Model Checking

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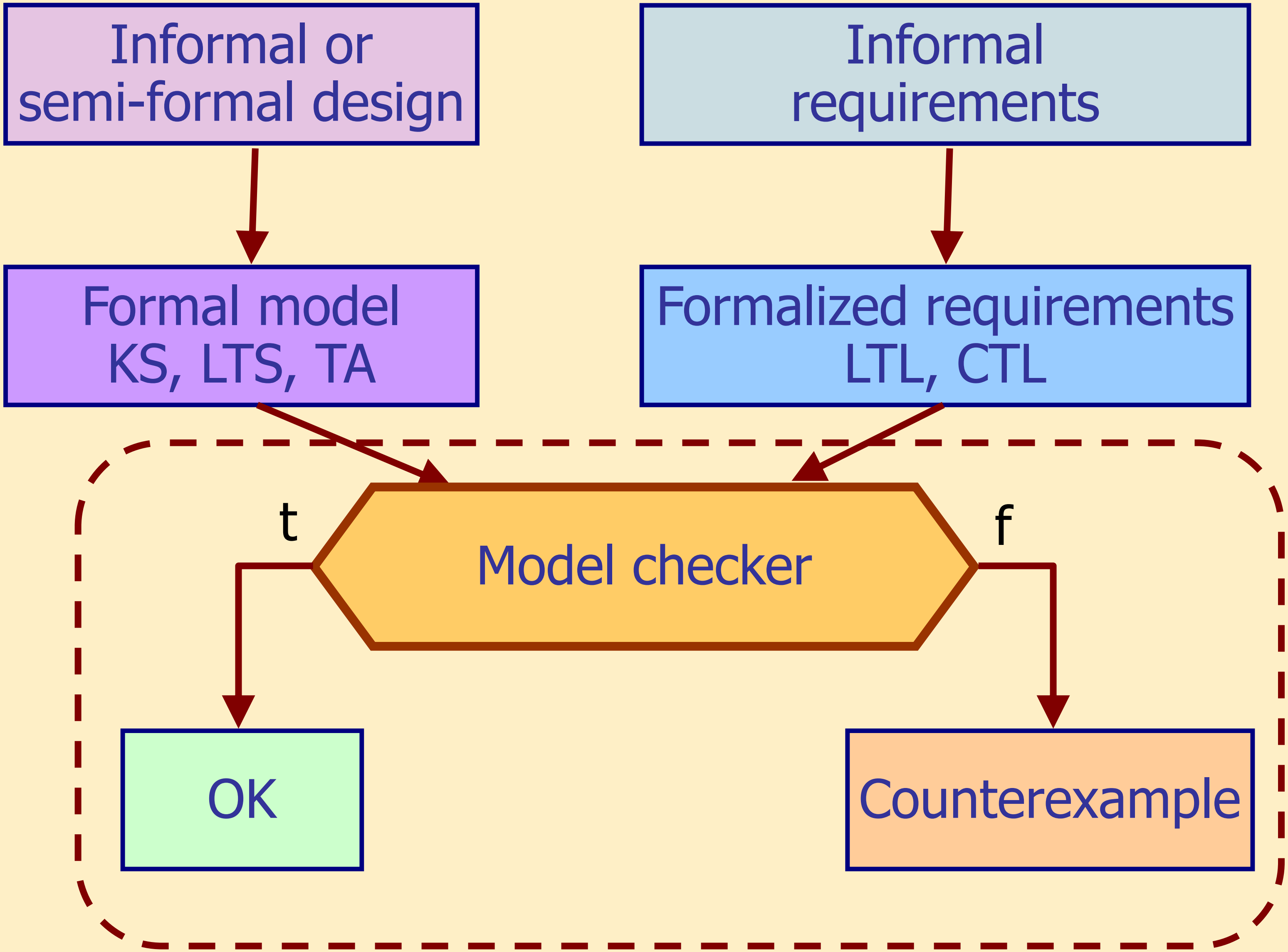
Our goal

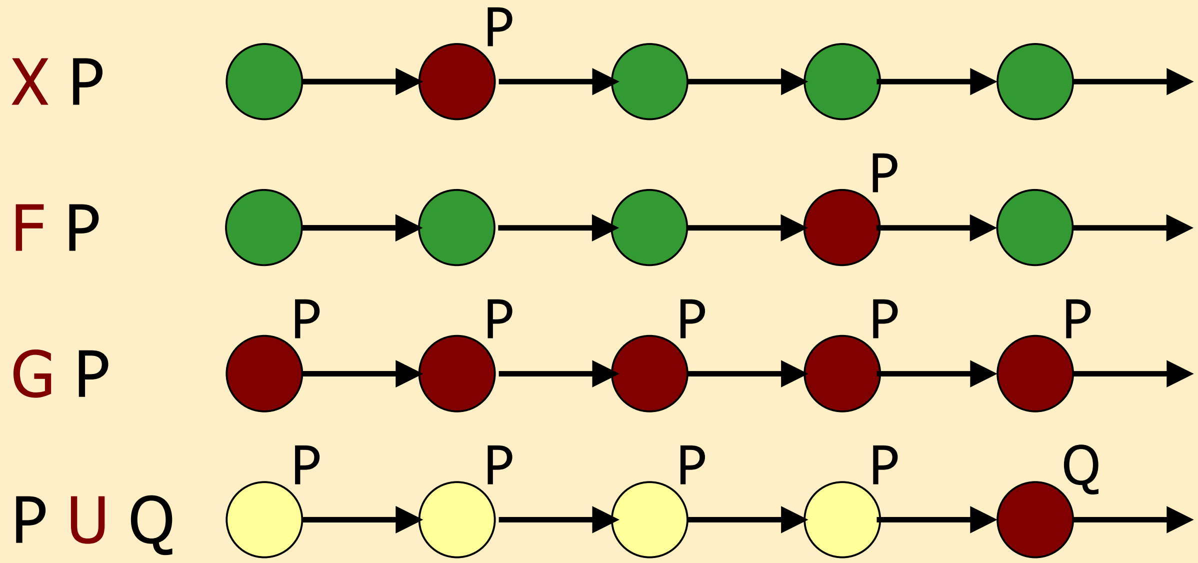


Informal or
semi-formal design
Informal
requirements
Formal model
KS, LTS, TA
Formalized requirements
LTL, CTL
t
Model checker
f
OK
Counterexample

Recap: linear temporal logic LTL

Elements of LTL:

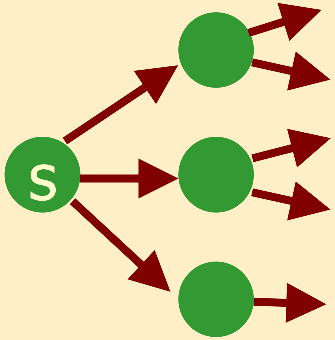
- Atomic propositions (elements of AP): P, Q, ...
- Boolean connectives: $\wedge$, $\vee$, $\neg$, $\Rightarrow$
  $\wedge$: conjunction, $\vee$: disjunction, $\neg$: negation , $\Rightarrow$: implication
- Temporal connectives: X, F, G, U:



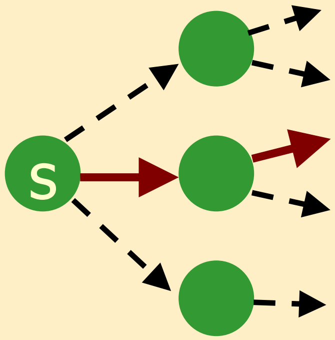
Recap: branching time temporal logic CTL*

Elements of CTL*:

- Path quantifiers:
  - A: for All paths
    starting from the current state



  - E: there Exists a path
    starting from the current state



- Path-specific operators (as in LTL):
  - X p, F p, G p, p U q

Recap: branching time temporal logic CTL

Elements of CTL:

Composite operators over states

- EX p: there exists a path where p holds in the next state
- EF p: there exists a path where p holds in the future
- EG p: there exists a path where p holds globally
- E(p U q): there exists a path where p holds until q eventually holds

- AX p: for all paths p holds in the next state
- AF p: for all paths p holds in the future
- AG p: for all paths p holds globally
- A(p U q): for all paths p holds until q eventually holds

