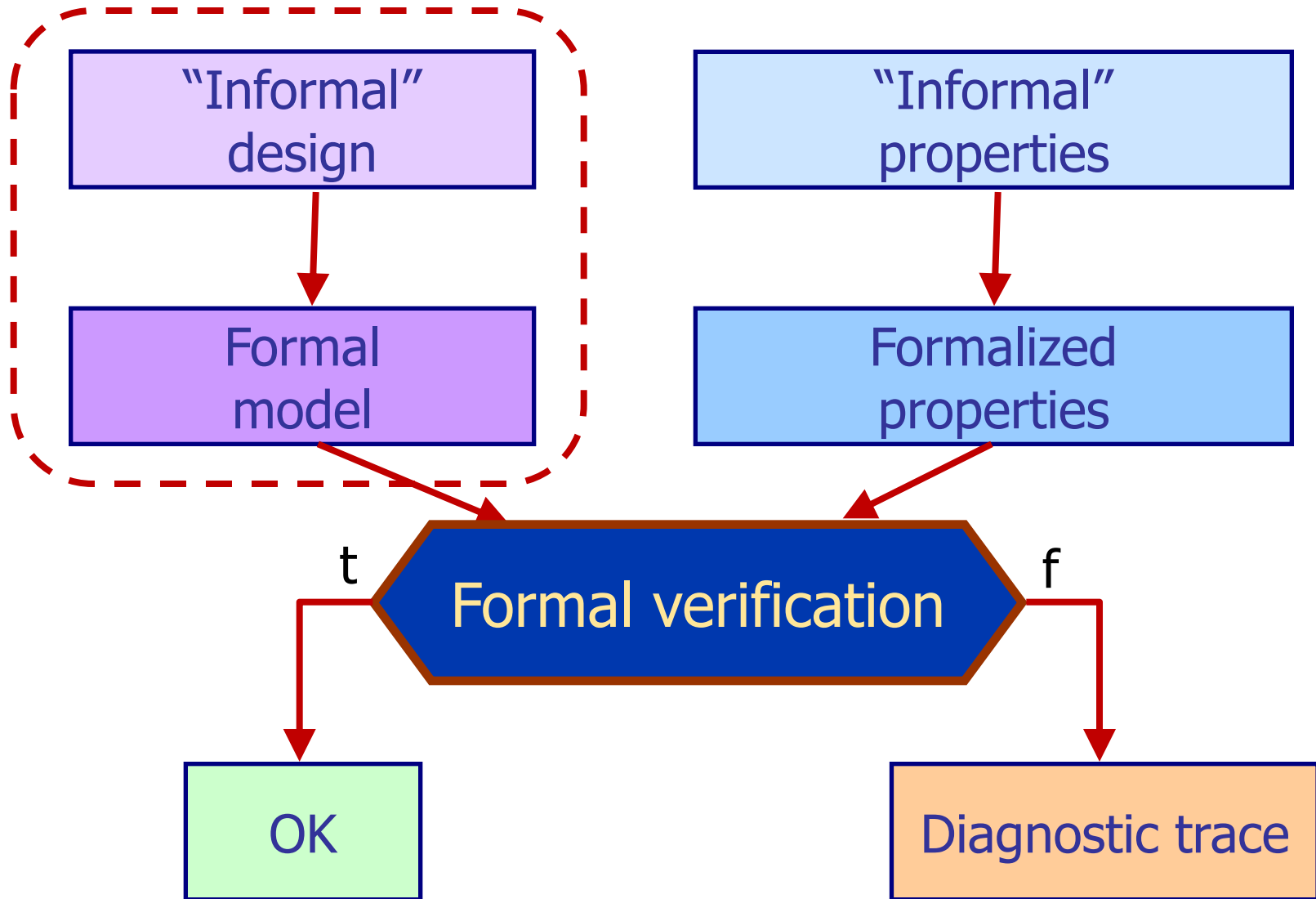


Formal verification: Basic formalisms

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Recap: The goal of formal verification



Basic structures

Kripke structure (KS)
Labeled transition system (LTS)
Kripke transition system (KTS)
Finite state automata (FSA)

1. Kripke structure

Basic characteristics:

- Capturing properties of **states**: labeling by **atomic propositions**
- Possibly more than one labels per state
- Application: description of behavior or algorithm

Definition:

