Formal Methods (VIMIM100)	**2016/2017. year II. semester**						4. May 2017.
Second Mid-term Exam	1.	2.	3.	4.	5.	6.	Σ
Name: ______________________							
NEPTUN code: ________________	14 points	8 points	6 points	8 points	6 points	8 points	50 points

1. Theoretical questions

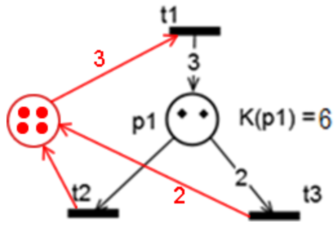
If you work on a separate sheet, please always indicate it!

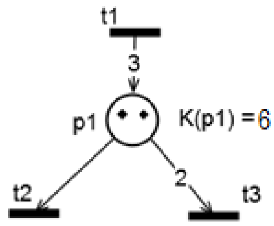
1.1. Give the formal definition of *T-invariants* in Petri nets! Give an example on the practical applications of T-invariants! **3 points**

A vector **σ** is a T-invariant if $\mathbf{W}^T\boldsymbol{\sigma} = \mathbf{0}$ holds. Firing the transitions of a T-invariant does not change the marking. In practice, this means that there is a cyclic behavior in the modeled system.

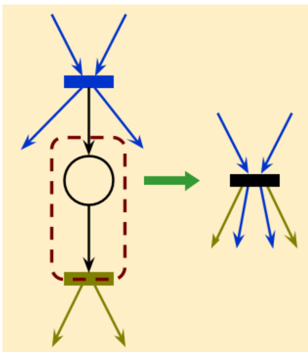
1.2. What does it mean when a place in a Petri net has *finite capacity*? Draw the equivalent, infinite capacity net, corresponding to the finite capacity net given below. **3 points**

The number of tokens **cannot exceed** the capacity (in any marking).

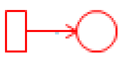




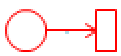
1.3. Draw the reduction rule corresponding to the *fusion of series transitions* (including the general initial structure and the reduced structure)! **2 points**



1.4. Draw a *source transition* and a *sink transition*! Explain the effect of such transitions on the liveness and safeness of a Petri net! **2 points**



Source: no input place. It can always fire, producing infinite tokens and preventing the net from being safe.

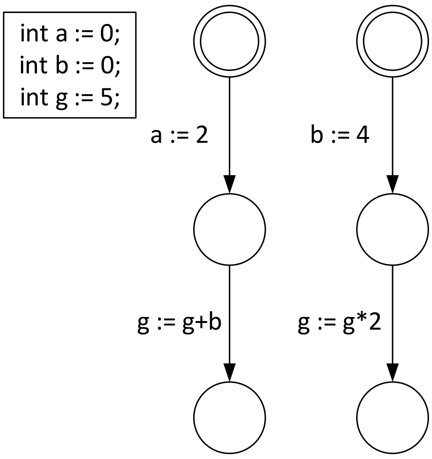


Sink: no output place. It can only consume tokens, possibly (but not always) causing a deadlock

1.5. The figure below shows two *Labeled Transition Systems* (LTS) corresponding to two parallel processes. Fill the cells of the table below with the dependencies between actions. Denote *independent* actions by I, *control dependency* by C and *data dependency* by D!