Overview

Mechanics of model checking

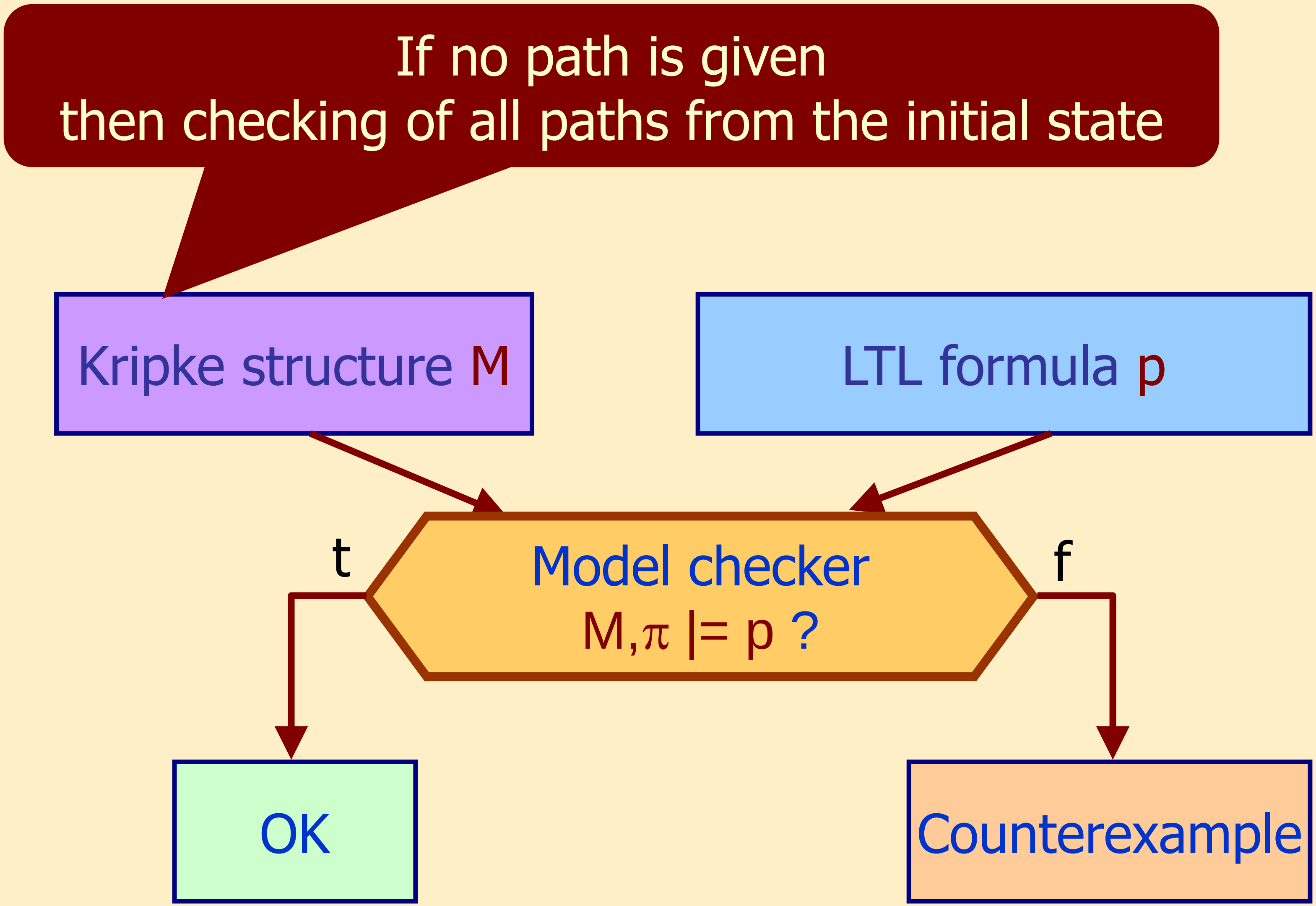
- Techniques for model checking
  - LTL: Semantic tableau
  - CTL: Labeling

Why is this useful?

- Possibilities, determining boundaries
  - Discovering boundaries (e.g. size of verifiable models)
  - Efficient implementation (10^{69000} states? – next lecture)
- Interesting applications (later)
  - Automatic test case generation
  - Synthesis of runtime monitors

LTL Model Checking using Semantic Tableau

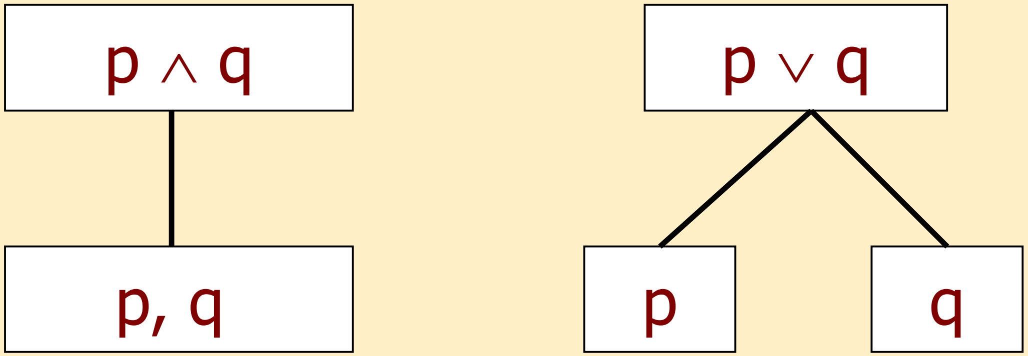
LTL model checking



Introduction: Semantic Tableau for Propositional Logic

Problem: satisfiability in propositional logic

- Idea: decomposition of the formula to a tree (the tableau)
  - Nodes: formulas to satisfy
  - Adding edges: decomposition rules based on the semantics of connectives
    Branching: more than one ways to satisfy a formula
- Before decomposition: negation normal form (NNF): negation only appears on atoms
  - de Morgan's law: $\neg(p \vee q) = (\neg p) \wedge (\neg q)$, $\neg(p \wedge q) = (\neg p) \vee (\neg q)$
- Decomposition rules for PL:



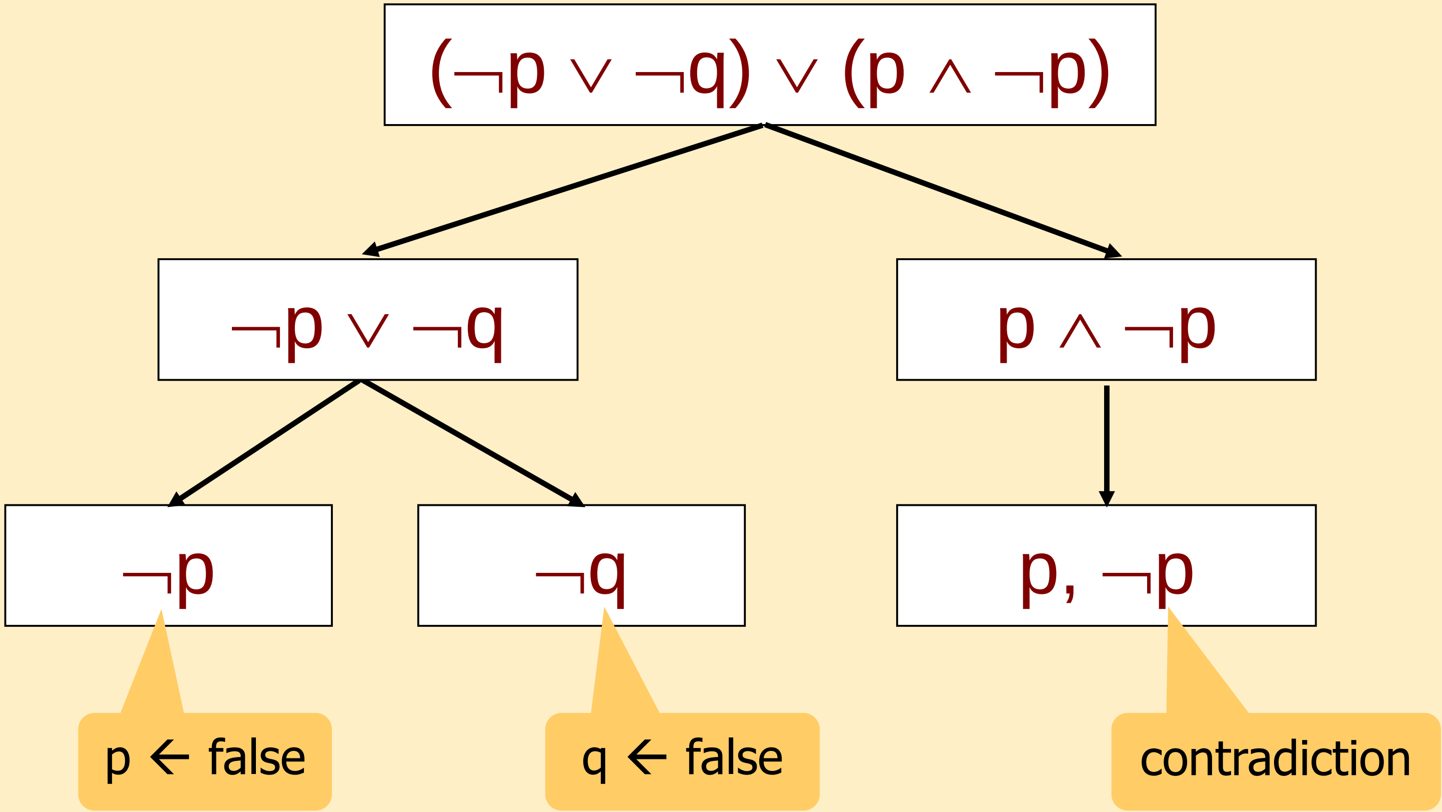
Introduction: Semantic Tableau for Propositional Logic