A Kripke structure KS over a set of atomic propositions

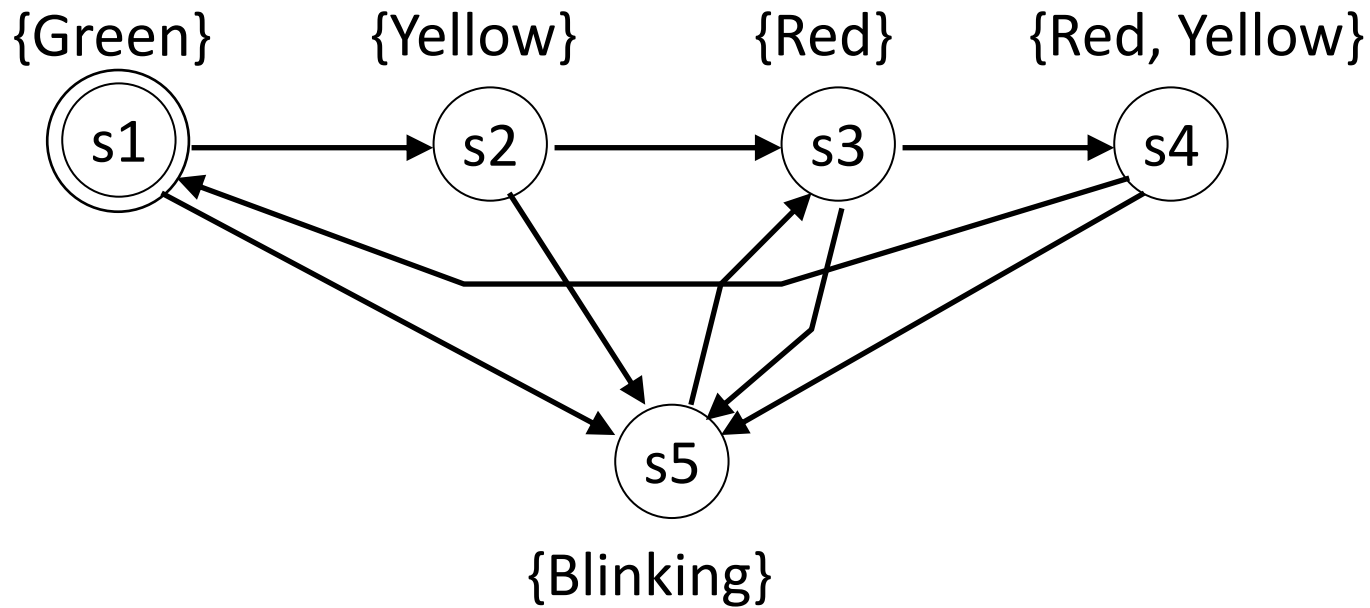
$AP = \{P, Q, R, \dots\}$ is a tuple (S, R, L) where

- $S = \{s_1, s_2, \dots, s_n\}$ is a finite set of states,
 $I \subseteq S$ is the set of initial states,
- $R \subseteq S \times S$ is the set of transitions and
- $L : S \rightarrow 2^{AP}$ is the labeling of states by atomic propositions

Example: Kripke structure

Traffic light controller

- $AP = \{\text{Green, Yellow, Red, Blinking}\}$
- $S = \{s_1, s_2, s_3, s_4, s_5\}$



2. Labeled transition system

Basic characteristics:

- Capturing properties of **transitions**: labeling by **actions**
- Exactly one action per transition
- Application: modeling of communication and protocols

Definition:

A labeled transition system *LTS* over a set of actions

Act = {*a, b, c, ...*} is a triple (*S, Act, →*) where

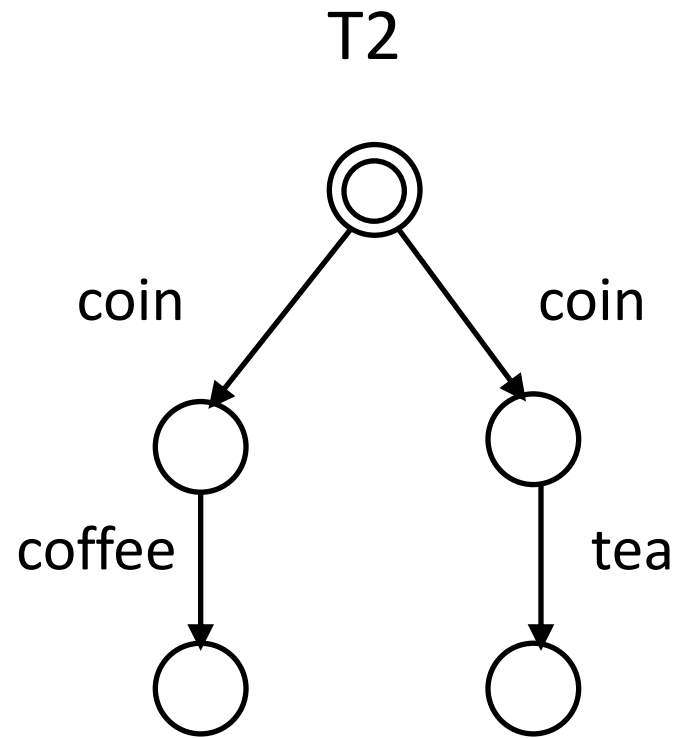
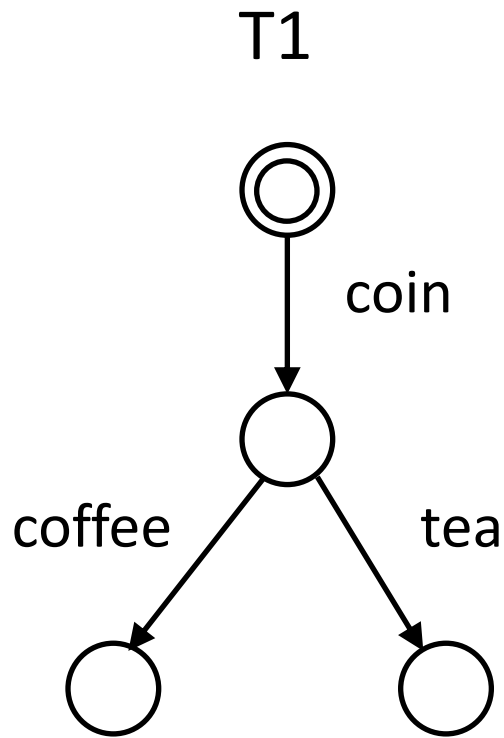
- *S* = {*s*₁, *s*₂, ..., *s*_{*n*}} is a finite set of states,
I ⊆ *S* is the set of initial states,
- *→* : *S* × *Act* × *S* is the set of transitions

We denote by $s \xrightarrow{a} s'$ iff $(s, a, s') \in \rightarrow$

Example: Labeled transition system

Vending machine

- Act = {coin, coffe, tea}



3. Kripke transition system

Basic characteristics:

- Capturing properties of both **states** and **transitions**: labeling by **atomic propositions** and **actions**
- Possibly more than one labels per state, exactly one action per transition

Definition:

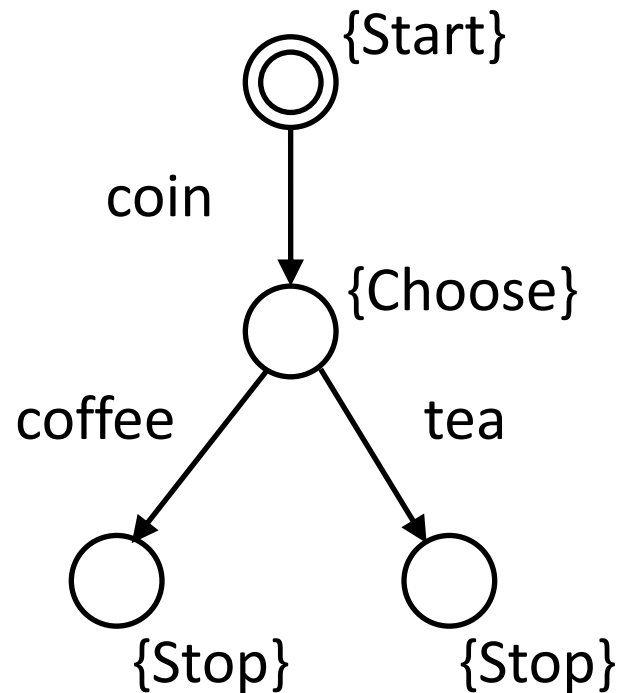
A Kripke transition system *KTS* over a set of atomic propositions *AP* and set of actions *Act* is a tuple (S, \rightarrow, L) where

- $S = \{s_1, s_2, \dots, s_n\}$ is a finite set of states,
 $I \subseteq S$ is the set of initial states,
- $\rightarrow : S \times Act \times S$ is the set of transitions
- $L : S \rightarrow 2^{AP}$ is the labeling of states by atomic propositions

Example: Kripke transition system

Vending machine with state labeling

- $Act = \{coin, coffee, tea\}$
- $AP = \{Start, Choose, Stop\}$



4. Automata on finite words

- $A=(\Sigma, S, S_0, \rho, F)$ where
 - Σ alphabet, S states, S_0 initial states
 - ρ state transition relation, $\rho: S \times \Sigma \rightarrow 2^S$
 - F set of accepting states
- Run of an automaton
 - State sequence $r=(s_0, s_1, s_2, \dots s_n)$ on the incoming word $w=(a_0, a_1, a_2, \dots a_n)$
 - r is an accepting run if $s_n \in F$
 - A word w is accepted by the automaton, if there is an accepting run over w
- Language L accepted by the automaton A :
$$L(A)=\{ w \in \Sigma^* \mid w \text{ accepted} \}$$