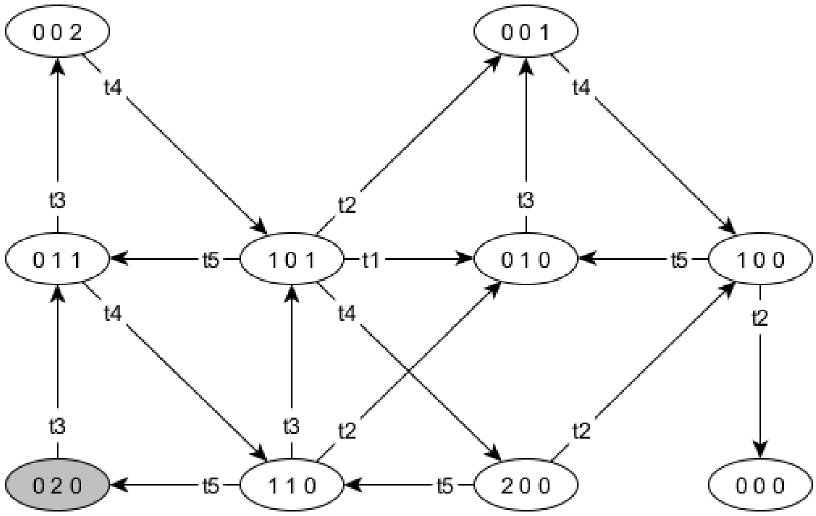
4 points

	a := 2	*b := 4*	*g := g+b*	*g := g*2*
a := 2		I	C	I
b := 4	I		D	C
g := g+b	C	D		D
*g := g*2*	I	C	D	



2. Dynamic properties

The figure below represents the state space of a Petri net as a *reachability graph*. The net contains 5 transitions denoted by t_1, ..., t_5. The states are denoted by token distribution vectors, for example the vector (0 2 0) represents $m(p_1) = 0$, $m(p_2) = 2$ and $m(p_3) = 0$. The initial state is marked with a darker background.



2.1. Answer the following questions based on the graph above. No explanation is needed here. 8 points

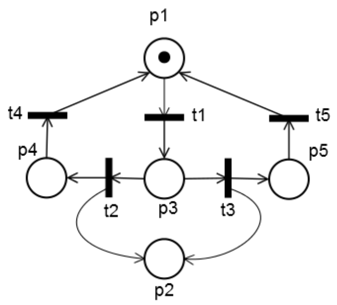
		True	False	Not decidable			True	False	Not decidable
(a)	The net is safe	☐	X	☐	(e)	Transition t_2 is L_2-live	☐	X	☐
(b)	The net is live	☐	X	☐	(f)	Transition t_3 is L_3-live	X	☐	☐
(c)	The net is bounded fair	☐	X	☐	(g)	Transition t_4 is L_4-live	☐	X	☐
(d)	The net is reversible	☐	X	☐	(h)	Transition t_1 is persistent	☐	X	☐

Explanations (not needed in the exam):

(a) There are markings where some places have more than one tokens, e.g., p2 in the initial marking.
(b) (0 0 0) is a deadlock.
(c) For example from the initial state t3, t4, t5 can fire infinitely many times without t2 being fired, which is enabled at (1 1 0).
(d) (0 0 0) can be reached from the initial state, but the initial state cannot be reached from (0 0 0).
(e) No loop contains t2 and the state space is finite.
(f) There are several loops that contain t3.
(g) t4 cannot fire from the marking (0 0 0).
(h) For example, t1 is enabled at (1 0 1), but after firing t2, it is no longer enabled at (0 0 1).

3. Invariants **Please provide the solution on a new sheet!**

The following Petri net is given.



3.1. Give the weighted *incidence matrix* of the net!

2 points

$W^T =$

	t1	t2	t3	t4	t5
p1	-1	0	0	1	1
p2	0	1	1	0	0
p3	1	-1	-1	0	0
p4	0	1	0	-1	0
p5	0	0	1	0	-1

3.2. Check if the following vector is a P-invariant of the net (explain your answer)!

2 points

$(1,0,1,1,1)^T$

Yes, because W multiplied by the vector is 0.

3.3. Check if the following vector is a T-invariant of the net (explain your answer)!

2 points

$(1,0,1,0,1)^T$

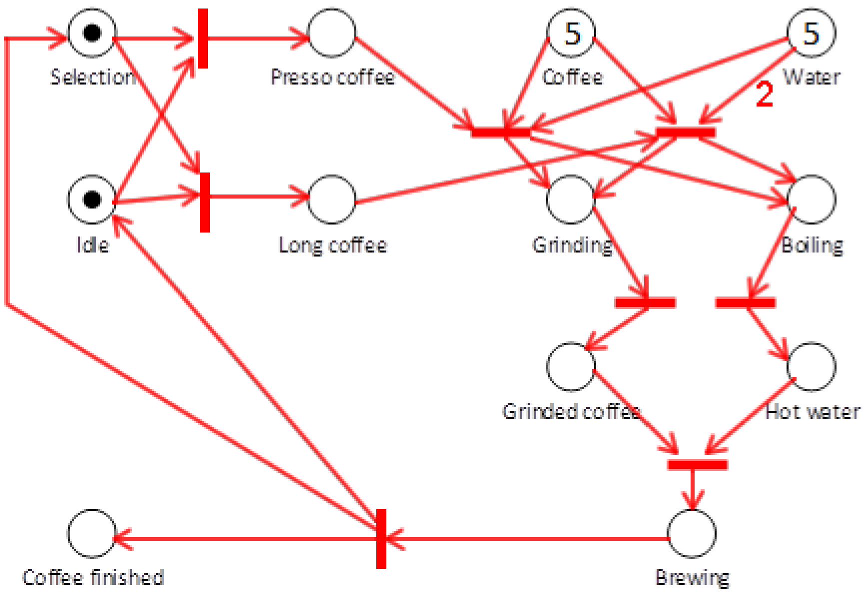
No, because W^T multiplied by the vector is not 0.