When to stop decomposing?

- Terminating a branch:
  - Only literals left
  - Each literal has to be satisfied by assigning values to variables
- After terminating a branch:
  - Contradiction: opposite literals
    - E.g. p, $\neg p$ is contradicting, no possible satisfying assignment
  - Successful branch: no contradiction
    - E.g.: for p, $\neg q$: $p \leftarrow$ true, $q \leftarrow$ false
    - This assignment is a model of the original formula
- Each successful branch corresponds to a satisfying assignment

Introduction: An example tableau for PL

- Original formula: $\neg(p \wedge q) \vee \neg(\neg p \vee p)$
- Pushing $\neg$ inwards: $(\neg p \vee \neg q) \vee (p \wedge \neg p)$
- Tableau construction:



Generalizing tableau construction to LTL

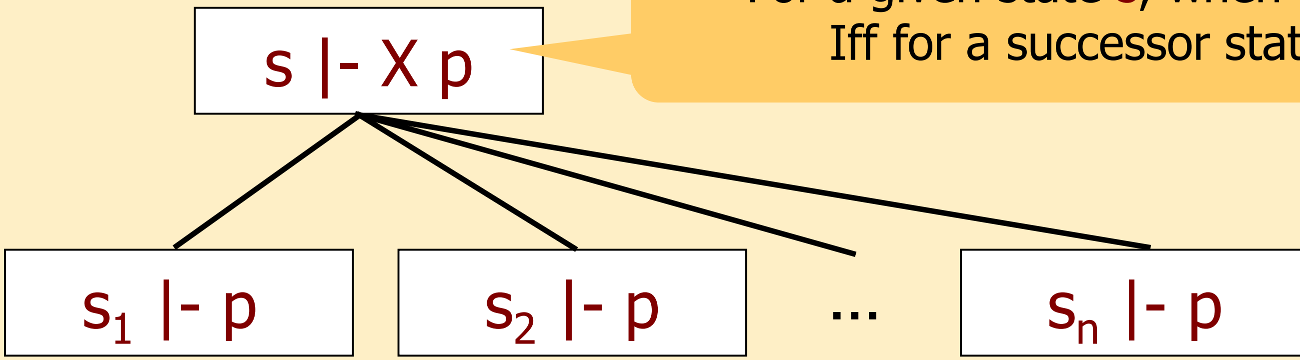
- Model checking: searches for a counterexample, thus

  The tableau is constructed for the negated formula!

  - The negated formula is transformed to NNF
  - If there exists a successful (not contradicting) branch, it induces a counterexample!
  - If all branches are contradicting, then the original property holds!
- Decomposition rules for temporal connectives
  - Novelty: Decomposition is performed based on the model
  - Notation: s |- p denotes that we evaluate p starting from state s
- Handling literals:
  - s |- P holds iff $P \in L(s)$
  - s |- $\neg$P holds iff $P \notin L(s)$
- Temporal operators:
  - Rules for X and U are sufficient (others can be derived)

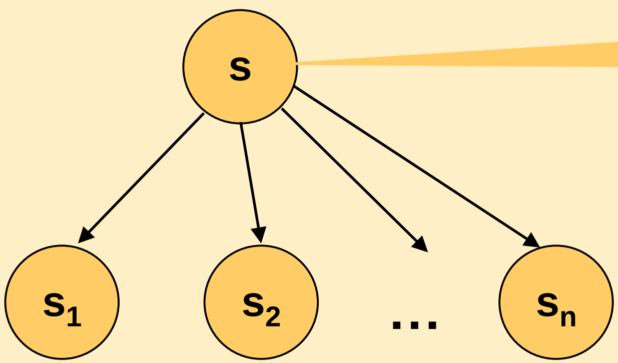
Decomposition for operator X

For a given state s, when does Xp hold?
Iff for a successor state p holds.



For model:

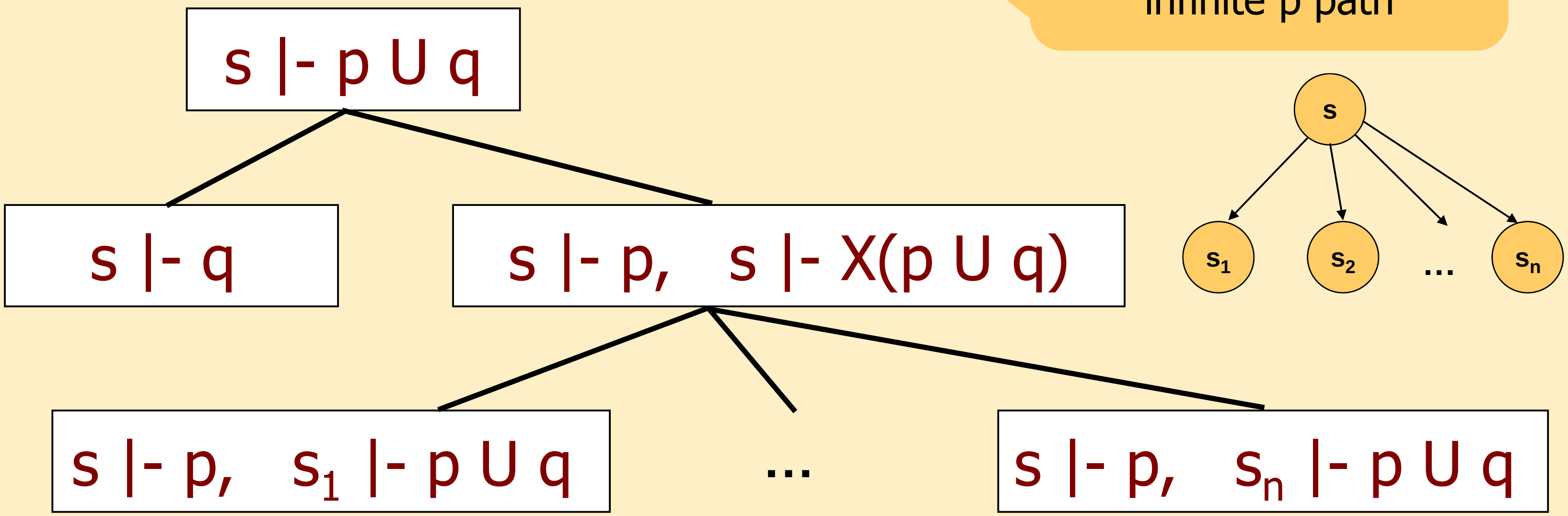
Direct contradiction,
if s has no successors