Automata on infinite words

- Infinite word: Accepting state at the end of an input word cannot be checked
- Büchi acceptance criterion:
 - On the incoming infinite word $w=(a_0, a_1, a_2, \dots)$ there is an $r=(s_0, s_1, s_2, \dots)$ infinite state sequence
 - $\text{lim}(r)=\{s \mid s \text{ occurs infinitely often, i.e., there is no } j, \text{ such that } \forall k>j:s \neq s_k\}$
 - Accepting run: $\text{lim}(r) \cap F \neq \emptyset$
 - A word w is **accepted** by the automaton, if there is an accepting run over w (i.e., accepting state occurs infinitely often along w)
- Language L accepted by the automaton A :
$$L(A)=\{ w \in \Sigma^* \mid w \text{ accepted} \}$$

Timed Automata: Finite State Automata with Time

Timed Automata in the UPPAAL model checker

Timed Automata: Extension with variables

- Basic formalism: Finite state automaton (FSA)
 - Control locations (named) – part of the state of the automaton
 - Edges – define state transitions
- Language extension: **integer variables**
 - Variables with restricted domain (e.g. **int[0, 10] id**)
 - Constants (e.g., **const int N = 6**)
 - Integer arithmetic
- Use of variables: on transitions
 - **Guard**: predicate over variables
 - The state transition can occur only if the predicate holds
 - **Action**: variable assignment

Timed automata: Extension with clock variables

- Goal: modeling time-dependent behavior
 - Time passes in given states of the component
 - Relative time measurement by resetting and reading timers, behavior depends on timer value (e.g., timeout)
- Language extension: clock variables
 - Measuring time elapse by a constant rate
- Use of clock variables on transitions
 - Guard: predicate over clock variables
 - Action: resetting clocks to zero
- Use of clock variables on locations
 - Location invariant: predicate over clock variables; being in a location is valid until its invariant holds

Timed automata in UPPAAL

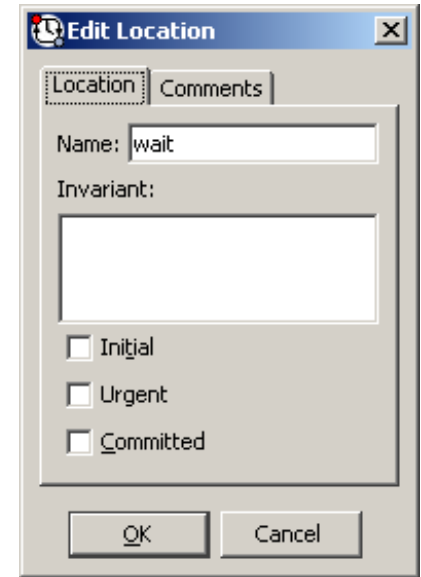
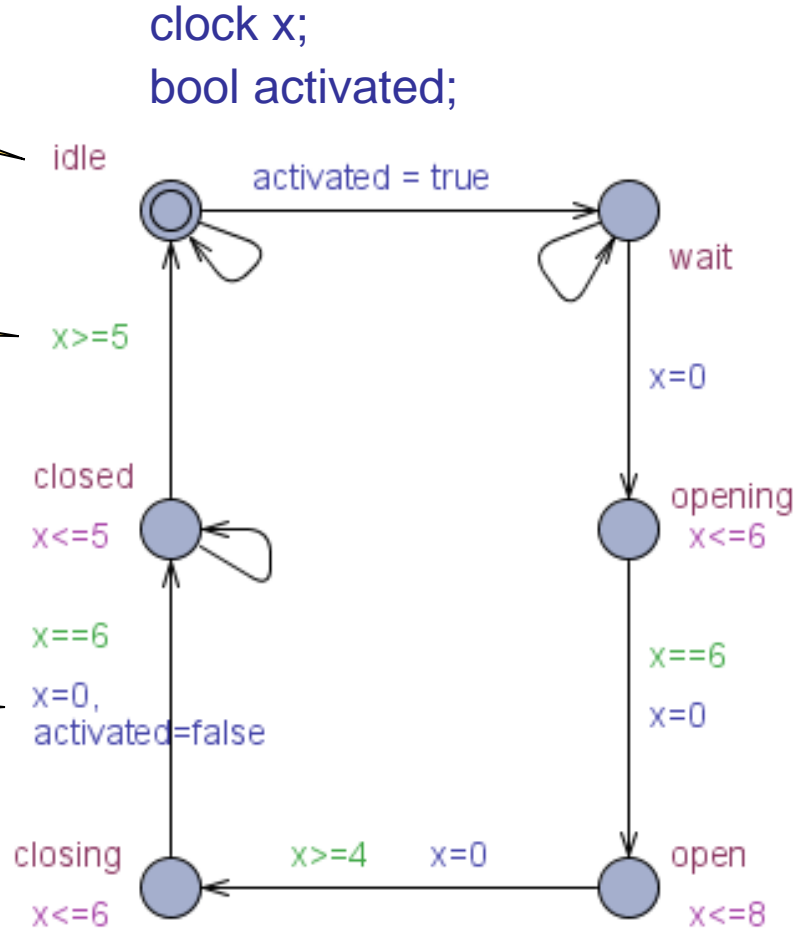
Example: revolving door