4. Modeling with Petri nets

4.1. Draw a (non-colored) Petri net model based on the following description by completing the partial model below with *transitions* and *arcs*! (If you are not sure, first draw a draft version on a separate sheet!)

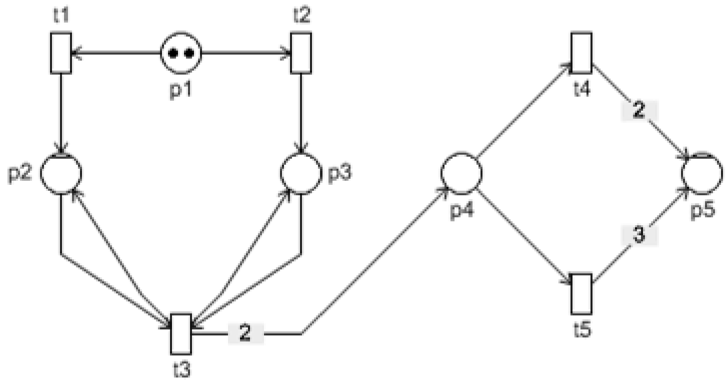
8 points

- The coffee machine of the department is initially *idle*, having 5 units of *coffee* and 5 units of *water*.
- If the machine is idle, we can press the buttons that pick *presso coffee* or *long coffee*.
- *Brewing* a presso coffee requires 1 unit of coffee and 1 unit of water. Long coffee requires 1 unit of coffee and 2 units of water.
- The machine starts *grinding* the required amount of coffee and starts *boiling* the required amount of water at the same time.
- We can assume, that the boiling process is identical for different types of coffee.
- If coffee is grinded and water is boiled, coffee *brewing* can be started.
- After brewing is complete, a unit of *coffee is finished* and the machine is idle again.