Decomposition for operator U

- We use: p U q = q ∨ (p ∧ X (p U q))

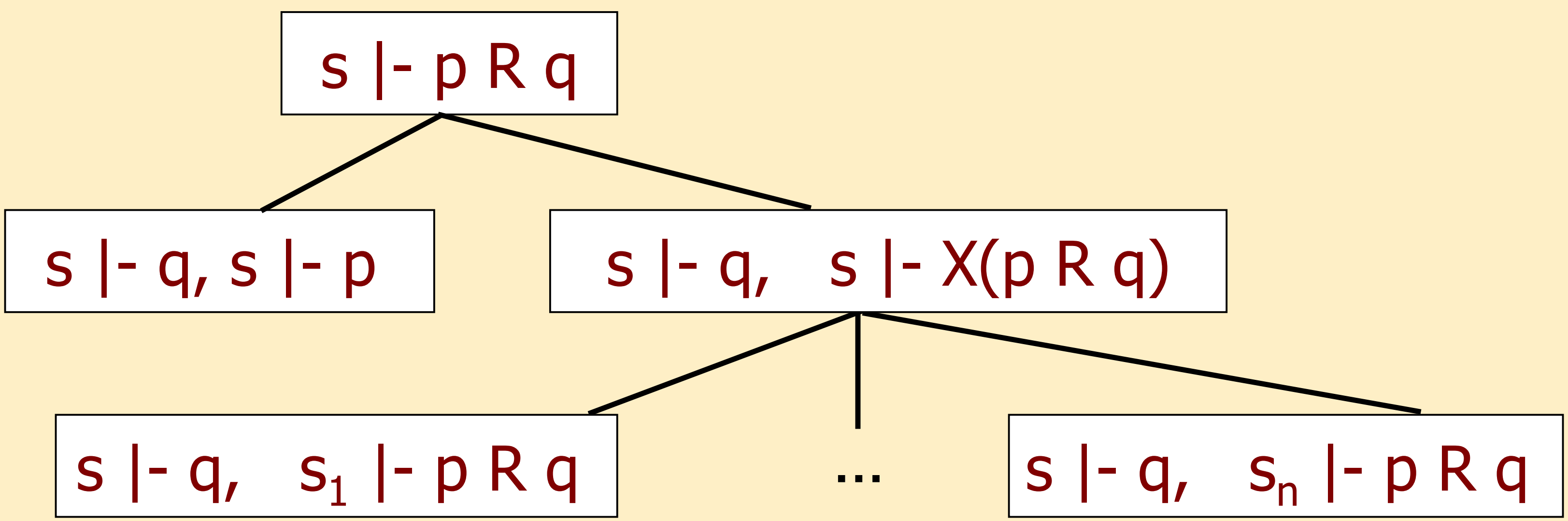
Needs extra attetnion:
finite path,
infinite p path



- When can we terminate?
  - Contradiction:
    - Atomic propositions contradict each other
    - Operator X – the path terminates without encountering q
    - Cycle of p states without encountering q
  - Successful branches:
    - Atomic propositions can be satisfied
    - Cycle without contradiction

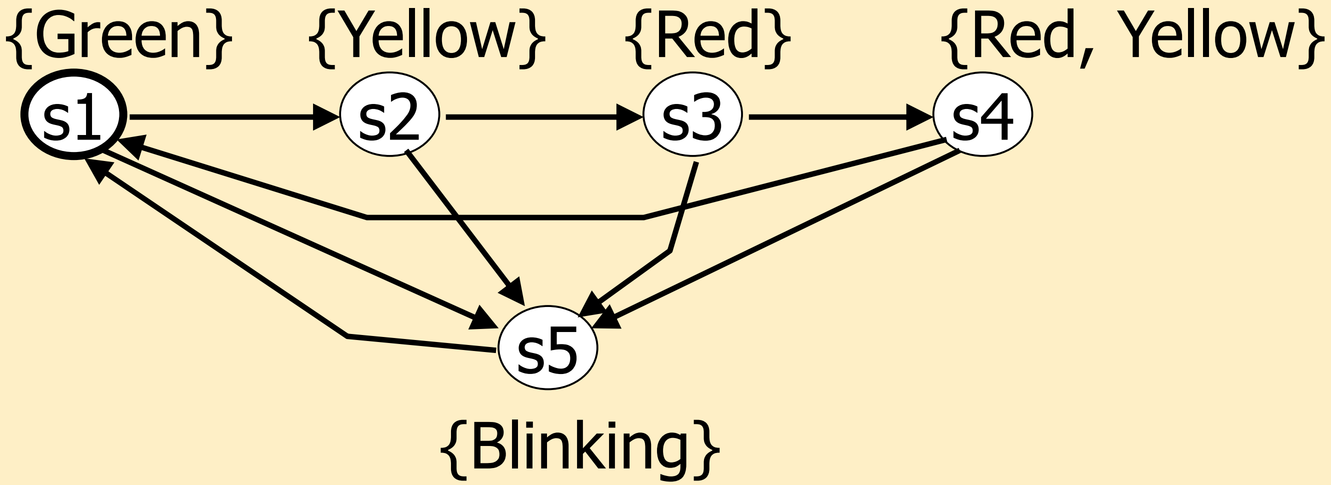
A special operator: R

- NNF for operator U:
  $\neg(p \; U \; q) = ?$
  - We introduce the dual of operator U: R (Release)
    $$\neg(p \; U \; q) = (\neg p) \; R \; (\neg q)$$
  - We use: $p \; R \; q = q \wedge (p \vee X \; (p \; R \; q))$
- The tableau for operator R:



An example

- Traffic light (KS)
- Is it true that if initially Green holds, then eventually Red will hold?
  - The formula to check: Green $\Rightarrow$ F Red



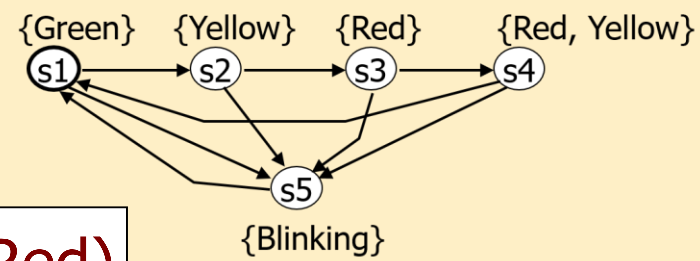
- Based on the model, can we construct a counterexample?

The tableau for the property

- Negation of the formula: $s_1 \mid\!- \neg(\text{Green} \Rightarrow \text{F Red})$
- NNF (based on $P \Rightarrow Q = \neg P \vee Q$):

$\neg(\text{Green} \Rightarrow \text{F Red}) = \text{Green} \wedge \neg\text{F Red} = \text{Green} \wedge \text{G}(\neg\text{Red})$

- Tableau construction:



$s1 \mid\!- \text{Green} \wedge \text{G}(\neg\text{Red})$

↓

$s1 \mid\!- \text{Green},\ s1 \mid\!- \text{G}(\neg\text{Red})$

↓

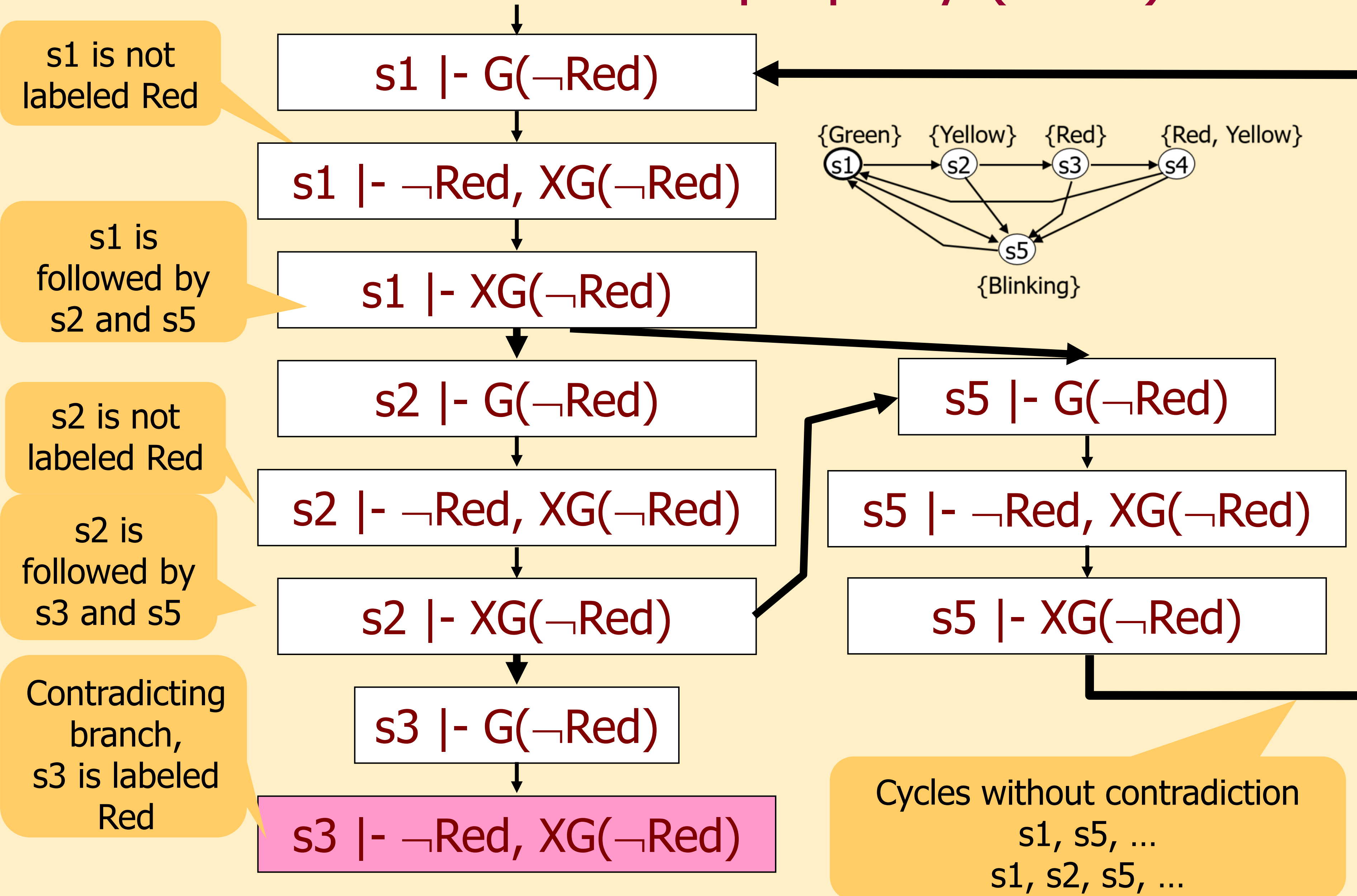
$s1 \mid\!- \text{G}(\neg\text{Red})$

↓

S1 is labeled Green

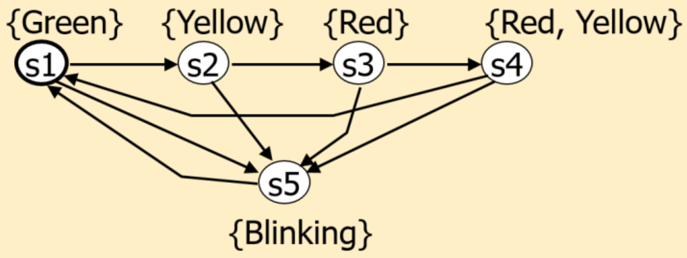
Simplification: $s1 \mid\!- \text{Green}$ removed

The tableau for the property (cont.)



The results of model checking

- The results of tableau for the negated formula:
  - A contradicting branch (here the property holds)
  - Two cycles without contradiction: counterexamples
- Conclusions:
  - There are executions where the negated property holds:
    Cycle 1: s1, s2, s5, …
    Cycle 2: s1, s5, …



- The original formula Green $\Rightarrow$ F Red thus fails
  - Counterexamples can be shown

Semantic tableau (summary)