Location

Guard

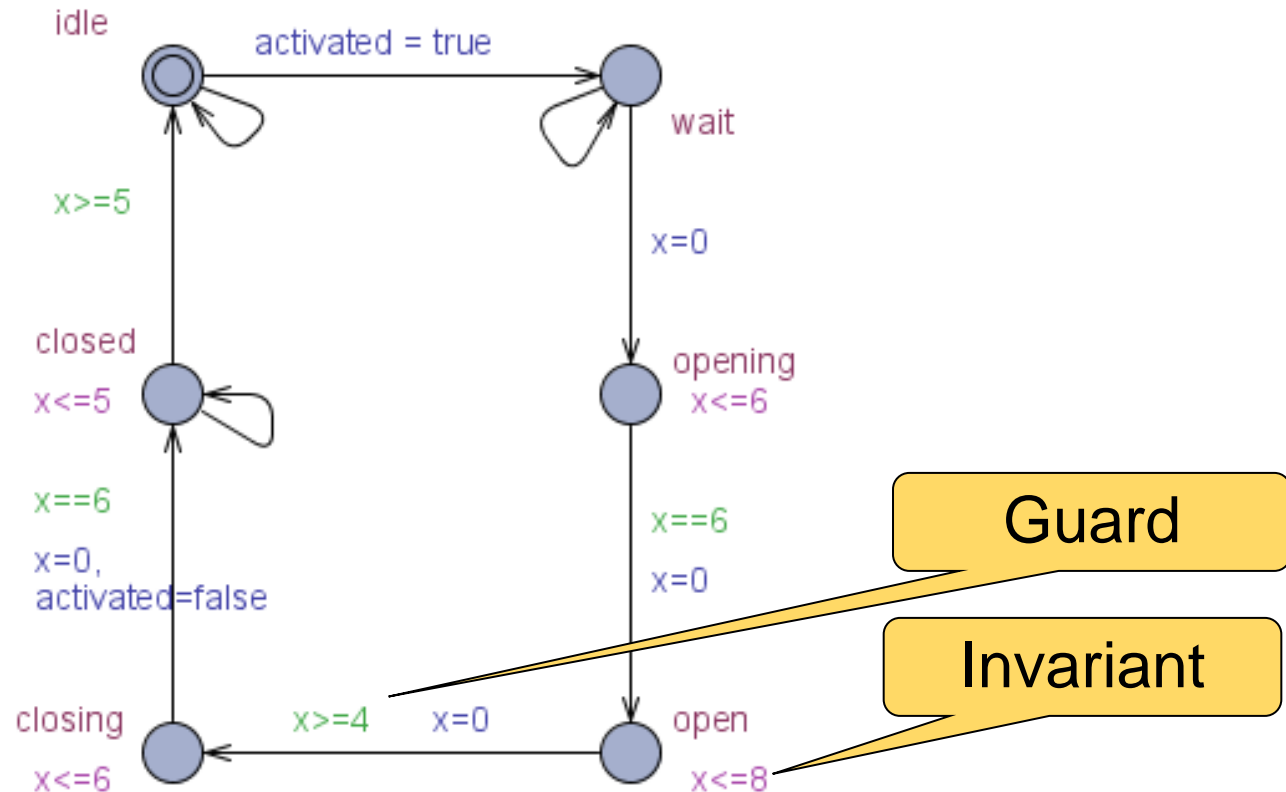
Invariant

Action



Role of guards and invariants

clock x;

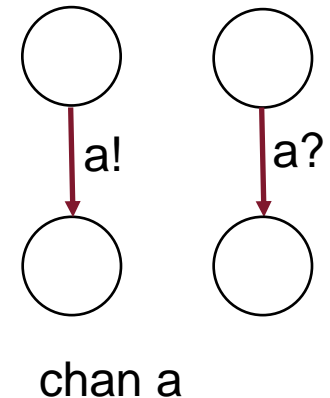


Upon exiting location **open**, the value of clock is in interval $[4, 8]$



Extensions for concurrency

- Goal: modeling **networks of automata**
 - Interaction: Synchronization between automata transitions
 - **Synchronous communication (handshake, rendezvous)**
 - Sending and receiving a message occurs at the same time
 - Modeling of asynchronous behavior is possible by modeling channels
- Language extension: **synchronized actions**
 - Declaring **channels** for sending messages
 - Sending a message: **!** operator
Receiving a message: **?** operator
 - E.g.: synchronization labels **a!** and **a?** for channel **a**
- Parameterization
 - Arrays of channels: E.g. channel **a[id]** for a variable **id**
 - Useful in case of several participants and interactions