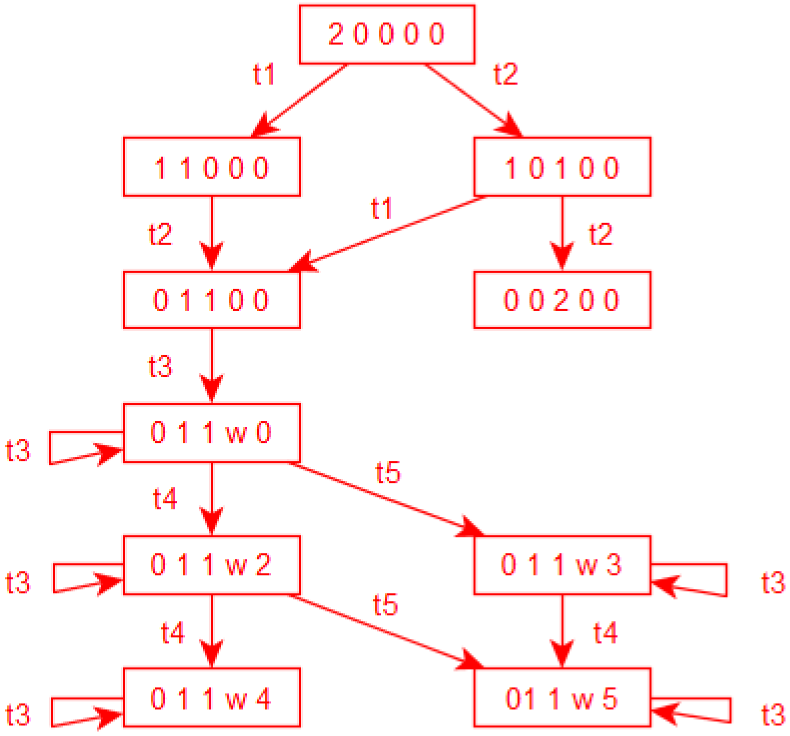


5. Coverability graph **Please provide the solution on a new sheet!**

The following Petri net is given where places *p2* and *p5* have finite capacity: *K(p2) = 1* and *K(p5) = 5*. All other places have infinite capacity. Numbers on the arcs denote arc weights.



5.1. Draw the *coverability graph* for the Petri net! Label arcs of the graph with transitions! 6 points

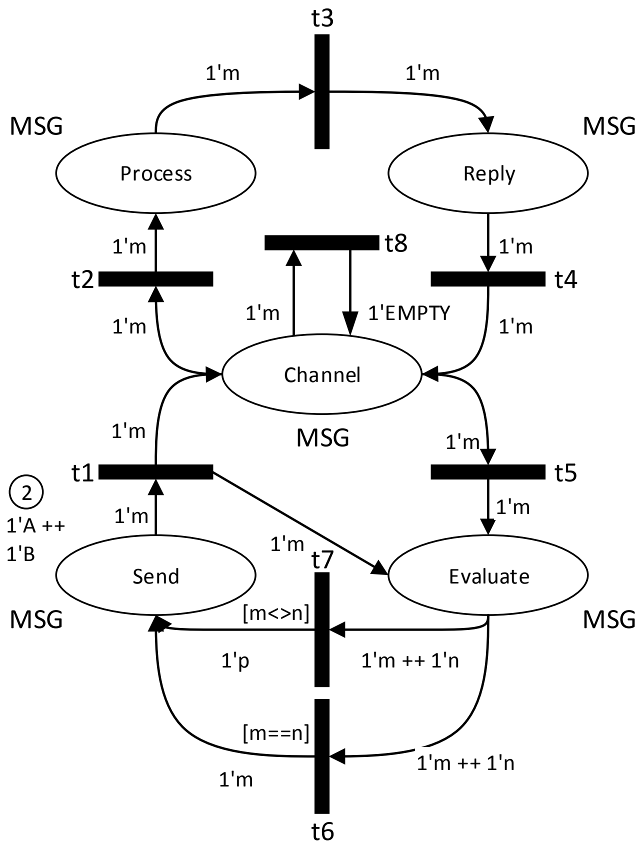


(The letter "w" represents the "ω" character.)

6. Colored Petri nets

The following Petri net is given with its definition block:

```
colset MSG = with A | B | C | EMPTY;
var m, n, p : MSG;
```



6.1. Answer the following questions (on a separate sheet):

a) Enumerate the enabled transition(s) with binding(s) under the actual marking!
t1 is enabled with two bindings: <m=A> and <m=B>

b) Give the markings reached after firing the enabled transition(s)!
- If t1 is fired with the binding <m=A>, then 1'A is removed from Send and put both to Channel and Evaluate: Send={1'B}, Channel={1'A}, Evaluate={1'A}, Process={}, Reply={}
- If t1 is fired with the binding <m=B>, then 1'B is removed from Send and put both to Channel and Evaluate: Send={1'A}, Channel={1'B}, Evaluate={1'B}, Process={}, Reply={}

c) Is the net bounded with the given initial state? Explain your answer!
Yes. Only t1 produces more tokens than it consumes, but only t6 and t7 can put tokens to the input place of t1 and they consume the extra tokens.

d) Is there a reachable state (from the given initial state) where transition *t6* is enabled? Explain your answer!
Yes, for example after t1 <m=A>, t2 <m=A>, t3 <m=A>, t4 <m=A>, t5 <m=A> there will be 2'A tokens in Evaluate, for which t6 is enabled by the binding <m=A, n=A>.

e) Is there a T-invariant in the net? Explain your answer!
Yes, for example the sequence t1 <m=A>, t2 <m=A>, t3 <m=A>, t4 <m=A>, t5 <m=A>, t6 <m=A, n=A> leads back to the initial state.

8 points