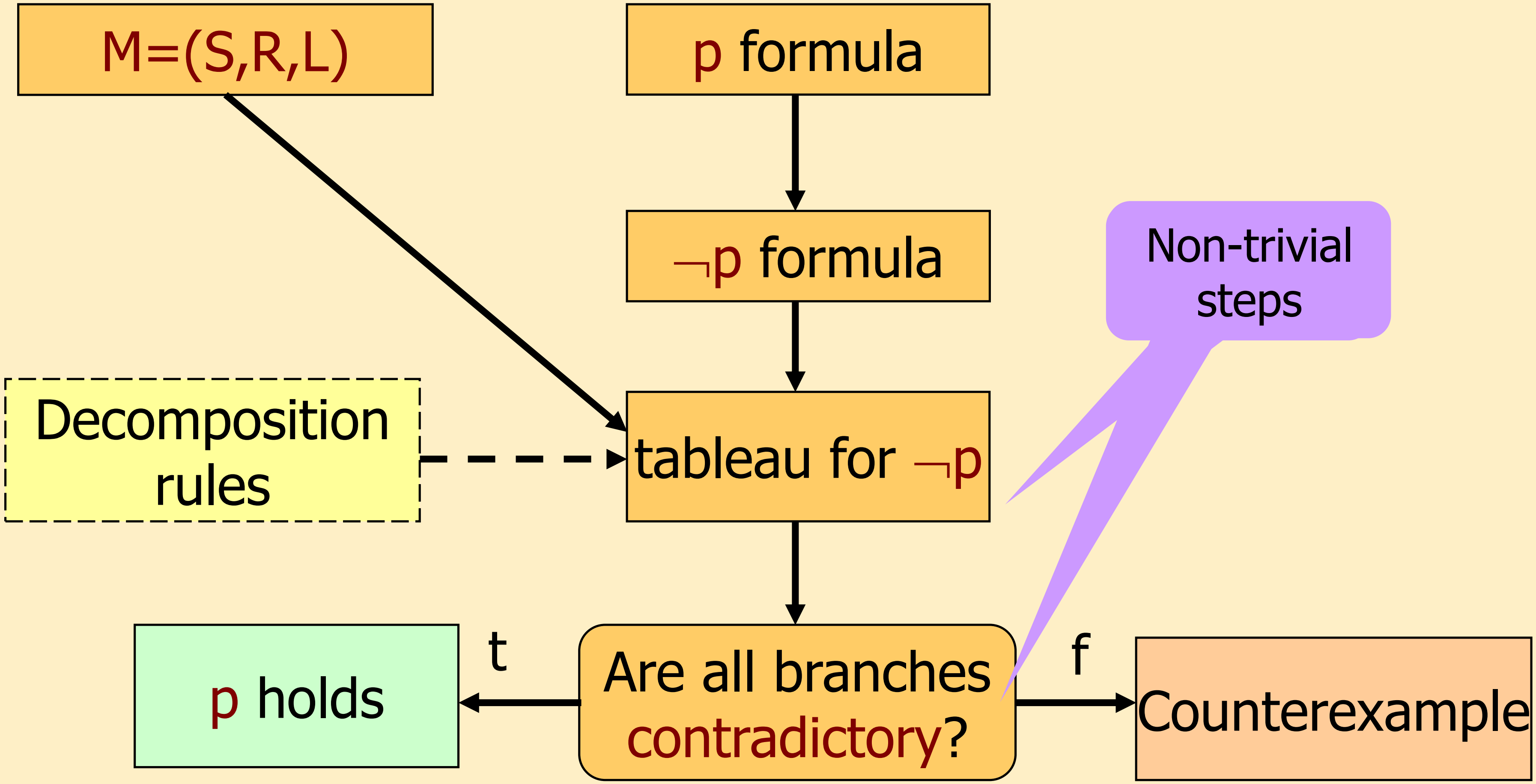
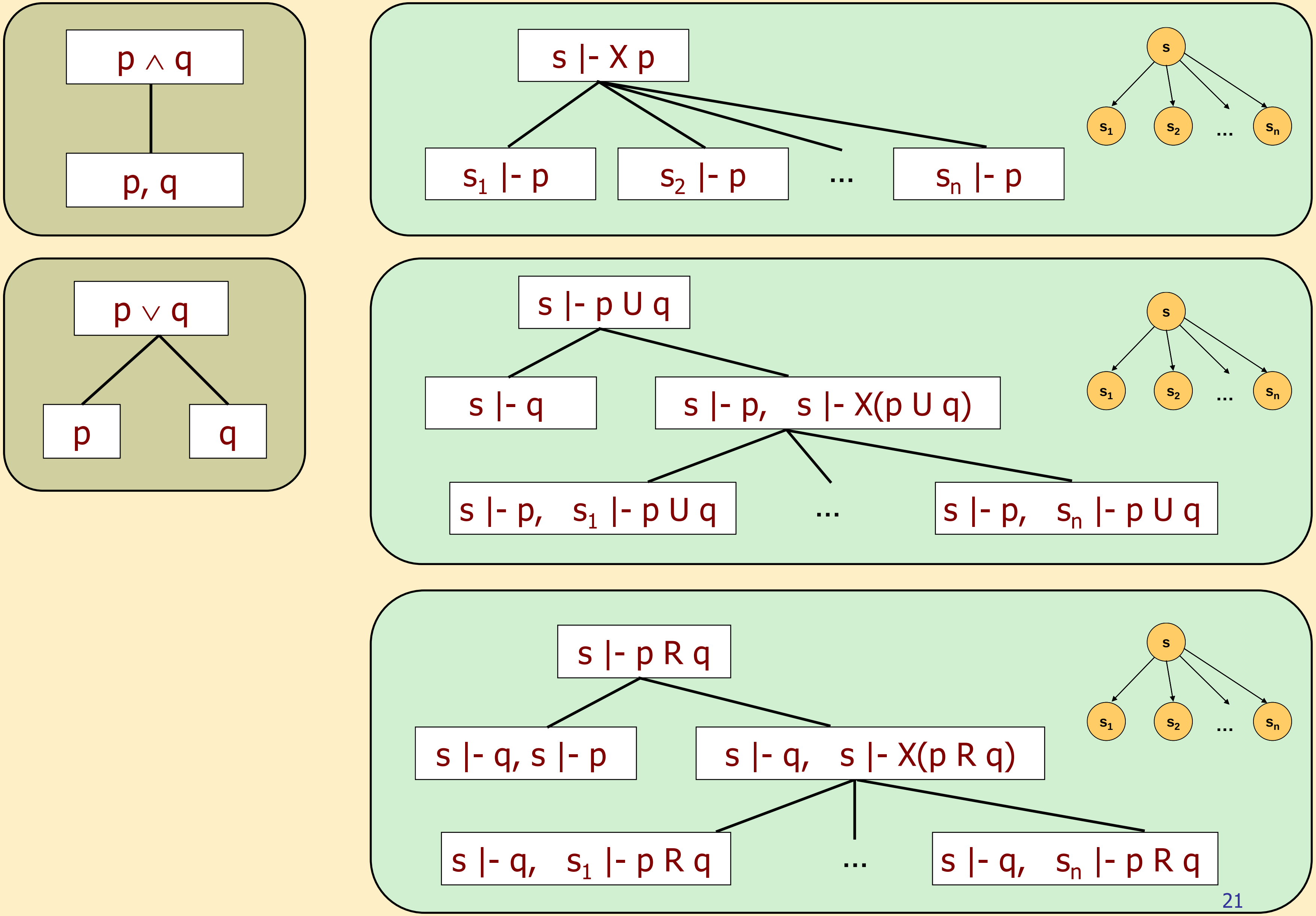
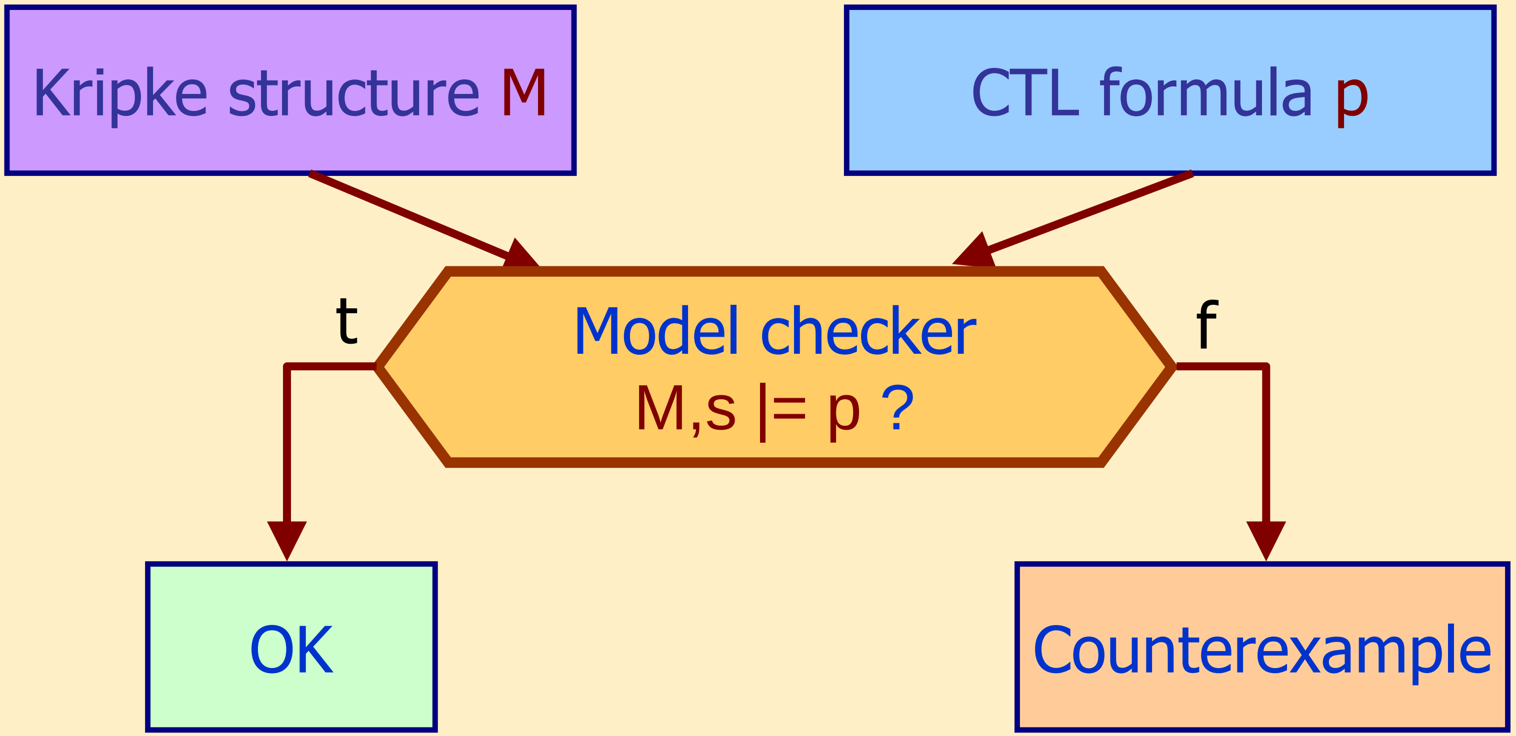


Tableau construction rules (summary)



CTL Model Checking Based on Labeling

CTL model checking



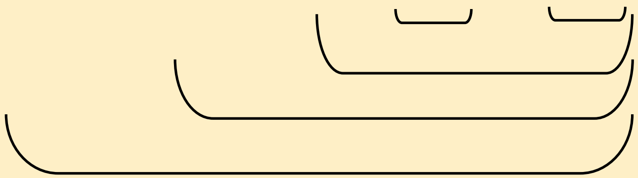
Idea: Labeling of states

- Global model checking:
  - Notation: Sat(p) denotes the set of states where CTL formula p holds
  - Labeling: we label these states by p
  - This way $s \in Sat(p)$ can be easily evaluated for a given state s (in particular for initial states): by checking whether it's labeled p
- The labeling, that is, Sat(p), is computed incrementally
  - We start from the labeling function L, and then expand it
  - The end of the iteration: fixed point reached