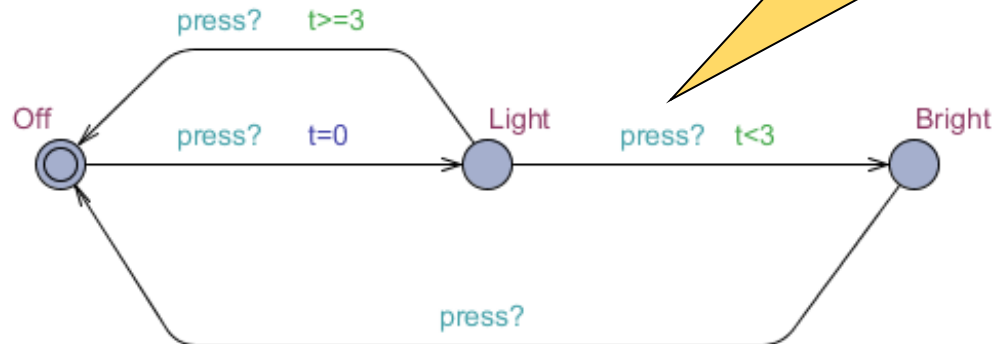
Example for clocks and synchronization

Declarations:

clock t, u ;

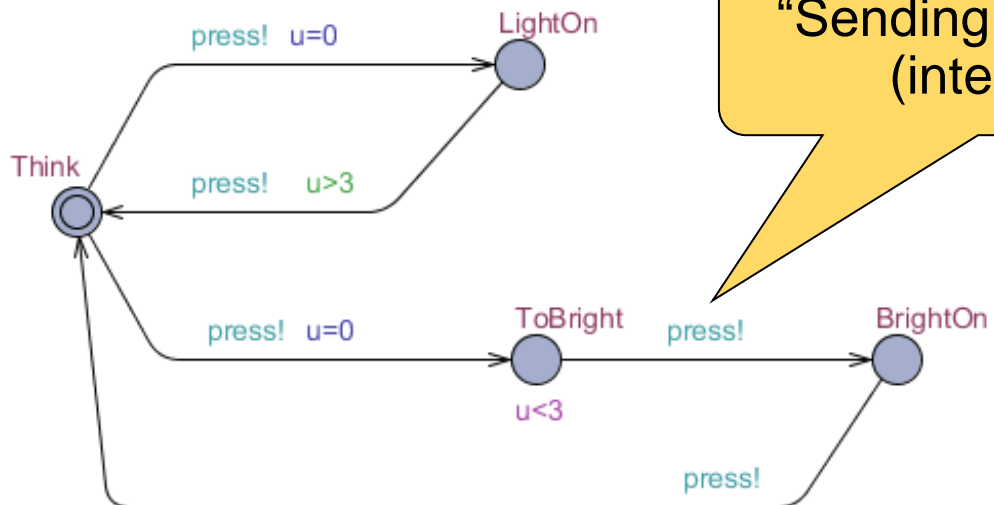
chan press;

Switch:



“Receiving a message”
(interaction)

User:

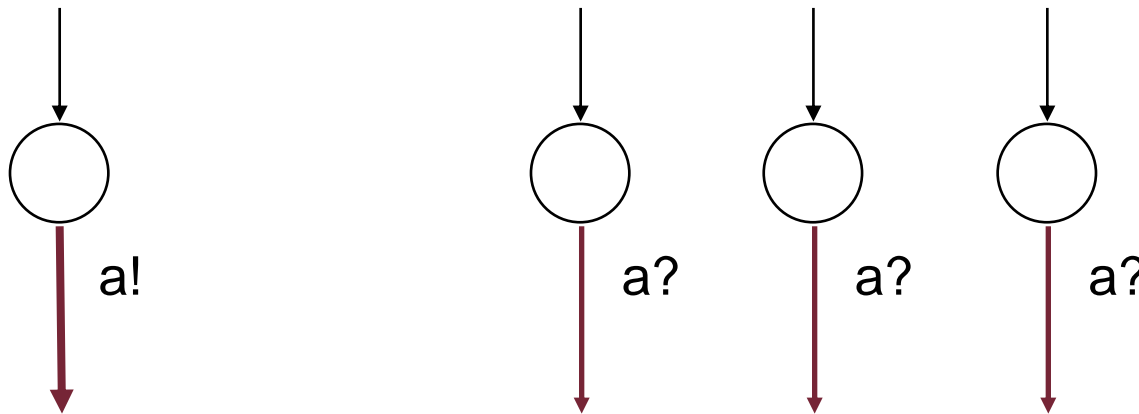


“Sending a message”
(interaction)

