t} 1$ ), ( $\mathrm{t} 17, \mathrm{t} 4$ ), ( $\mathrm{t} 17, \mathrm{t} 6),(\mathrm{t} 17, \mathrm{t} 10),(\mathrm{t} 10, \mathrm{t} 6),(\mathrm{t} 1, \mathrm{t} 4)$
6.3. What is the set of fireable transitions after resolving the conflicts? If there are multiple sets of fireable transitions, give all sets!
$\{\mathrm{t} 1, \mathrm{t} 6\},\{\mathrm{t} 4, \mathrm{t} 6\}$
6.4. What is (are) the next stable state configuration(s)? If there are more than one possible stable state configurations give only one of them! Give the actions and their order during firing the transition! Do not forget to include the entry and exit actions!

1 point

3 points

For $\{\mathrm{t} 1, \mathrm{t} 6\}: \quad$ Next: $\{\mathrm{s} 0, \mathrm{~s} 3, \mathrm{~s} 4, \mathrm{~s} 8\}$ Actions: ( $\mathrm{s} 2 . e x i t, \mathrm{i}, \mathrm{s} 3 . e n t r y)|\mid(\mathrm{s} 6 . e x i t, \mathrm{~s} 5 . e x i t, ~ i, ~ s 8 . e n t r y)$
For $\{\mathrm{t} 4, \mathrm{t} 6\}:$ Next: $\{\mathrm{s} 0, \mathrm{~s} 1, \mathrm{~s} 4, \mathrm{~s} 8\}$ Actions: (s2.exit, i, s1.entry) || (s6.exit, s5.exit, i, s8.entry)
6.5. The next incoming event is again " $y$ ". Give the set of fireable transitions (after resolving conflicts) and the next stable configuration! If there are more than one fireable sets and next stable configurations, give all of them!

From $\{\mathrm{s} 0, \mathrm{~s} 3, \mathrm{~s} 4, \mathrm{~s} 8\}:$ Fireable: t 14 Next: $\{\mathrm{s} 0, \mathrm{~s} 3, \mathrm{~s} 10, \mathrm{~s} 12\}$
From $\{\mathrm{s} 0, \mathrm{~s} 1, \mathrm{~s} 4, \mathrm{~s} 8\}:$ Fireable: t 14 Next: $\{\mathrm{s} 0, \mathrm{~s} 1, \mathrm{~s} 10, \mathrm{~s} 12\}$