CTL model checking with state labeling

- Labeling of states: where the formula holds
- Labeling with complex formulas?
  - Decomposition of the formula based on its structure, and computing Sat() for subformulas (from the inside outwards):

$$AF\ (\ P \wedge E\ (Q\ U\ R))$$



- Algorithm based on the decomposition of the formula:
  - Base case: KS is labeled by atomic propositions
  - Continuation: labeling with more complex formulas
  - Rules: if we have established labels p and q then we can establish where we have labels
    $\neg p$, $p \wedge q$, EX p, AX p, E(p U q), A(p U q)
    This way we progress outwards from the inside of a complex formula

Rules: Atomic propositions and Boolean connectives

- P holds in a state s iff $P \in L(s)$
  - Here, P is already a label of s
- $\neg P$ holds in a state s iff $P \notin L(s)$
  - These states can be labeled $\neg P$
- $p \wedge q$ holds in a state s where p and q holds
  - A state can be labeled $p \wedge q$ iff it is already labeled p and q

- Temporal operators: EX, AX, E(U), A(U)
  - More complex labeling rules

Rules: AX, EX

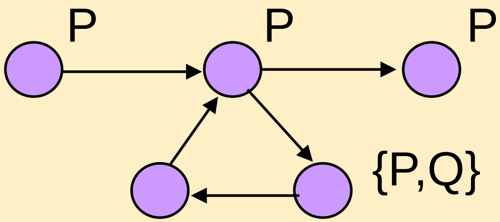
- EX p holds in a state s iff it has a successor where p holds
  - A state can be labeled EX p iff it has a successor labeled p

- AX p holds in a state s iff for all its successors p holds
  - A state can be labeled AX p iff all its successors are labeled p

Rules: E(p U q)

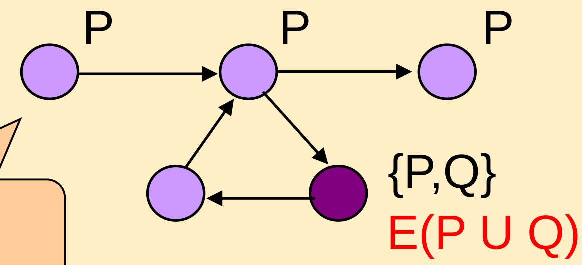
- Where does E(p U q) hold?
  - We use: E(p U q) = q $\vee$ (p $\wedge$ EX E(p U q))
  - "Recursive" formula
- So when can a state s be labeled E(p U q)?
  - if s is labeled q, or
  - if s is labeled p and there is at least one succeeding state (EX) that is already labeled E(p U q)
- An iteration arises:
  - States labeled q are the states where label E(p U q) first appears
  - We consider the predecessors of these states:
    If it is labeled p, we can add label E(p U q)
  - This way we traverse those paths backwards that lead to states labeled q through states labeled p

Labeling by E(P U Q)

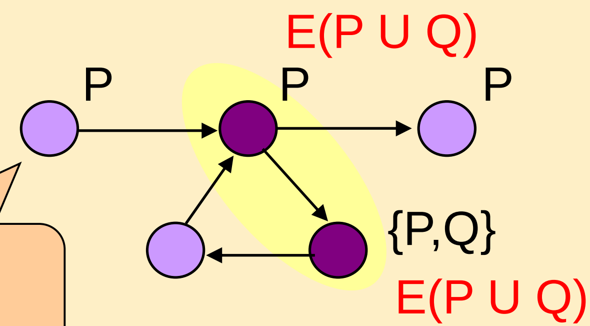


Kripke structure with initial labeling

First step: Q

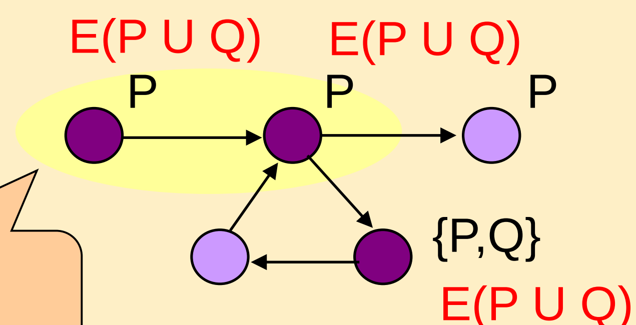


Second step: "P∧ EX"



- We iterate until a fixpoint is reached

Third step: "P∧ EX"



Rules: A(p U q)

- Where does A(p U q) hold?
  - We use: $A(p\ U\ q) = q \vee (p \wedge AX\ A(p\ U\ q))$
  - "Recursive" formula
- So when can a state s be labeled A(p U q)?
  - if s is labeled q, or
  - if s is labeled p and all succeeding states (AX) are already labeled A(p U q)
- An iteration arises:
  - States labeled q are the states where label A(p U q) first appears
  - We consider the predecessors of these states:
If it is labeled p, and all its successors are labeled A(p U q), we can add label A(p U q)

This way we covered all operators defined in the syntax.

An additional rule: AF p

- Where does AF p hold?
  - We use: AF p = p ∨ AX AF p
  - "Recursive" formula
- So when can a state s be labeled AF p?
  - if s is labeled p, or
  - all its successors (AX) are labeled AF p
- An iteration arises:
  - States labeled p are the states where label AF p first appears
  - We consider the predecessors of these states:
    If all its successors are AF p, we can add label AF p
  - This way we traverse those paths backwards that lead to a state labeled p

Iteration using set operations

- We expand the labeling using operations on sets
  - Initial set: states already labeled by subformulas
  - Expanding the labeling:
    - E(p U q): "At least one successor is labeled …"
    - A(p U q): "All successors are labeled …"
  - This way we can label preceding states
- How can we define the set of preceding states?
  - Based on set of already labeled states Z:

    $pre_E(Z) = \{s \in S \mid$ there exists s' such that $(s,s') \in R$ and $s' \in Z\}$

    $pre_A(Z) = \{s \in S \mid$ for all s' such that $(s,s') \in R$ we have $s' \in Z\}$

At least one successor is labeled

All successors are labeled

- Example: E(P U Q):
  - Initial set: $Z_0 = \{s \mid Q \in L(s)\}$
  - Expansion: $Z_{i+1} = Z_i \cup (pre_E(Z_i) \cap \{s \mid P \in L(s)\})$

Labeled so far

Predecessors of already labeled states

labeled P

  - End of the iteration: if $Z_{i+1} = Z_i$ (fixedpoint)

CTL model checking – summary

- Global model checking:
  - Labeling of states by (sub)formulas that hold in the state
- Labeling by increasingly complex formulas
  - Starting from atomic formulas to more complex formulas, from the inside outwards
  - Using the labeling obtained in the previous iteration based on rules derived from operator semantics
- EX, AX: Examining and labeling predecessors
- E(p U q), A(p U q): Incremental labeling
  - Initial set:
    - State sets determined by the innermost formulas (p, q)
    - Iteration: based on semantics (applied to predecessors)
  - End of iteration: no more labels can be added to the labeling

Example

- Decomposition of formulas:

$AF\ (\ P \land E\ (Q\ U\ R))$

Q and R are labels in KS

Incremental labeling: E(. U .) at the end of the iteration, label E(Q U R)

The intersection of states labeled P and E(Q U R) (treating E(Q U R) as atomic): We add the label $P \land E(Q\ U\ R)$

Incremental labeling: based on AF (treating $P \land E(Q\ U\ R)$ as atomic): We add the label $AF(P \land E(Q\ U\ R))$. This can be evaluated on the initial state.