Further extensions: broadcast channel

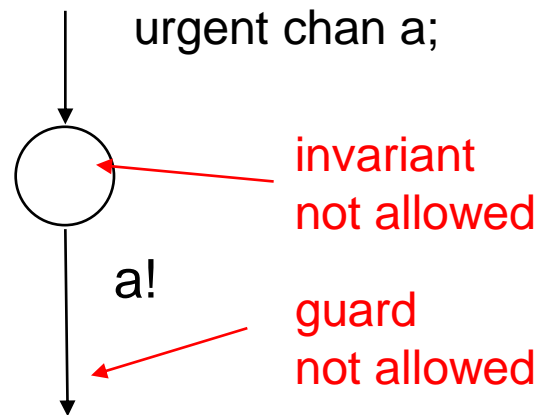
- **Broadcast** channel: one-to-many communication
 - Sending a message: unconditional
 - No handshake needed
 - Receiving a message: synchronized to the sender
 - **All processes** that are ready to receive the message will synchronize
 - Restriction: no guard on receiving edge

broadcast chan a;



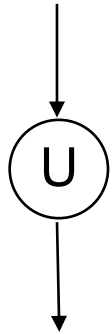
Further extensions: urgent channel

- **Urgent** channel: prohibit time delay (waiting for synchronization)
 - The synchronization is executed **without delay** (other edges might be traversed before, but only instantly)
 - Restrictions:
 - No guard is allowed on an edge labeled with the name of an urgent channel
 - No invariant is allowed on a location that is the source of an edge labeled with the name of an urgent channel

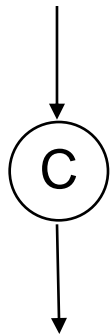


Further extensions: special locations

- **Urgent** location: prohibit time delay (waiting in location)
 - Time is not allowed to progress in the location
 - Equivalent model:
 - Introduce a clock variable: **clock x**
 - Reset clock on all incoming edges: **$x:=0$**
 - Add invariant: **$x \leq 0$**



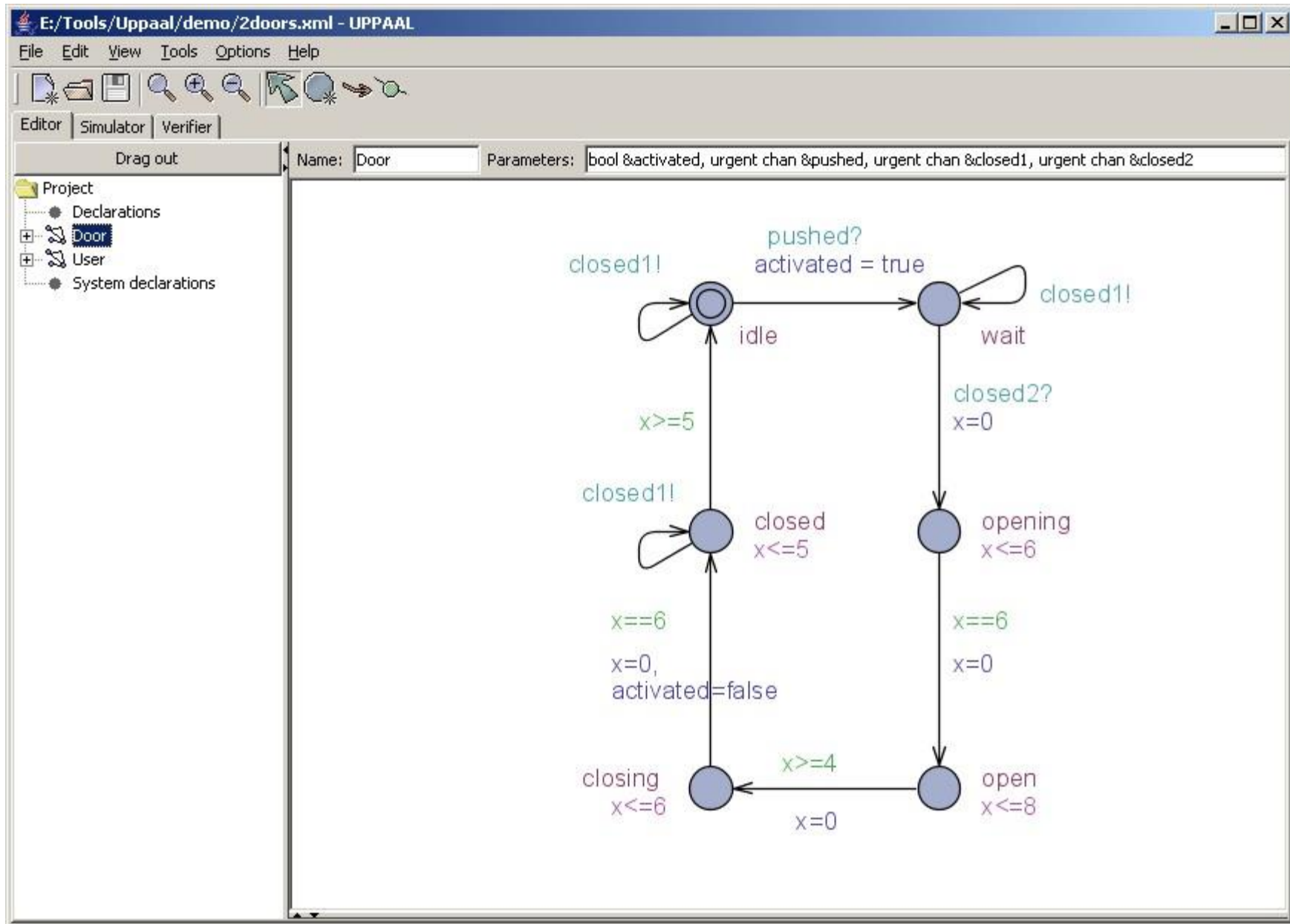
- **Committed** location: even more restrictive
 - A committed location is urgent
 - **Committed state**: at least one committed location is active
 - The next transition from a committed state must involve at least one out-edge of an active committed location
 - Simpler case: If only one committed location is active then its out-edge shall immediately follow its in-edge