Exercise

- A traffic light has three aspects:
  red, yellow and green.
  - Initially all aspects are off.
  - After turning the light on, the red aspect is on.
  - From this, there are two ways to proceed:
    red-yellow (both are on), and
    green.
  - Red-yellow is followed by green, and green is followed by red again. From this, the behavior is the same as before.
- Check whether the following formula holds for the initial state of the model: $E((\neg red)\ U\ (EX\ green))$

Summary

- LTL model checking
  - Tableau construction
    - Propositional logic: contradictory and successful branches
    - LTL: searching for a counterexample (witness for negated formula)
- CTL model checking
  - Iterative labeling
    - Incremental labeling with increasingly complex formulas (global model checking)
    - Set operations

How can these algorithms be implemented efficiently?

LTL model checking:
Automata theoretic approach

(Supplementary)

Automata for finite words

- $A=(\Sigma, S, S_0, \rho, F)$ where
  - Σ is the alphabet, S are states, S_0 are initial states
  - ρ is the transition relation, $\rho: S \times \Sigma \rightarrow 2^S$
  - F is the set of accepting states
- A run of the automaton:
  - For a sequence of symbols from the alphabet – a word $w=(a_0, a_1, a_2, \ldots a_n)$ – a sequence of states $r=(s_0, s_1, s_2, \ldots s_n)$
  - r is an accepting run iff $s_n \in F$
  - Word w is accepted iff there exists an accepting run
- $L(A)=\{ w \in \Sigma^* \mid w \text{ is accepted} \}$ the language accepted by the automaton

Auxiliary material

Automata on infinite words

- Application: continuously operating systems
  - No final state – can not be checked for acceptance
- Büchi acceptance condition:
  - For a word $w=(a_0, a_1, a_2, \dots)$
    a sequence of states $r=(s_0, s_1, s_2, \dots)$
  - $lim(r) = \{s \mid s$ occurs infinitely many times,
    that is, there is no j such that $\forall k>j: s \neq s_k\}$
  - A run is accepting iff $lim(r) \cap F \neq 0$
  - A word w is accepted iff there exists an accepting run over it
    (an accepting state is encountered infinitely many times)
- $L(A)=\{w \in \Sigma^* \mid w \text{ accepted}\}$
  the language accepted by the automaton

Auxiliary material

Automata theoretic approach

- For a state s of KS: L(s) is a symbol of alphabet 2^{AP}
  E.g. {Red, Yellow} is a symbol of the alphabet
- A path $\pi=(s_0, s_1, s_2, \ldots s_n)$ induces a word

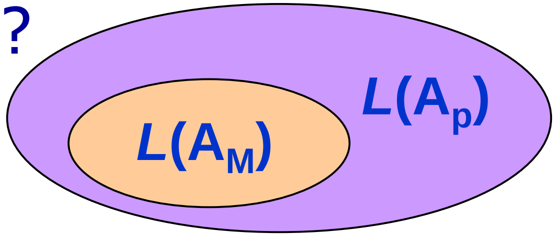
  $(L(s_0), L(s_1), L(s_2), \ldots L(s_n))$

- We construct two automata:
  - Based on Kripke structure M=(S,R,L) an automaton A_M can be constructed that accepts exactly those words that correspond to paths of M.
  - Based on formula p an automaton A_p can be constructed that accepts exactly those words that characterize paths for which p holds
    Tableau construction rules can be used: what must hold in the current state, and what for the successor states

Auxiliary material

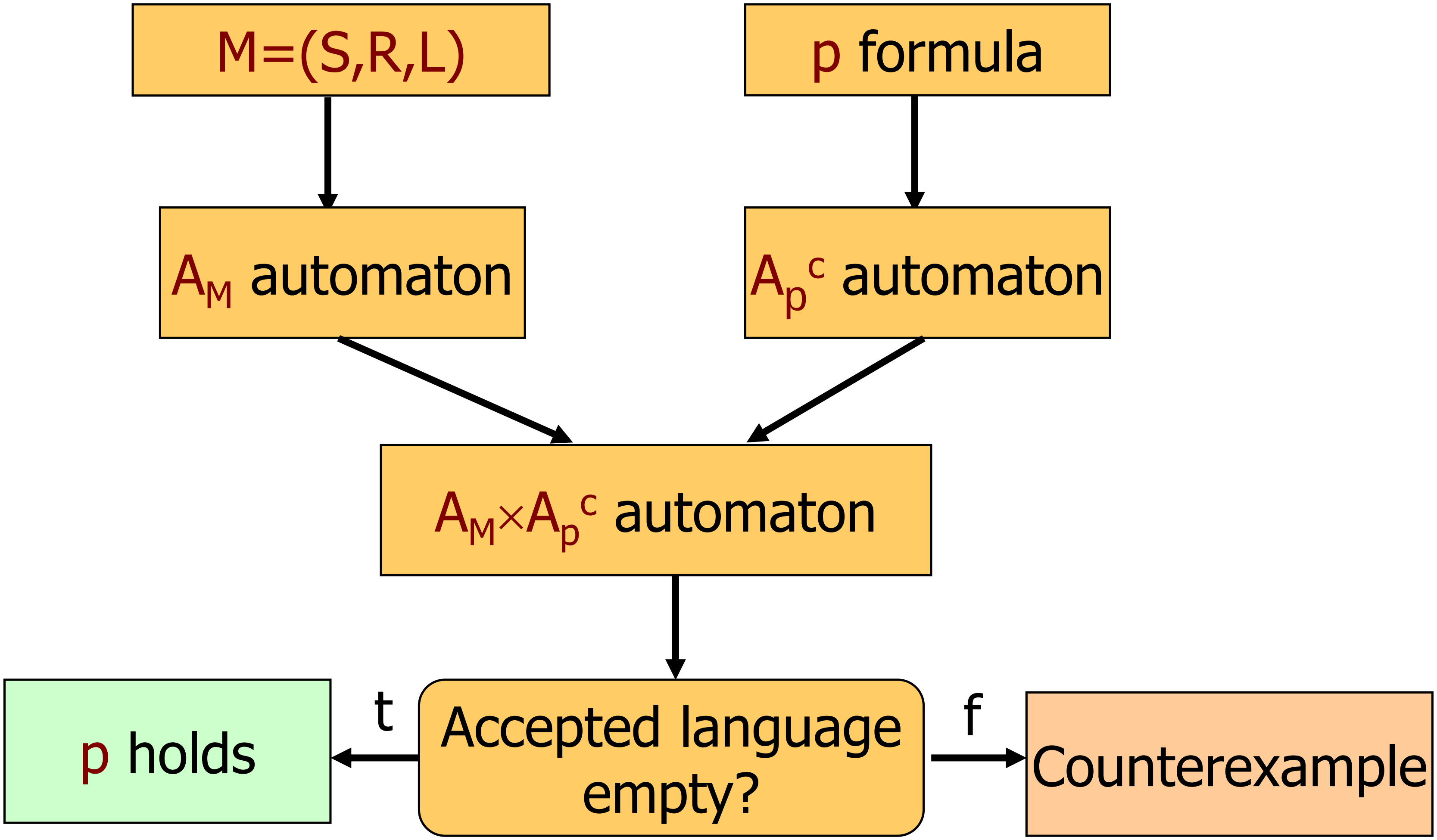
Model checking using automata

- Model checking problem: $\mathcal{L}(A_M) \subseteq \mathcal{L}(A_p)$, i.e., is the model's language part of the property's language?
  - If so then $M \models p$



- Reformulating the problem:
  - Checking emptiness of intersection of languages: $\mathcal{L}(A_M) \cap \mathcal{L}(A_p)^c = 0$, here $\mathcal{L}(A_p)^c$ is the complement of the language
  - Is the language accepted by the synchronous product automaton $A_M \times A_p^c$, induced by the model automaton A_M and the complement automaton of the property A_p^c, empty?
    - If so then $M, \pi \models p$
    - The accepted language is empty iff there is no reachable accepting state
- Continuously operating systems
  - Automata on infinite words;
    Büchi acceptance condition: searching for loops

Automata theoretic model checking



Auxiliary material

"On-the-fly" model checking

- Idea:
  - During construction of automaton A_p the synchronous product can be constructed
- Construction of synchronous product automaton
  - Directed by the property to verify:
    as the states of the automaton A_p are established,
    the states of A_M has to be "looked up"
  - The generation of the full state space is not necessary
    - E.g. when deriving from a higher level formalism