The UPPAAL model checker

- Development (1999-):
 - Uppsala University, Sweden
 - Aalborg University, Denmark
- Web page (information, examples, download):
<http://www.uppaal.org/>
- Related tools:
 - UPPAAL CoVer: Test generation
 - UPPAAL TRON: On-line testing
 - UPPAAL PORT: Component based modeling
 - ...
- Commercial version:
<http://www.uppaal.com/>

Automaton model



Simulator

E:/Tools/Uppaal/demo/2doors.xml - UPPAAL

File Edit View Tools Options Help

Editor Simulator Verifier

Drag out

Enabled Transitions

User2

closed2: Door2 --> Door1

Next Reset

Simulation Trace

(idle, idle, idle, idle)

User1

(idle, idle, -, idle)

pushed1: User1 --> Door1

(wait, idle, idle, idle)

Trace File:

Prev Next Replay

Open Save Random

Slow Fast

Drag out

activated1 = 1
activated2 = 0
Door1.x >= 0
Door2.x >= 0
User1.w = 0
User2.w >= 0
Door1.x = Door2.x
Door2.x = User2.w
User2.w = Door1.x

Door1

Door2

User1

User2

Door1 Door2 User1 User2

Verification

The screenshot shows the UPPAAL software interface with the following components:

- File:** FTapps/Uppaal/demo/2doors.xml - UPPAAL
- Menu:** File, Edit, View, Tools, Options, Help
- Toolbar:** Includes icons for file operations (new, open, save), search, and navigation.
- Editor/Simulator/Verifier:** The active tab is 'Verifier'.
- Overview:** A list of properties with status indicators (green for satisfied, grey for not checked).
 - `A[] not (Door1.open and Door2.open)` (Green)
 - `A[] (Door1.opening imply User1.w<=31) and (Door2.opening imply User2.w<=31)` (Green)
 - `E<> Door1.open` (Grey)
 - `E<> Door2.open` (Green)
- Query:** `A[] not (Door1.open and Door2.open)`
- Comment:** Mutex: The two doors are never open at the same time.
- Status:** A log of verification results.
 - Established direct connection to local server.
 - (Academic) UPPAAL version 4.0.7 (rev. 4140), November 2008 -- server.
 - Disconnected.
 - Established direct connection to local server.
 - (Academic) UPPAAL version 4.0.7 (rev. 4140), November 2008 -- server.
 - `A[] not (Door1.open and Door2.open)`
Property is satisfied.
 - `A[] (Door1.opening imply User1.w<=31) and (Door2.opening imply User2.w<=31)`
Property is satisfied.
 - `E<> Door2.open`
Property is satisfied.
 - `A[] not deadlock`
Property is satisfied.
 - `Door2.wait --> Door2.open`
Property is satisfied.
 - `Door1.wait --> Door1.open`
Property is satisfied.